As with definitional equivalence between models, definitional equivalence between theories is not transitive (and hence not an equivalence relation).

Example 1.13 Consider the theories

$$T_1 = \{\exists x \forall y (y = x), \forall x P x\} \tag{1.14}$$

$$T_2 = \{\exists x \forall y (y = x), \forall x \neg P x\} \tag{1.15}$$

$$T_3 = \{\exists x \forall y (y = x), \forall x Q x\} \tag{1.16}$$

where T_1 and T_2 are both in signature $\{P^{(1)}\}$, while T_3 is in signature $\{Q^{(1)}\}$. Then T_1 and T_2 are definitionally equivalent to T_3, but not definitionally equivalent to one another.

1.3 Theories and Models

Finally, we consider what kinds of relationships we can draw between syntactic and semantic notions of definability. In this context, a significant idea is that of *implicit definition*.[13]

Definition 1.14 (Implicit definition) Let Σ be a signature, and let $\Sigma^+ = \Sigma \cup \{P^{(n)}\}$. A Σ^+-theory T *implicitly defines* P *in terms of* Σ if for any two models $\mathtt{A}, \mathtt{B}$ of T, if $\mathtt{A}|_\Sigma = \mathtt{B}|_\Sigma$ then $\mathtt{A} = \mathtt{B}$.

Intuitively, the idea of implicit definition is that the extensions of the Σ-predicates uniquely 'fix' the extension of P: provided that two models of T agree on the Σ-extensions, they have to agree on the extension of P. This idea is closely related to the notion of *supervenience*, as discussed in the metaphysics literature. In particular, McLaughlin and Bennett (2018) define 'strong global supervenience' as follows: 'A-properties *strongly globally supervene* on B-properties iff for any worlds w_1 and w_2, every B-preserving isomorphism between w_1 and w_2 is an A-preserving isomorphism between them.' Adjusting to the model-theoretic context, we can formulate the following definition:

Definition 1.15 Let Σ be a signature, and let $\Sigma^+ = \Sigma \cup \{P\}$. P *strongly globally supervenes on* Σ, relative to the Σ^+-theory T, iff for any models $\mathtt{A}, \mathtt{B}$ of T, any isomorphism $f : \mathtt{A}|_\Sigma \to \mathtt{B}|_\Sigma$ is an isomorphism from $\mathtt{A}$ to $\mathtt{B}$ (i.e. is such that f preserves the extension of P).

[13] This is not to be confused with the idea that a set of axioms, such as Hilbert's axioms for geometry, 'implicitly define' the meanings of the terms occurring therein.