

Contents

<i>Preface</i>	<i>page ix</i>
PART I ARITHMETIC GROUPS IN THE GENERAL LINEAR GROUP	1
1 Modules, Lattices, and Orders	3
1.1 Modules	4
1.2 Projective R -modules	7
1.3 Modules of fractions and localisation	10
1.4 Lattices	14
1.5 Integrality properties	18
1.6 Discrete valuation rings, Dedekind domains, and overrings	20
1.7 Modules over Dedekind domains	26
1.8 Central simple algebras	28
1.9 Orders in algebras	37
1.10 Non-abelian Galois cohomology	43
2 The General Linear Group over Rings	46
2.1 Elementary matrices	47
2.2 The stable structure of GL_n	53
2.3 The stable range of a Dedekind domain	59
2.4 The stable range of orders in division algebras	61
2.5 An application: Mennicke symbols and their properties	63
3 A Menagerie of Examples: A Historical Perspective	70
3.1 Reduction theory of quadratic forms	71
3.2 Lattices in Euclidean space	78
3.3 Unit groups of number fields	82

vi	<i>Contents</i>	
	3.4 Unit groups in division algebras: the theorem of Käthe Hey	83
	3.5 The modular group	85
4	Arithmetic Groups	90
	4.1 Rings of \mathcal{S} -integers: general concept and results	91
	4.2 Global fields	93
	4.3 Rings of \mathcal{S} -integers in global fields	97
	4.4 Arithmetic and \mathcal{S} -arithmetic groups	102
	4.5 Arithmetic groups: their ambient Lie groups	104
	4.6 \mathcal{S} -arithmetic groups: their ambient Lie groups	105
	4.7 The general linear group over the ring of adèles	108
	4.8 Strong approximation property and consequences	113
	4.9 Elements of finite order in $\mathrm{GL}_n(\mathbb{Z})$	118
5	Arithmetically Defined Kleinian Groups and Hyperbolic 3-Space	122
	5.1 Kleinian groups acting on hyperbolic 3-space	122
	5.2 Bianchi groups	124
	5.3 Reduction theory for Bianchi groups	126
	5.4 Arithmetic groups originating from orders in quaternion division algebras	135
	PART II ARITHMETIC GROUPS OVER GLOBAL FIELDS	139
6	Lattices: Reduction Theory for GL_n	141
	6.1 The basic cases of a global field: the number field \mathbb{Q} and a rational function field $F_q(t)$	142
	6.2 Minkowski inequalities or successive minima	143
	6.3 Mahler's compactness criterion	150
7	Reduction Theory and (Semi)-Stable Lattices	152
	7.1 Euclidean \mathbb{Z} -lattices	153
	7.2 Arithmetic \mathcal{O}_k -lattices	159
	7.3 Canonical filtration, (semi)-stable lattices	164
	7.4 Reduction theory and the canonical filtration for \mathbb{Z} -lattices	167
	7.5 Comparison	171
8	Arithmetic Groups in Algebraic k-Groups	174
	8.1 Arithmetic groups	175
	8.2 Chevalley group schemes over \mathbb{Z}	178
	8.3 Integral structures for inner k -forms of SL_n	186
	8.4 Integral structures for arbitrary k -forms of SL_n	191
	8.5 Division algebras with prescribed local behaviour	199

9	Arithmetic Groups, Ambient Lie Groups, and Related Geometric Objects	202
9.1	Homogeneous spaces, locally symmetric spaces	203
9.2	S -arithmetic groups and affine buildings	205
9.3	Arithmetic groups in unipotent groups	206
9.4	Arithmetic groups in algebraic k -tori	211
9.5	Godement's compactness criterion	219
9.6	Constructions of compact or non-compact arithmetic quotients	232
10	Geometric Cycles	237
10.1	Construction of geometric cycles	238
10.2	Orientability	241
10.3	Intersection numbers, excess bundles, and Euler numbers	245
11	Geometric Cycles via Rational Automorphisms	249
11.1	Prelude	250
11.2	Fixed points and non-abelian Galois cohomology	252
11.3	Intersection numbers of special geometric cycles	256
11.4	The Euler number of the excess bundle	258
11.5	Non-vanishing of the intersection number of two geometric cycles	268
11.6	Construction of cohomology classes: an outlook	272
12	Reduction Theory for Adelic Coset Spaces	274
12.1	Preliminaries: adelic coset spaces	275
12.2	The adèle groups $G(\mathbb{A}_k)$ and $G(\mathbb{A}_k)^1$	276
12.3	Adelic heights and their properties	279
12.4	Reduction theory for GL_n : Minkowski revisited	285
12.5	Compactness criterion and Siegel domains	289
12.6	The case of connected reductive k -split groups: a sketch	291
	PART III APPENDICES	297
<i>Appendix A</i>	Linear Algebraic Groups: A Review	299
A.1	Affine k -group schemes with k a ring	299
A.2	Hopf algebras and affine k -group schemes	304
A.3	Smoothness	307
A.4	Operations and representations	308
A.5	Restriction and induction of G -modules	311
A.6	Cohomology of G -modules	313
A.7	Weil restriction or restriction of scalars	314
A.8	Unipotent groups	317
A.9	Diagonalisable and multiplicative groups	319

viii	<i>Contents</i>	
	A.10 Algebraic k -tori	321
	A.11 Reductive groups	322
	A.12 Forms of algebraic groups	324
Appendix B	Global Fields	326
	B.1 Absolute values and local fields	326
	B.2 Global fields	328
	B.3 Restricted products of topological spaces	330
	B.4 The ring of adèles	331
	B.5 The idele group	333
	B.6 S -integers and S -units	334
Appendix C	Topological Groups, Homogeneous Spaces, and Proper Actions	335
	C.1 Topological groups	335
	C.2 Topological transformation groups	339
	C.3 Locally compact transformation groups	341
	C.4 Proper maps	341
	C.5 Proper actions of topological groups	343
	C.6 Characterisation of proper actions	346
References		350
Index		357