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978-1-108-83144-4 — Equivariant Stable Homotopy Theory and the Kervaire Invariant Problem

Michael A. Hill , Michael J. Hopkins , Douglas C. Ravenel

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