

To Heather and Carolin





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## **Preface**

Over the last couple of decades, enhancements of computational power and denser seismic networks have made a considerable impact on seismological practice. As a result, it is now possible to examine and model complex structures at relatively high frequencies. Further, the development of correlation techniques has allowed the exploitation of ambient noise and reduced dependence on fortuitous placement of earthquakes. Waveform inversion techniques enable the exploitation of much more of the seismogram than hitherto, but the small-scale structures that determine the high-frequency character of seismograms remain beyond reach of direct investigation.

This book provides an account of the use of correlation concepts in a broad range of applications in seismology, and the use of higher-frequency waves to examine the finer-scale aspects of the heterogeneity of the Earth. One of the major objectives of seismology has always been to extract as much information as possible from seismograms about the seismic source and the structure of the Earth. The growth of computational power means that it is now possible to undertake direct calculations of the seismic wavefield for realistic three-dimensional models and to use these to invert for complex structure, so we provide a full discussion of the inversion of seismic waveforms.

In recent years the density of seismometers available in some parts of the world for earthquake studies has reached the point where signal enhancement using multi-sensor techniques can exploit experience gained in the exploration field. The work therefore endeavours to provide links between the applications of seismology to earthquakes, ambient noise, regional and global studies, and seismic exploration. With numerical simulation we can approach the complexity of unfiltered observed seismograms, and the best results come when the physical processes controlling the behaviour of the wavefield are well understood. Our aim in this book is therefore to provide a suitable background for the appreciation of recent developments in seismic wave analysis.

An introductory chapter discusses the background to the work and the way in which it builds on and integrates material from prior studies in seismology. This

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is followed by a summary of the structure of the book, indicating the nature of the topics to be covered and the way that they interact.

To keep the work in bounds, we have assumed a reasonable acquaintance with the principles of seismology, and provide a concise recapitulation of important results in Part I that are exploited in later parts.

The treatment in this book draws on the fundamentals developed in the two volumes of *The Seismic Wavefield* (Kennett, 2001; 2002), and thus does not attempt to provide a comprehensive treatment of basic topics. References to sections in these volumes are indicated using a section marker (e.g., § SWI:3.1.2). For equations the volume number is represented explicitly as in (SWII:17.2.5). Where reference is made to the book *Geophysical Continua* (Kennett & Bunge, 2008) the designator GC is employed (e.g., § GC:11.3.2). Occasional use is also made of *Seismic Wave Propagation in Stratified Media* (Kennett, 1983; 2009) designated SM.

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