Cambridge University Press
978-1-108-83074-4 — Exploiting Seismic Waveforms
Brian Kennett , Andreas Fichtner
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Exploiting Seismic Waveforms

Exploiting Seismic Waveforms introduces a range of recent developments in seismology including the application of correlation techniques, understanding of multi-scale heterogeneity and the extraction of structure and source information by seismic waveform inversion. It provides a full treatment of correlation methods for seismic noise and event signals and develops inverse methods for both sources and structure. Higher frequency components of seismograms are frequently neglected, or removed by filtering, but they contain information about seismic structure on scales that cannot be revealed by seismic tomography. Sufficient computational resources are now available for waveform inversion for 3-D structure to be a practical procedure and this book describes suitable algorithms and examples reflecting current best practice. Intended for students and researchers in seismology, this book provides a physical understanding of seismic waveforms and the way that different aspects of the seismic wavefield are revealed by the way that seismic data are handled.

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Exploiting Seismic Waveforms

Correlations, Heterogeneity and Inversion

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CAMBRIDGE
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University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

79 Anson Road, #06–04/06, Singapore 079906

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www.cambridge.org
Information on this title: www.cambridge.org/9781108830744
DOI: 10.1017/9781108903035

First published 2021

Printed in the United Kingdom by TJ Books Limited, Padstow Cornwall

A catalogue record for this publication is available from the British Library.

ISBN 978-1-108-83074-4 Hardback
ISBN 978-1-108-82878-9 Paperback

To Heather and Carolin

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Preface

Over the last couple of decades, enhancements of computational power and denser seismic networks have made a considerable impact on seismological practice. As a result, it is now possible to examine and model complex structures at relatively high frequencies. Further, the development of correlation techniques has allowed the exploitation of ambient noise and reduced dependence on fortuitous placement of earthquakes. Waveform inversion techniques enable the exploitation of much more of the seismogram than hitherto, but the small-scale structures that determine the high-frequency character of seismograms remain beyond reach of direct investigation.

This book provides an account of the use of correlation concepts in a broad range of applications in seismology, and the use of higher-frequency waves to examine the finer-scale aspects of the heterogeneity of the Earth. One of the major objectives of seismology has always been to extract as much information as possible from seismograms about the seismic source and the structure of the Earth. The growth of computational power means that it is now possible to undertake direct calculations of the seismic wavefield for realistic three-dimensional models and to use these to invert for complex structure, so we provide a full discussion of the inversion of seismic waveforms.

In recent years the density of seismometers available in some parts of the world for earthquake studies has reached the point where signal enhancement using multi-sensor techniques can exploit experience gained in the exploration field. The work therefore endeavours to provide links between the applications of seismology to earthquakes, ambient noise, regional and global studies, and seismic exploration. With numerical simulation we can approach the complexity of unfiltered observed seismograms, and the best results come when the physical processes controlling the behaviour of the wavefield are well understood. Our aim in this book is therefore to provide a suitable background for the appreciation of recent developments in seismic wave analysis.

An introductory chapter discusses the background to the work and the way in which it builds on and integrates material from prior studies in seismology. This is followed by a summary of the structure of the book, indicating the nature of the topics to be covered and the way that they interact.

To keep the work in bounds, we have assumed a reasonable acquaintance with the principles of seismology, and provide a concise recapitulation of important results in Part I that are exploited in later parts.

The treatment in this book draws on the fundamentals developed in the two volumes of *The Seismic Wavefield* (Kennett, 2001; 2002), and thus does not attempt to provide a comprehensive treatment of basic topics. References to sections in these volumes are indicated using a section marker (e.g., § SWI:3.1.2). For equations the volume number is represented explicitly as in (SWII:17.2.5). Where reference is made to the book *Geophysical Continua* (Kennett & Bunge, 2008) the designator GC is employed (e.g., § GC:11.3.2). Occasional use is also made of *Seismic Wave Propagation in Stratified Media* (Kennett, 1983; 2009) designated SM.

Acknowledgments

We are grateful to many people who have been kind enough to provide material for figures including M. Afanasiev, N. Blom, C. Bösch, P. Boué, Y. Chen, A. El-Sharawy, M. de Kool, T.-K. Hong, E. Kissling, T. Meier, T.-S. Pham, K. Priestley, N. Rawlinson, A.M. Reading, J. Ritsema, K. Sager, E. Saygin, B. Tauzin, H. Tkalčić, and D. Thompson. Takashi Furumura kindly produced some new numerical simulations for the work, as well as contributing figures.

1

Introduction

The early advances in understanding the internal structure of the Earth and the nature of earthquakes were achieved with a very limited number of seismic stations. Yet, by 1914, not only the presence of a core was recognised by Oldham (1906) from his analysis of the travel times of seismic waves from 14 earthquakes across the globe, but also a good estimate of its radius had been made (Gutenberg, 1913).

Many advances came from painstaking analysis of large numbers of earthquake records at the same stations; such as the discovery of the presence of the inner core by Inge Lehmann from the records of the Copenhagen station (Lehmann, 1936) – fortunately the plentiful events on the Pacific Rim were at a suitable distance.

The basic theory of linear elastic waves was well established by the 1850's, but seismograms did not conform to the expectations for simple models such as a uniform half space with simple P, S and Rayleigh waves (Lamb, 1904). Love (1911) introduced the possibility of dispersion for observed seismograms, though the details remained sketchy because the necessary calculations for multilayered structures were difficult even with early digital computers (e.g., Ewing, Jardetsky & Press, 1957).

In this book we explore the aspects of seismic waves that have been made accessible by the dramatic increase in the availability and quality of seismic data across a broad range of frequencies and the computational power needed for modelling and analysis. Indeed we have reached a situation where numerical simulation is capable of generating seismograms that rival observations in their complexity. In consequence we still need suitable tools to disentangle the complex physical processes that control both observations and synthetics.

Our focus is on recent developments that reach beyond a radially stratified model of the Earth. We try to develop concepts that bridge scales and help develop understanding of the seismic wavefield in three-dimensions (3-D). We examine the application of correlations between seismic waveforms in a wide range of contexts, from the estimation of time and phase delays, through the exploitation of ambient noise and earthquake coda for structural studies, to receiver studies.

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Improved sampling of the seismic wavefield has shed light on the smaller scales of heterogeneity. Rather than just simplifying seismograms by low-pass filtering, it is possible to utilise the higher-frequency components of the coda of seismic phase to gain information about structure.

Parametric stochastic models provide a convenient description of the smaller-scale features of heterogeneity via spatial correlation functions. We use such models to discuss the effects of heterogeneity on seismic waves and the way in which multiple scales of heterogeneity interact. The capacity to simulate realistic seismograms opens the opportunity for using waveform inversion methods to extract structural information directly from observed seismograms. This approach is facilitated by the use of adjoint techniques that reduce computational demands, and we show how waveform inversion can be used for both event records and correlation data.

1.1 Growth of Recording Networks

Since the recognition that waves from earthquakes could be detected thousands of kilometres away from the source, the development of earthquake seismology has had two major strands. The first strand comes from the deployment of seismographs in regions of earthquake activity with the aim of understanding the nature of local events and their implications for earthquake hazard. The second strand utilises seismic stations across the globe and exploits distant recordings to determine source characteristics of larger events and to understand the internal structure of the Earth. From the 1960s efforts to monitor nuclear testing led to coherent global networks and the development of seismic arrays.

The advent of digital recording encouraged the development of broadband sensors that could cover the full frequency range encompassed by seismic waves. Now it was no longer necessary to have two separate seismometer systems to focus on the signals above and below the dominant microseismic peak around 0.12 Hz. The versatility of the new instrumentation meant that it was possible to capture both local and global signals with a single sensor, and so markedly enlarge the scope of seismological studies. The result has been a fusion of the two strands of enquiry, with enhanced national networks also able to be used for global studies. The increased suite of permanent stations has been accompanied by extensive deployments of portable instrumentation directed towards structural studies.

The increase in the number of seismic stations in the world has been dramatic. For example for events with moment magnitude Mw 7 in the New Ireland region of Papua New Guinea, the number of stations reporting time readings to the International Seismological Centre increased from 324 in 1978, 639 in 1999, to 1255 in 2015. The volume of waveform data also continues to grow at a rapid rate: for Mw 7.7 events at a very similar location in this region, around 1200 stations were potentially available in 2000, compared with more than 3000 stations in 2016.

One of the side effects of the introduction of broadband sensors is that the

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spectrum of seismic noise is faithfully recorded, rather being suppressed by selective frequency response. Before long, methods for exploiting seismic noise were developed building on ideas from acoustics (e.g., Campillo & Paul, 2003). The stacking of correlation of seismic noise records between two different seismic stations over a substantial period of time was found to be equivalent to having virtual sources and receivers at each location. This advance opened up new ways of examining seismic structure, particularly with higher-frequency surface waves, since pairs of stations could be used without need for alignment to the great-circle path from a distant earthquake that was a major restriction on early structural studies (Knopoff, 1972). The exploitation of ambient noise correlation has become a major part of the toolkit of seismology with many different applications (see, e.g., Nakata, Gualtieri & Fichtner, 2019). Since the number of stations pairs in a network of N stations is $\frac{1}{2}N(N-1)$, very large numbers of virtual propagation paths can be explored via correlation methods.

One of the developments in the use of portable broadband instrumentation has been the creation of broad areal coverage through multi-year experiments with progressive movement of stations. An early example was the SKIPPY experiment in Australia that achieved reconnaissance coverage of the continent at 400 km station spacing over a period of 4 years (van der Hilst et al., 1994). With a limited number of stations, a sequence of 6-month deployments were made to progressively cover the continent from east to west, with deployments in the south in the austral summer to avoid the ‘wet’ in the tropics. The relatively short duration of each deployment was sufficient to collect numerous events in the earthquake belts around Australia for surface wave studies, but less effective for body wave work.

Subsequent more ambitious projects with much larger numbers of instruments have typically used more than 2 years recording at each site. The most comprehensive project was USArray in which around 400 stations in the Transportable Array were moved like a shutter blind across the continental United States over a period of 7 years from 2007–2014. At the end of their recording interval, stations were moved from the western limb of the array to the east to maintain continuous coverage. Following the successful deployment, the transportable stations were moved to the Alaskan region. Other major portable deployments have been the ongoing ChinArray in China, and the IberArray that spanned from Morocco to northern Spain in three deployments in the period 2009–2014. The AlpArray in Europe (2016-2019) has been a multi-national effort with 45 research institutions from 18 countries cooperating to link the permanent stations in the Alpine area with a broad-scale deployment of portable instruments, with the objective of studying the structure and evolution of the lithosphere in the entire Alpine region of Europe. The AlpArray experiment has included both land-based stations and sea-bottom seismometers (Figure 1.1).

Even without any major experiment, a combination of smaller experiments can

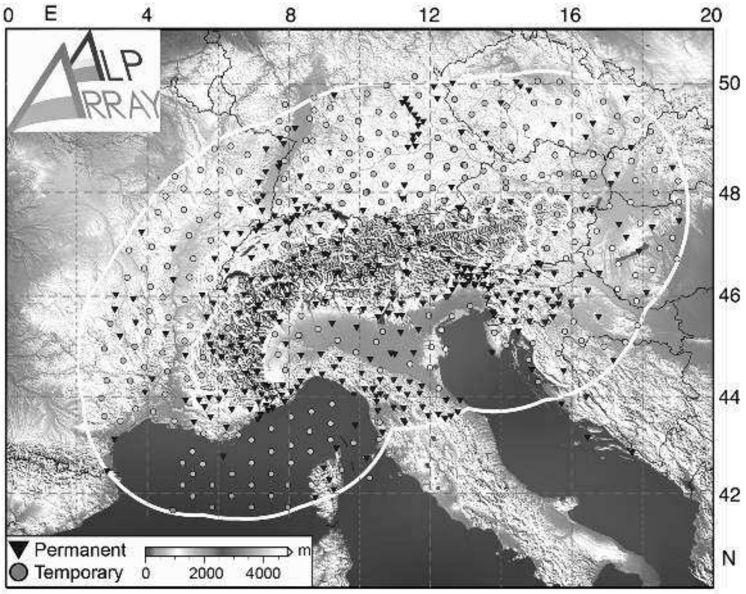

Figure 1.1 Configuration of the AlpArray experiment in Europe, covering the entire Alpine area with a combination of permanent and portable stations. [Courtesy of AlpArray Working Group.]

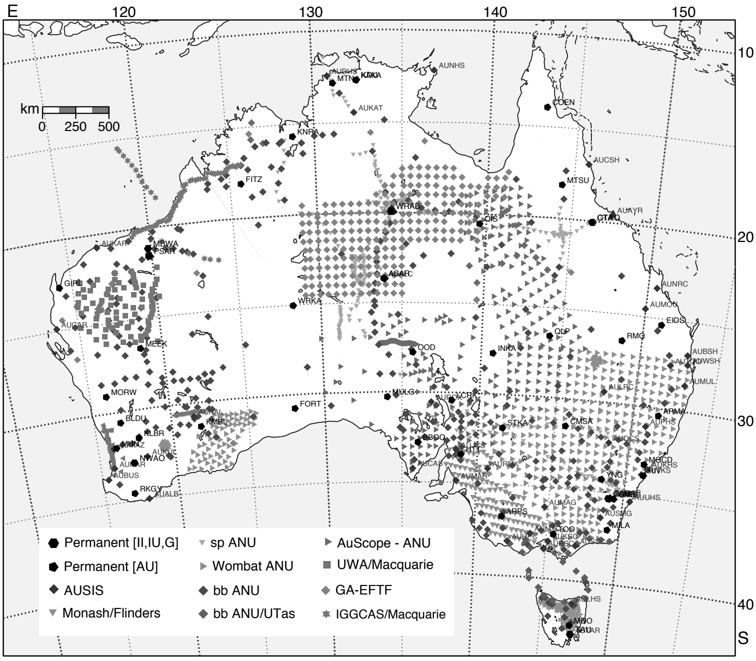

Figure 1.2 Station coverage in Australia from both permanent stations and portable deployments to the end of 2019. [Courtesy of AuScope.]

over time produce substantial coverage. In Australia, the early SKIPPY project was followed by a number of more localised broadband experiments, and a sequence of portable deployments in southeastern Australia, collectively known as the WOMBAT project. Additional recent deployments in northern and western Australia have produced reasonable coverage at the continental scale (Figure 1.2). Substantial areas of desert on the Australian continent remain undersampled, since access and logistics are difficult.

The progress from initially sparse sensors to dense networks has been even more marked in exploration seismology. The amount of energy reflected from depth is rather small, but can be enhanced if multiple coverage of the subsurface can be combined. Thus stacking procedures of increasing sophistication became an essential part of the reflection surveys. Where possible, multiple line profiles have been replaced by full surveys with an areal distribution of source and receivers and intensive processing of enormous data sets to extract three-dimensional structure. On land, the use of sensors linked by long cables is often now replaced by large numbers of autonomous sensors that allow more flexible configurations. Thousands of sensors are routinely deployed with the aim of securing sufficient sampling of the wavefield that the details of reflections are faithfully recorded

Both aspects of seismology are converging towards a goal of full rendering of the seismic wavefield over a broad range of frequencies, so that the maximum structural information can be extracted. We therefore provide links between the treatment of the reflection wavefield and techniques for the study of larger-scale structure.

1.2 Theoretical and Computational Developments

Theoretical developments in seismology have drawn heavily on the tools developed by mathematical physicists, exploiting separation of variables and the inter-relations between plane, cylindrical, and spherical waves. Lamb (1904) formulated the response of an infinite half-space to a surface source in terms of double integrals over wavenumber and frequency. The elegant work of Lapwood (1948) used steepest-descent and stationary-phase methods to examine the seismic phases arising from a buried source in a half-space. Pekeris (1948) also used stationary phase techniques and recognised the Airy phase in dispersive wave propagation associated with extrema in group slowness that tends to dominate in long-distance propagation. Cagniard (1939) developed a way of recasting the integral contributions for an individual arrival so that the time response was more readily obtained. This approach was simplified somewhat by de Hoop (1958), and the modified form has been extensively used in the generalised ray method (see, e.g., Aki & Richards, 2002).

In 1968, two articles appeared that showed the application of numerical seismograms to understanding active source experiments. Helmberger (1968) used a development of the wavefield in terms of generalised ray contributions

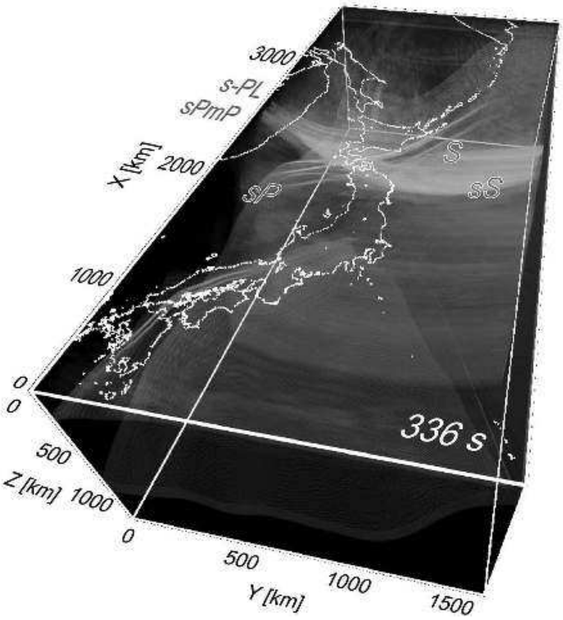

Figure 1.3 3-D modelling using the finite-difference technique for an event in the Sea of Okhotsk recorded across Japan. [Courtesy of T. Furumura.]

whose time signatures were summed to yield seismograms at various distances. Fuchs (1968) worked directly with the integral formulations for a multi-layered medium with numerical integration over wavenumber and a fast Fourier transform to recover the time response. Fuchs & Müller (1971) introduced a shallow zone with just transmission overlying layering for which the full reflection response was calculated. This *reflectivity* method was used to calculate a full record section for the interpretation of a refraction profile. Active source experiments provided early applications of synthetic seismograms because they had relatively dense sampling for the time. It was not long though before earthquake observations were being used and the structure employed extended through the upper mantle transition zone (Helmberger & Wiggins, 1971). Kennett (1975) pointed out the important role of attenuation on the character of the wavefield and the potential trade-offs between structure and zones of low Q.

Alongside the use of layered models, direct numerical solutions were developed for the underlying differential equations (e.g., Alterman & Loewenthal, 1972). Fully numerical approaches emerged at around the same time and by 1972 finite-difference techniques had been applied to simple two-dimensional (2-D) heterogeneous models (Boore, 1970), and a finite-element approach used for surface wave propagation across complex structures (Lysmer & Drake, 1972). In these years a major limitation for all the numerical procedures was the availability of computer memory, since even with the most powerful machines of the 1970s only relatively small grids could be used.

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Figure 1.4 Variable-density grid employed with spectral element modelling.

For single processor systems, finite-difference methods tended to concentrate on higher-order difference schemes or pseudospectral methods (e.g., Furumura et al, 1998) since these minimised numerical dispersion for coarser grids. But, once parallel computation became fully established, it was recognised that simpler schemes, e.g., 4th order in space and 2nd order in time were more effective for inter-processor communications. The finite-difference approach for 3-D models exploits staggered grids where displacements and tractions are defined at different grid positions. These displaced grids minimise numerical dispersion effects, but at the cost of a complex rendering of interface conditions and the free surface. With thousands of compute nodes that have gigabytes of local memory, it has become possible to undertake finite-difference numerical simulations for relatively high frequencies across a large spatial domain (Figure 1.3). The computational effort for such complex calculations remains high, and care needs to be taken in constructing 3-D models. Two-dimensional simulations remain valuable for exploration of the effects of fine-scale heterogeneity.

The finite-element method exploits an integral equation formulation of elastic wave propagation and so automatically satisfies the free-surface boundary condition even in the presence of topography. The computational domain is divided into discrete elements so irregular geometries can be accommodated. With low-order polynomial representations in each element and explicit continuity conditions, the solution of the resulting system of linear equations requires considerable effort and numerical dispersion can be significant. As a result the direct finite-element approach has not been extensively used in seismic applications.

However, a variant of the finite-element technique has achieved widespread

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use. This spectral element method again uses non-overlapping elements that can be adapted to complex geometries. Within each element a high-order spectral representation is used. With a suitable choice of polynomial and quadrature procedure within each element, the effort of solving the requisite linear equations is much reduced. Since the free-surface condition is built in, the spectral element method is very effective for modelling surface wave propagation and similar phenomena (e.g., Komatitsch & Tromp, 2002a,b). Spectral element procedures can be adapted to a wide range of conditions, with substantial variations in the size of mesh as illustrated in Figure 1.4. The spectral representation carries the implicit assumption that the variation in wavespeed in each element is smooth. Thus rather small elements are needed for rapidly varying wavespeeds.

The advent of full 3-D computations opens the possibility of exploiting much more of the seismic wavefield than in the past. We can simulate entire seismic waveforms from events for moderate frequencies, and make use of such results in the inversion of observed seismograms. We still need good starting models, which can exploit the wide range of results obtained with travel-time tomography as well as earlier waveform inversions. The importance of scattered energy increases rapidly with frequency, and so the presence of unresolvable fine-scale structure is likely to impose limits on the applications of full waveform inversion.

1.3 Structure of the Book

In this book we provide a treatment of a broad range of topics that relate to the nature of seismic wave propagation in complex structures and the ways in which understanding of 3-D structure can be extracted from observations. We exploit recent developments, but draw on a range of techniques and modes of analysis. We endeavour to provide links between the applications of seismology to earthquakes, ambient noise, regional and global studies, and seismic exploration. We use both observations and numerical simulations to illustrate the topics and provide insight into the character of the seismic wavefield.

In Part I we develop the representations of the seismic wavefield needed for the later topics. In Chapter 2, we provide a discussion of the way that the higher-frequency body waves and surface wave components emerge from the normal mode spectra of the Earth to link global and local concepts. In Chapter 3, we consider integral representations of the wavefield and develop the concepts of reflection and transmission operators that allow insight gained in the discussion of stratified media to be transferred to a more general 3-D environment. This operator approach also provides a convenient link between larger-scale seismology and seismic reflection studies. Chapter 4 provides a discussion of the reflection field in an exploration context, which allows links to be drawn to current developments for exploiting teleseismic arrivals. We consider propagation issues using the operator development and show how this leads to understanding of migration procedures, with links to remapping of reflectivity as is currently being exploited in Marchenko techniques.

Part II is concerned with the exploitation of correlations in seismology in a wide range of circumstances. In Chapter 5 we introduce the concepts of waveform correlations, and the way in which they can be exploited to extract information from seismograms. We then consider the closely related topic of transfer functions between aspects of the wavefield, and this leads into a discussion of the ways by which seismograms can be compared – a topic of importance in the comparison of observations and simulations. We also consider the nature of receiver functions and the correlation of teleseismic signals at a receiver. Chapter 6 addresses the nature of the correlation wavefield and its relation to the group of techniques collectively known as *seismic interferometry*. We establish a direct representation of the cross-correlation of the seismic signals between two stations and show how, with a suitable distribution of sources, this correlation can provide a virtual source–receiver pair whose phase properties arise from differencing. We then discuss the concept of generalised interferometry with an arbitrary distribution of sources, and illustrate the way in which processing procedures can affect the nature of correlated signals. Having laid down the fundamentals, in Chapter 7 we examine the application of correlation procedures to the exploitation of the ambient noise field where the dominant component of the correlation field comes from surface waves, though body waves can be extracted in some circumstances. In Chapter 8 we turn attention to applications of correlation techniques to the coda of seismic source signals where steeply travelling body waves are the main contributors to the correlation wavefield. The correlation wavefield emphasises seismic phases that are difficult to detect in direct excitation by a source and so can provide new information on internal structure, e.g., an improved estimate of the shear wavespeed in the inner core. In Chapter 9 we consider correlation methods applied to surface recordings with the objective of extracting information on subsurface reflectivity. We show how the auto-correlation of seismic signals can provide information on reflections without conversions, and can be exploited to provide indirect imaging of heterogeneous structure. Correlations between signals at different sensors can also be exploited in reflection work to provide virtual sources that provide new ways of imaging complex structure.

In Part III we examine the interaction of seismic waves with heterogeneity at all scales, with an emphasis on the influence of structure on multiple scales. We discuss ways in which numerical simulations and inversions can exploit data with differing station density to provide maximum resolution of structure. The finer scales of variation within the Earth lie beyond any capacity for direct imaging, but the scattered wavefield that they produce contains important information on structure. In Chapter 10 we contrast deterministic and stochastic representations of heterogeneity, and look at the way that ensemble results can be exploited for Earth structure that is time invariant. We also consider the way that effective media, with simpler structure, can be extracted from complex models by the process of wavespeed upscaling. In Chapter 11 we examine the ways in which the effects of heterogeneity can be handled. We first consider perturbations of the wavefield using Born series and show how such concepts can be combined with the use of reflection and transmission operators to provide a flexible treatment of structures with varying heterogeneity in different zones of the model. Although the various modes of surface waves propagate independently in simple structure, the presence of heterogeneity induces cross-coupling that modifies the wavefield. In Chapter 12 we examine the processes of scattering in the Earth, the various zones where it is important and the way that these influence observed seismograms. Guided waves in heterogeneous structures play an important role at high frequencies. We discuss examples from the propagation of deep earthquakes in subduction zone environments, and for the oceanic and continental lithosphere. Chapter 13 discusses the interaction of seismic waves with multiple scales of heterogeneity, particularly in the lithosphere.

Part IV introduces the concepts and developments needed for the inversion of seismic waveforms, with a discussion of algorithms and illustrations of practical inversions. Chapter 14 presents the basic inversion framework in a Bayesian context, leading into the formulation of the nonlinear inversion process in terms of optimisation of a measure of misfit. In Chapter 15 we provide a broad survey of methods for inversion relevant to waveforms, showing how inversions can be performed for models described by very large numbers of parameters. Chapter 16 first describes the way in which adjoint techniques allow the computation of derivatives in complex models and so enable practical non-linear inversion. This is followed by a discussion of sensitivity kernels associated with the variation of critical parameters for both structure and sources. We show how such kernels can provide insight into the nature of Earth structure and potential resolution. In Chapter 17 we describe the process of waveform inversion for earthquake data to extract 3-D structure, including computational aspects. We show how, as a model is improved, it becomes possible to incorporate further data and hence achieve model refinement. We examine the issues of practical resolution assessment, and validation of proposed models. In Chapter 18 we turn attention from event data to the exploitation of the correlation wavefield, demonstrating that it is possible to achieve joint inversion for noise sources and Earth structure and include a discussion of the relevant sensitivity kernels. The final chapter (Chapter 19) addresses a range of topics that hold considerable promise for future developments. We start by considering nested inversions that allow definition of heterogeneity across a wide range of length scales from local through regional to global. This is followed by discussion of adaptive numerical gridding, exploitation of data redundancy, the development of efficient random sampling methods for inversion, and the use of Hamiltonian Monte Carlo techniques for efficient searching of high-dimensional spaces.