Equivariant Stable Homotopy Theory and the Kervaire Invariant Problem

The long-standing Kervaire invariant problem in homotopy theory arose from geometric and differential topology in the 1960s and was quickly recognized as one of the most important problems in the field. In 2009, the authors of this book announced a solution to the problem, which was published to wide acclaim in a landmark *Annals of Mathematics* paper.

The proof is long and involved, using many sophisticated tools of modern (equivariant) stable homotopy theory that are unfamiliar to nonexperts. This book presents the proof together with a full development of all the background material to make it accessible to a graduate student with an elementary knowledge of algebraic topology. There are dozens of explicit examples of constructions and concepts used in solving the problem, as well as detailed accounts of all the computations needed along the way.

Also featuring a motivating history of the problem and numerous conceptual and expository improvements on the proof, this is the definitive account of the resolution of the Kervaire invariant problem.

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Douglas C. Ravenel is the Fayerweather Professor of Mathematics at the University of Rochester. He is the author of two influential previous books in homotopy theory and roughly 75 journal articles on stable homotopy theory and algebraic topology.

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To Tim, Rose, Vivienne and Elizabeth,
the twinkles in our eyes

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