

Dynamic Systems and Control Engineering

Using a step-by-step approach, this textbook provides a modern treatment of the fundamental concepts, analytical techniques, and software tools used to perform multi-domain modeling, system analysis, and simulation, linear control system design and implementation, and advanced control engineering. The chapters follow a progressive structure, which builds from modeling fundamentals to analysis and advanced control while showing the interconnections between topics, and solved problems and examples are included throughout. Students can easily recall key topics and test understanding using Review Note and Concept Quiz boxes, and over 200 end-of-chapter homework exercises with accompanying Concept Keys are included. Focusing on practical understanding, students will gain hands-on experience of many modern MATLAB® tools, including Simulink® and physical modeling in Simscape™. With a solutions manual, MATLAB® code, and Simulink®/Simscape™ files available online, this is ideal for senior undergraduates taking courses on the modeling, analysis, and control of dynamic systems, as well as for graduates studying control engineering.

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To Fati, my life, my light and my love – for her continued support and sacrifices.

To my children and parents for their unconditional love and support.

Nader Jalili

To my family.

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Dynamic Systems and Control Engineering

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The Big Picture

Overview

Crafted from course-tested materials and proven methods – refined by over 25 years of careful observation and feedback from thousands of students – the presentation of material in *Dynamic Systems and Control Engineering* guides readers through their first encounter with the truly interdisciplinary concentrations of dynamic systems and feedback control design. In developing this text, we have attempted to maintain a focus on fostering accurate understanding of core concepts while providing practical insights for improved knowledge retention and inspiration for further pursuit of this important field. In most cases, we have opted for detailed explanations, graphical depictions, worked examples, and solved problems over heavy mathematical rigor. Accordingly, we hope students, instructors, and independent readers find this text to be enjoyable, useful, and self-contained.

Audience

This book is intended to serve as the primary course text for either a one-semester upper-class undergraduate courses in modeling, analysis, and control of dynamic systems or a one-semester first-year graduate course in control engineering. Moreover, owing to the linear and constructive organization of material, the text is also suitable for two-semester courses at the undergraduate level, with a first semester on modeling and analysis and a second semester focused on feedback control systems.

Book Structure and Approach

To understand the structure of this text and our approach, let us first consider the diagram in Figure 0.1, which provides the roadmap for the coming work.

Progressing through the text, we will become familiar with the details of this diagram as well as the reasons for its peculiar and ubiquitous design. For now, let us focus on the large roman numerals near the “plant” (I), “output” (II), “measurement signal” (III), and entire feedback control system (IV). These numerals indicate the focal points of the four distinct parts of this text:

- I. Modeling of Multi-Domain Dynamic Systems
- II. Analysis of Multi-Domain Dynamic Systems

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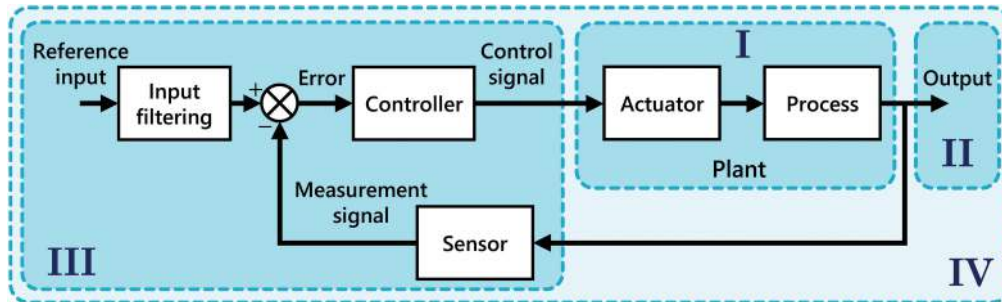


Figure 0.1 Architecture of the book.

III Introduction to Feedback Systems

IV Analysis and Feedback Control of Modern Systems

Organization of the Text

Philosophically, the four-part structure of the text (e.g., see Figure 0.1) puts the focus on achieving physical understanding, formalized through established mathematical principles that are introduced, as needed, without the distraction of rigorous proofs and theorems – which typically detract from the conceptual nature of a first contemplation. Pedagogically, we follow a progressive, fundamentals-first approach, wherein each part of the text builds on the preceding materials while carefully avoiding premature discussions of future or advanced topics. Thus, chapter by chapter we build a unified framework for modeling, analyzing, and controlling dynamic systems from many disparate domains.

Inevitably, our tools and techniques for modeling systems, or “plants,” developed in Part I play a key role in understanding how systems and their “outputs” respond to excitations, or “inputs,” in Part II. In turn, the combination of topics from modeling and analysis provides the basis for understanding and designing control systems in Part III, where several additional methods relevant to control systems are introduced. Finally, these core concepts of feedback control guide our discussion of modern and advanced control techniques in Part IV.

Notably, all these topics are covered in a unified and consistent manner, ultimately coming together to form a comprehensive introduction to dynamic systems and feedback control, as depicted in Figure 0.1 and laid out as follows.

Part I: Chapters 1–4

In Part I the main text commences with a treatment of mechanical, electrical, thermal, fluidic, hydraulic, multi-domain, and contemporary dynamic modeling (including MATLAB, Simulink, and Simscape) that is one of the most complete in comparison with any similar

text. Through discussions of the system decomposition and mechanical modeling techniques, we review a primarily physics-based approach to system modeling, which leads to mathematical representations of various systems in the form of ordinary differential equations.

Part II: Chapters 5–7

Once a model has been obtained using the techniques discussed in Part I, the underlying physics of the system is set aside, and the system is investigated through the properties of its mathematical model. This approach allows any physical system to be treated with the same set of solution techniques and analysis tools. Accordingly, in Part II, analytical and numerical solution techniques are progressively introduced to obtain system responses, and analytical tools are developed to classify systems according to properties of their mathematical models and their responses in both the time and frequency domains. Subsequently, Laplace-domain techniques are introduced, leading to the detailed treatment of transfer functions, block diagrams, poles, zeros, stability, and eventually feedback, which is the cornerstone of the classical control system techniques discussed in Part III.

Part III: Chapters 8–11

Through the developments in Parts I and II, which review important modeling and analysis techniques, readers are well prepared for the discussions on control systems and controller design that are progressively developed in Part III. Leveraging our toolkit developed in Parts I and II, Chs. 8–10 introduce open-loop and closed-loop control strategies and provide appropriately comprehensive treatments of the Routh–Hurwitz root-locus, Bode, and Nyquist techniques used in the analysis and design of control systems. Following these discussions, Ch. 11 provides closure to Part III with a review of several important considerations regarding the real-world implementation of control systems. Together, the techniques in Parts I–III provide the basis for development of the intermediate and advanced control engineering techniques in Part IV.

Part IV: Chapters 12–14

Continuing the discussion of control systems initiated in Part III, the final chapters of this text introduce the field of modern control as well as several advanced topics including digital, robust, adaptive, and nonlinear control systems. Together, the methods discussed in these chapters represent a set of tools that are essential for practicing as a modern control engineer and that provide a basis for continued study in the analysis and control of an even more general class of complex dynamic systems.

Pedagogical Elements

On the basis of our experience, there is value in the repetition and reinforcement of essential concepts. Reflecting this principle, the text not only includes a significant number

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of worked examples and solved problems but is also interspersed with Concept Quizzes, Review Notes, and Enhanced Reviews as elements to reinforce learning. For the reader's convenience, these elements are color coded to distinguish and highlight their use throughout the text. We refer to and label the first of these elements as **Concept Quizzes**; they comprise a set of brief conceptual problems and are contained within blue-green boxes. As shown in the example below, these Concept Quizzes consist of basic questions with concise final answers and are intended to provide opportunities for readers to test their own fundamental understanding while progressing through the text.

Concept Quiz 1: Considering the architecture shown in Figure 0.1 and discussion above, what are the four parts of the text?

Answer: The four parts of the text are: (I) Modeling of Multi-Domain Dynamic Systems; (II) Analysis of Multi-Domain Dynamic Systems; (III) Introduction to Feedback Systems; and (IV) Analysis and Feedback Control of Modern Systems.

Review Note 1

Throughout this text, we use **dot notation** to indicate time derivatives of system variables. In dot notation, the number of dots over a variable indicates the order of the time derivative being represented; for example the first time derivative of a variable $x(t)$ is given as

$$\dot{x}(t) = \frac{dx(t)}{dt}.$$

In addition to Concept Quizzes, the text also includes two elements designed to provide review materials for refreshing the reader's recall of fundamental topics or results from the fields of science, mathematics, and other domains of interest. The simpler of these two elements comprises the **Review Notes**, which are contained inside boxes with gray backgrounds and provide timely explanations or details that may help readers understand discussions without having to refer to other texts or materials.

On the other hand, **Enhanced Reviews**, contained in blue-green boxes with prominent headings, provide more extensive background on selected topics. It is our belief that these Enhanced Reviews will help readers connect discussions in this text with other important engineering topics, providing a more robust and integrated learning environment that helps to locate our discussions on dynamic systems and control engineering within the broader scope of science, technology, and engineering.

Enhanced Review 1 – Mathematical Rigor

Throughout the text, we will both review and, potentially, introduce concepts from mathematics. In doing so, we will apply only as much mathematical rigor and detail as deemed appropriate and necessary for solving the problem at hand. There are many graduate-level texts on dynamic systems and control engineering that delve more deeply into these topics using proofs and extensive mathematical notation.

In this text, however, we avoid burdening readers with mathematical proofs and, instead, opt for greater conceptual and pragmatic coverage of materials to provide a first introduction to the fundamental principles of dynamic systems and control engineering that can be readily applied to real-world problems. Where possible, software tools, especially those available in MATLAB, Simulink, and Simscape, are discussed as a practical alternative to certain mathematical operations (e.g., obtaining numerical, rather than analytical, solutions).

Together, we believe the practical aspects of our approach provide significant benefits over a more mathematically focused methodology, making this text more accessible to a broad readership with a focus on undergraduate engineering students and first-year graduate students.

Introduction to Dynamics Systems and Control Engineering

As motivation for the 14 chapters that follow, we will proceed with an example of the very common and seemingly simple task of designing a system that drives the direct current (DC) motor in Figure 0.2 from one constant (possibly stationary) speed to another within a specified timeframe. In doing so, we will touch on many topics. Some of these topics may be familiar, and other topics may be entirely new. At this point, we encourage the reader not to worry too much about the underlying physics, terminology, or mathematics – whether familiar or not (each will be addressed in due time) – and instead focus on the intuitive dynamic systems approach and the natural evolution from a very realistic problem to an appropriately detailed and robust solution.

Now, rather than setting out to control this motor's speed by performing a series of trial-and-error tests energizing it, logging results, making well-intended adjustments, and eventually developing/implementing a set of rules for switching between different speeds, we will begin the dynamic systems approach by setting three objectives for the motor's behavior.

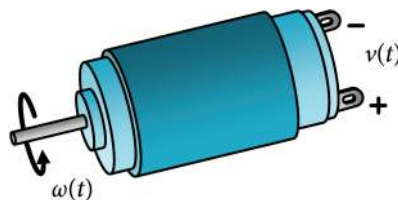


Figure 0.2 Direct current motor.

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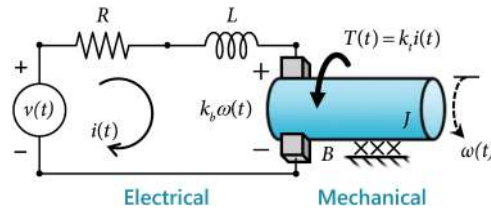


Figure 0.3 Combined electrical (left) and mechanical (right) diagram of a DC motor.

- (1) *Stability*: The motor should be able to reach and maintain the desired operating point (i.e., shaft velocity).
- (2) *Performance*: The motor should respond rapidly enough to changes in the desired speed to satisfy the requirements of a given application.
- (3) *Robustness*: Small disturbances to the motor's operation (e.g., noise in the supply voltage) and minor deviations from the expected properties (e.g., some additional friction) should not significantly impact the stability or performance of the motor.

To achieve these goals, we will have to understand some of the important electrical and mechanical properties of the motor. Furthermore, because we have set requirements on the motor's performance (i.e., it must switch from one speed to the next within a certain timeframe), knowledge of the motor's static (or **steady-state**) behavior is insufficient – we need to know both the speed it is to reach and how long it will take to get there. Thus, we are interested in both the static and the dynamic properties of this motor, such as how the motor shaft (which is not massless but has inertia and must likely overcome some significant friction as well) responds to the applied torque (i.e., over time).

Step 1: Modeling the Motor Dynamics

In our approach, we will start modeling the motor by drawing diagrams of its electrical and mechanical systems, as shown in Figure 0.3.

Using the mechanical part of the diagram in Figure 0.3, we can proceed to develop a mathematical expression describing the motor's **angular velocity** $\omega(t)$ and **angular acceleration** $\dot{\omega}(t)$ as functions of the **inertia** J , **damping** B , and applied **torque** $T(t)$:

$$J\dot{\omega}(t) + B\omega(t) = T(t).$$

Now, we must account for how this torque $T(t)$ is developed over time as the **voltage** $v(t)$ causes electrical current to pass through the motor – exciting electromagnetic interactions, generating heat, and turning the motor shaft. These interactions are captured by the electrical diagram on the left in Figure 0.3. Therefore, we will use this diagram to develop two additional mathematical expressions. The first relates the flow of **current** $i(t)$ to the motor's **inductance** L , **resistance** R , velocity $\omega(t)$ (through the proportionality constant k_b , known as the **back electromotive force (EMF) constant**), and the applied voltage $v(t)$ as

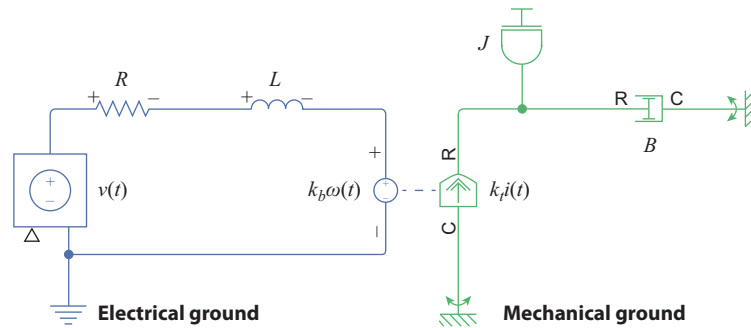


Figure 0.4 Physical model of the DC motor with no load in Simscape.

$$L \frac{di(t)}{dt} + Ri(t) + k_b\omega(t) = v(t).$$

The second expression relates this current $i(t)$ to the applied torque $T(t)$ resulting from electromagnetic interactions as

$$T(t) = k_t i(t),$$

where k_t is a proportionality constant known as the motor's **torque constant**.

Next, to create a mathematical representation of the entire motor, we will collect these **governing equations**

$$J\dot{\omega}(t) + B\omega(t) = k_t i(t),$$

$$L \frac{di(t)}{dt} + Ri(t) + k_b\omega(t) = v(t),$$

and we may gather the parameter values – necessary for investigating the behavior of this **mathematical model** – from the motor's manual or specification sheet.

Without further mathematical analysis, we could, at this point, resort to simulations (e.g., using **physical modeling** in MATLAB/Simscape as shown in Figure 0.4) to graph the dynamic response of the motor's shaft speed when various constant voltages are applied. Notably, the physical modeling approach in Simscape could be applied at an earlier phase, as it does not require an explicit mathematical model of the system but rather creates a numerical model by mimicking the physical laws that govern the system's elements, as discussed in Appendix B and Chs. 2–4. While numerical simulations, such as those obtained through Simscape, can provide some analytical insight, we will see shortly that a mathematical model is more valuable for detailed analysis and subsequent controller design.

Considering the case where the motor shaft is at rest when the applied voltage is suddenly changed from 0 V to some constant value, the physical model in Figure 0.4 can be used to plot the motor shaft's velocity response, as shown for a few characteristic cases in Figure 0.5. From this response, we see that applying a constant input (e.g., 1 V) causes the motor to approach a constant speed over the course of about 1.5 s and the motor then maintains this constant speed –

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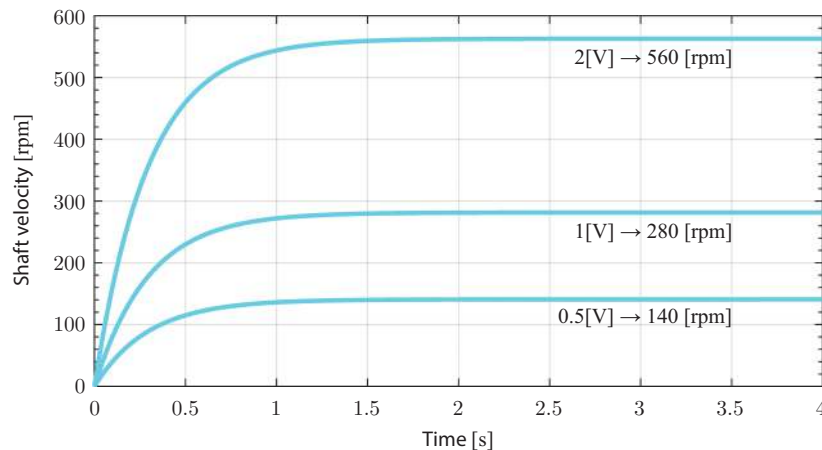


Figure 0.5 Response of the DC motor to constant 0.5 V, 1 V, and 2 V inputs.

satisfying our stability objective. In some cases, this may be acceptable. However, let us assume that we would like to reach and stay at 280 rpm in less than 0.5 s. In this case, the 1.5 s that it takes the motor to settle at the final speed in Figure 0.5 fails to meet our performance objective.

Intuitively, we might recognize that the motor can reach 280 rpm faster if we apply more voltage. However, this faster response comes with the cost that the input can no longer be a constant voltage and must itself be dynamic. Accordingly, we will likely want the voltage $v(t)$ to be higher than 1 V initially (i.e., to elicit a fast response) and then decrease to 1 V to maintain the desired speed (i.e., assuming the steady-state behavior in Figure 0.5 still holds with a non-constant input).

Unfortunately, this simulation model alone does not tell us enough about the behavior of the motor to infer whether this is an acceptable approach or how to go about achieving the desired response without significant trial and error. Further, without investigating the properties of the mathematical model, it is difficult to say whether the results of such a trial-and-error approach could be generalized to other desired speeds. Therefore, our best course of action is to analyze the system properties on the basis of the mathematical model before moving forward.

Step 2: Analyzing the Motor Behavior

Now, as will be reviewed later in Ch. 1, we may recognize that our mathematical model is equivalent to a set of **linear time-invariant ordinary differential equations**. Therefore, we have several analytical methods available to investigate the system response and help us systematically design the voltage $v(t)$. However, when we are concerned with the properties of the motor itself rather than its response to a particular input, the **Laplace transform** has a distinct advantage over other methods. As we will see, this transform allows us to study the

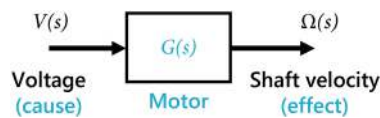


Figure 0.6 Block diagram representation of the motor transfer function model; s is the Laplace variable

motor in the **Laplace domain** (i.e., an alternative to the **time domain**), where our mathematical model is given by a set of algebraic, rather than differential, equations:

$$\begin{aligned} J\Omega(s)s + B\Omega(s) &= k_t I(s), \\ LI(s)s + RI(s) + k_b\Omega(s) &= V(s), \end{aligned}$$

where s is the Laplace variable (the time variable t does not exist in the Laplace domain) and $\Omega(s)$, $I(s)$, and $V(s)$, respectively, represent the Laplace-domain shaft velocity, motor current, and applied voltage.

In this domain, substitutions can easily be performed between these algebraic equations. Therefore, we can eliminate the intermediate variable $I(s)$ and develop a Laplace-domain input–output model known as a **transfer function**

$$G(s) = \frac{\text{Output}}{\text{Input}} = \frac{\Omega(s)}{V(s)} = \frac{k_t}{(LJ)s^2 + (LB + RJ)s + RB + k_b k_t},$$

which directly describes the dynamic cause-and-effect relationship between the applied voltage $V(s)$ and the shaft velocity $\Omega(s)$ as an algebraic ratio $G(s)$, often represented by a special form of **block diagram**, as shown in Figure 0.6. The dynamic relationship expressed by this transfer function represents how the motor converts the voltage $v(t)$ into a shaft velocity $\omega(t)$. This remains true regardless of the voltage $v(t)$ we apply.

Step 3: Controlling the Motor Velocity

Now, staying in the Laplace domain, we will find that we can easily develop additional algebraic equations, normally in the form of transfer functions, that modify this system. In fact, once we are familiar with the Laplace domain, it is quite straightforward to create transfer functions that represent our own added dynamics, such as

$$D(s) = \frac{\text{Output}}{\text{Input}} = \frac{V(s)}{\Omega_D(s)},$$

which relates our **desired velocity** profile $\Omega_D(s)$ to the motor's input voltage in Figure 0.7. As shown by this figure, cascading the output of this transfer function $D(s)$ with the input of the original motor dynamics $G(s)$ results in a chain of cause-and-effect relationships that is easily represented using block diagrams.

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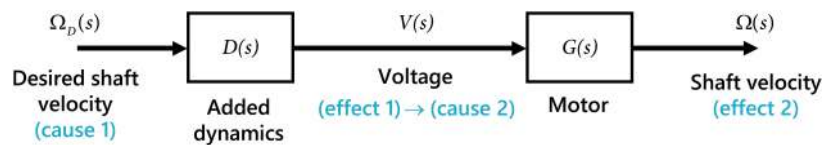


Figure 0.7 Cascade of multiple cause-and-effect relationships.

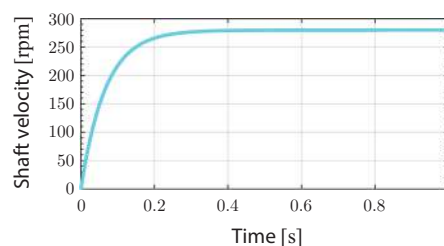


Figure 0.8 Response of the DC motor to the desired shaft velocity.

Now, assuming we design $D(s)$ appropriately, we will achieve a very close relationship between the desired shaft velocity $\omega_D(t)$ and the actual shaft velocity $\omega(t)$, such that the motor speed follows the desired reference profile almost exactly. For instance, we might design $D(s)$ in such a way that providing an immediate change from 0 to 280 rpm, the desired shaft velocity $\omega_D(t)$, results in the motor speed response $\omega(t)$ shown in Figure 0.8, which is much faster than the response achieved by applying a constant 1 V input as in Figure 0.5.

This process of cascading the output of the added dynamics $D(s)$ into the input of the motor $G(s)$, as in Figure 0.7, is a form of rudimentary control system known as **open-loop control**, and the $D(s)$ relationship that we have designed is known as a **controller**. To see how this cascade method operates, it is useful to look at plots of the desired shaft velocity, voltage, and actual shaft velocity together, as in Figure 0.9.

Notice that the voltage used to achieve a shaft velocity of 280 rpm in under 0.5 s initially spikes above 4 V and then decreases to 1 V – thereby maintaining the constant 280 rpm speed. This exactly follows our intuition. Furthermore, because we have altered the behavior of the system by adding the controller dynamics $D(s)$, we can now get the motor to quickly reach any desired velocity and possibly even track some velocity profiles that are not constant. In essence, we have gained control over the motor's behavior by making it part of a larger dynamic (control) system.

Unfortunately, the process of open-loop control depends heavily on our exact knowledge of the motor model, and its performance is not robust to model variations. For example, if we consider a case where the motor's resistance is provided with $\pm 10\%$ accuracy from the nominal value R_{NOM} , and everything else is exactly known, we may still see large deviations in steady-state velocity, as shown in Figure 0.10.

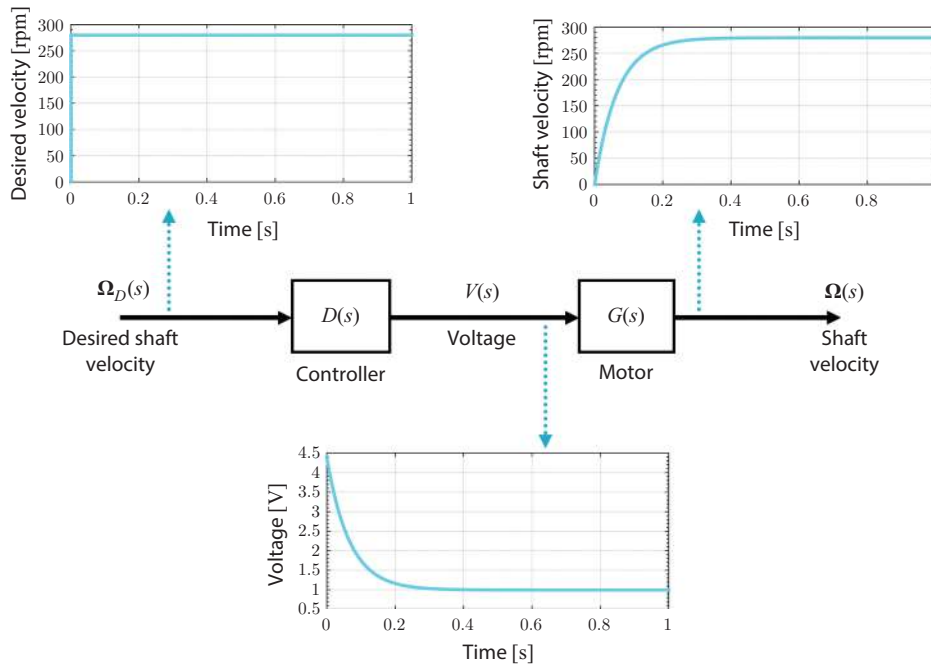


Figure 0.9 Using open-loop control to manipulate the motor shaft velocity $\omega(t)$.

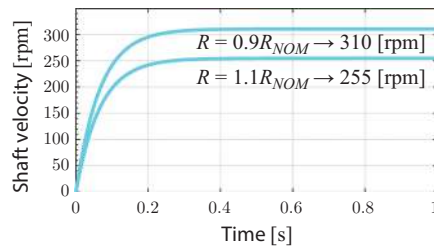


Figure 0.10 Open-loop control of motors with $\pm 10\%$ error in the modeled resistance.

Compared with the case in Figure 0.8, only the motor dynamics have changed – both the desired shaft velocity $\Omega_D(s)$ and controller $D(s)$ are the same as before. Therefore, we are applying the same voltage as that in Figure 0.9 to characteristically different motors. Then it may not be surprising that the shaft velocities are not the same as we obtained using our nominal model.

As it turns out, however, a more robust solution can be achieved by developing a **feedback control system**, like those shown in Figure 0.1 and Figure 0.11. In this case, we are still adding a controller $D(s)$, but now we are also measuring the motor’s velocity in real time and adjusting the voltage on the basis of the difference between the desired and measured shaft velocities, known as the error $E(s)$. That is, even if there are inaccuracies in the motor model used for design of the controller $D(s)$, a properly designed feedback control system will still be capable of

xxii The Big Picture

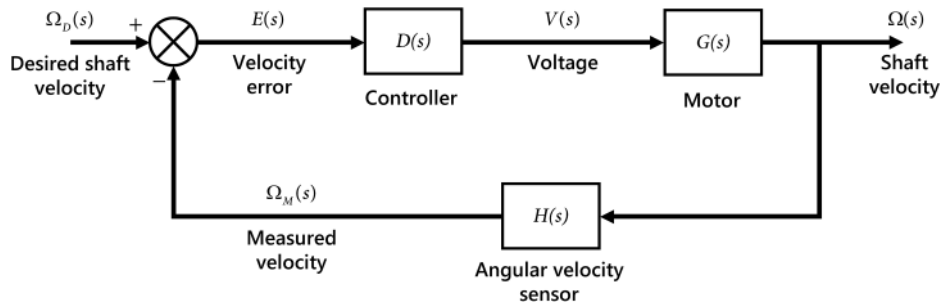


Figure 0.11 Feedback control system for the DC motor.

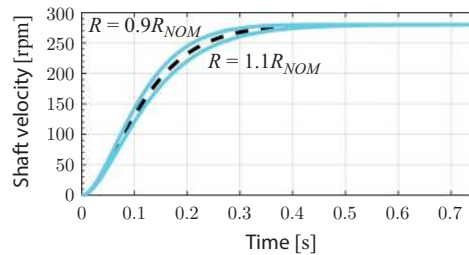


Figure 0.12 Feedback control of three motors with $R = R_{NOM}$ (dashed), $R = 0.9R_{NOM}$, and $R = 1.1R_{NOM}$.

achieving the desired shaft velocity because the applied voltage $V(s)$ is no longer a fixed function of the desired shaft velocity.

Now, assuming the same $\pm 10\%$ variations from the nominal motor resistance, Figure 0.12 shows the much tighter control that is achievable by using feedback, or **closed-loop**, control instead of an open-loop approach.

Since the responses in Figure 0.12 meet both our stability and performance objectives, despite variations in the motor model, we see that feedback offers a robust method of controlling the motor shaft velocity. In fact, in later chapters, we will see that it is even possible to design feedback controllers that meet our stability and performance objectives when the motor or, more generally, **plant** being controlled is affected by a combination of **model uncertainty** (such as inaccurate parameters), **external disturbances** (such as stray voltages or loads on the motor shaft), and **measurement noise**, as shown in Figure 0.13.

Looking Ahead

In summary, the process taken to control the speed of the DC motor was undertaken in three steps:

- I. *System Modeling*, to represent the system dynamics in a useful mathematical form;
- II. *System Analysis*, to identify the input–output behavior of the system ; and
- III. *Controller Design*, to add dynamics that improve the input–output behavior of the combined control system.

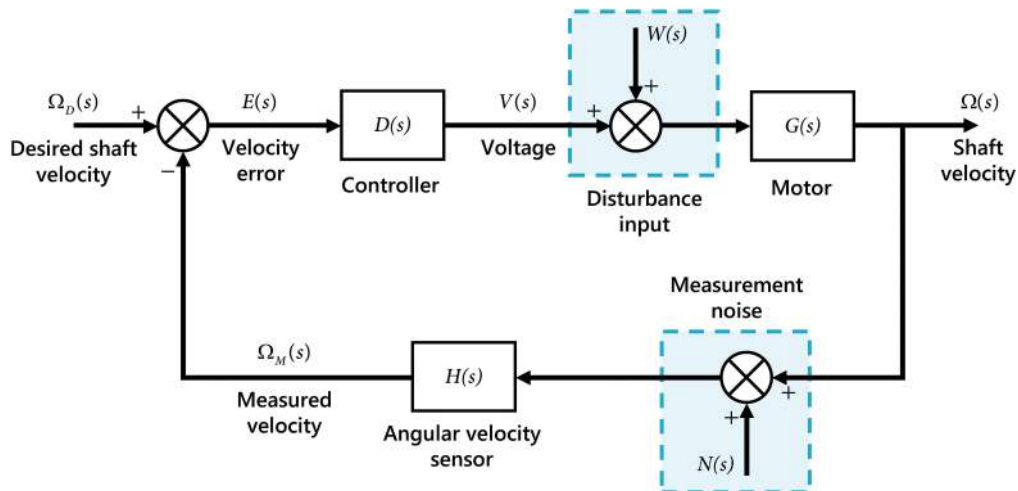


Figure 0.13 Feedback control system for the DC motor subjected to external disturbances $W(s)$ and measurement noise $N(s)$.

While there are different ways of generating mathematical models of system behaviors, various mathematical tools for performing analysis and design, and alternative methods for achieving a desired system response, the framework developed in this text applies a unified approach for modeling, analyzing, and controlling dynamic systems without regard to their underlying physical composition. This approach is ubiquitous in solving real-world problems, and even our simple task of controlling the speed of a DC motor is readily generalized to innumerable applications, from regulating fan, pump, propeller, tool, and vehicle speeds, to reading from and writing to compact disks, hard drives, and tapes, or even carrying out complex manipulations with articulated robots in surgical or manufacturing environments – to name but a few.

In what follows, we present a four-part, 14-chapter structure, in which we have dedicated complete discussions to each of these three topics (i.e., modeling, analysis, and control), as we progressively build toward a formal yet pragmatic understanding of the tools and techniques leveraged in the fields of dynamic systems and feedback control.

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