NOTATION INDEX

Notation	Meaning	Page
a, b, c	Typical symbols from an alphabet	1
A^{T}	Transposed matrix	39
$A > 0$	Positive matrix	100
$A \geqslant 0$	Nonnegative matrix	100
$A \geqslant B$	Matrix inequality	100
$A > B$	Strict matrix inequality	100
$A \lessapprox B$	Elementary equivalence	226
$A \approx B$	Strong shift equivalence	227
$A \sim B$	Shift equivalence	235
$\mathcal{A}$	Alphabet	1
$\mathcal{A}^k$	k-blocks over $\mathcal{A}$	2
$\mathcal{A}^{\mathbb{Z}}$	Full shift over $\mathcal{A}$	2
A_G, $A(G)$	Adjacency matrix of graph G	35
$A_{\mathcal{G}}$	Symbolic adjacency matrix	65
$\operatorname{adj} A$	Adjugate of A	113
$\mathcal{B}(X)$	Language of X	9
$\mathcal{B}_n(X)$	n-blocks in X	9
β_N	Higher block map	12
$BF(A)$	Bowen–Franks group	248
$BF_p(A)$	Generalized Bowen–Franks group	250
C_f	Companion matrix of f	373
$C_k(u)$	Cylinder set using block u	179
$\chi_A(t)$	Characteristic polynomial of A	100
$(\Delta_A, \Delta_A^+, \delta_A)$	Dimension triple of A	252
d_ϕ	Degree of sliding block code ϕ	303
d_ϕ^*	Minimal number of pre-image symbols	304
$\mathcal{E}(G)$, $\mathcal{E}$	Edge set of graph	33
$\mathcal{E}_I$	Edges with initial state I	34
$\mathcal{E}^J$	Edges with terminal state J	34
e, f, g	Typical edges of a graph	33
$\langle E \rangle_{\mathbb{Q}}$	Rational vector space generated by E	397
$\langle E \rangle_{\mathfrak{R}}$	$\mathfrak{R}$-ideal generated by E	417
$\mathcal{F}$	Set of forbidden blocks	5
$F_X(w)$	Follower set of a word	72
$F_{\mathcal{G}}(I)$	Follower set of a state	78
f_λ	Minimal polynomial of λ	370
f_Λ	Polynomial associated to list Λ	385
$[[\phi]]$	Topological full group	476
$(\partial\Phi, \Phi)$	Graph homomorphism	34
G	Graph	33
G^{T}	Transposed graph	39
$G(A)$, G_A	Graph with adjacency matrix A	35
$G \cong H$	Graph isomorphism	34
$G^{[N]}$	Higher edge graph	42
G^N	Higher power graph	45
$\mathcal{G} = (G, \mathcal{L})$	Labeled graph	64
$\mathcal{G}_X$	Minimal presentation of sofic shift	83
$G_1 \times G_2$	Product of graphs	87

$\mathcal{G}_1 \times \mathcal{G}_2$	Product of labeled graphs	88
$\mathcal{G}_1 * \mathcal{G}_2$	Label product	89
$h(X)$	Entropy of a shift space	101
$h(\phi)$	Entropy of a map	191
$\mathcal{I}(A)$	Ideal class of A	417
$i(e)$	Initial state of edge e	34
I, J, K	Typical states of a graph	33
$J^\times(A)$	Jordan form away from zero	247
$J^\times(B) \subseteq J^\times(A)$	Containment of Jordan forms	408
K_θ	Cone of "size" θ	375
λ_A	Perron eigenvalue of A	110,121
$\mathcal{L}$	Labeling on graph	64
$\mathcal{L}^+$	Label applied to one-sided shifts	141
$\mu(I), \mu(e\|I)$	Markov chain probabilities	47
$\mu(n)$	Möbius function	192
(M, ρ)	Metric space	173
$\mathcal{M}(X)$	Marker coordinates	346
$p_n(X)$	Number of points of period n in X	106
$p_n(\phi)$	Number of points of period n under ϕ	189
$P(X)$	Periodic points of X	339
$P(X) \hookrightarrow P(Y)$	Embedding periodic point condition	339
$P(X) \searrow P(Y)$	Factor periodic point condition	359
$\mathrm{per}(\cdot)$	Period of state, graph, or shift	126
$\mathrm{per}(z)$	Least period of z	343
Φ	Block map	15
$\phi = \Phi_\infty^{[-m,n]}$	Sliding block code	15
π, τ, ω	Typical paths on a graph	38
$q_n(X)$	Number of points of least period n in X	116
$\mathcal{R}_A$	Eventual range of A	243
σ, σ_A	Shift map	3, 7
$\mathrm{sgn}(\pi)$	Sign of a permutation	198
$\mathrm{sp}^\times(A)$	Nonzero spectrum of A	197
$t(e)$	Terminal state of edge d	34
$\mathbb{T}$	Circle	175
$\mathrm{tr}(\Lambda)$	Trace of list Λ	349
$\mathrm{tr}_n(\Lambda)$	nth net trace of a list Λ	350
$\mathrm{tr}_n(A)$	nth net trace of a matrix A	349
u,v,w	Typical blocks over an alphabet	2
$\|u\|$	Length of block u	2
$\mathbf{u},\mathbf{v},\mathbf{w}$	Typical vectors	107
$\mathcal{V}(G), \mathcal{V}$	Vertex set of graph	33
$\mathbf{v}_A$	Perron eigenvector of A	110
x,y,z	Typical points from a shift space	2
$x_{[i,j]}$	Subblock from a point x	3
$X_\mathcal{F}$	Shift space with forbidden blocks $\mathcal{F}$	5
$X(d,k)$	Run-length limited shift	6
$X(S)$	S-gap shift	7
$X^{[N]}$	Nth higher block shift	12
X^N	Nth higher power shift	14
X_A or X_G	Edge shift of matrix A or graph G	36
$X_{[r]}$	Full r-shift	36
$\widehat{X}_B$	Vertex shift of 0-1 matrix	43

X^+	One-sided sequences from X	141
$X^\intercal$	Transposed shift	39
$X^+_{G,I}$	One-sided infinite paths starting at I	141
$X \cong Y$	Conjugacy between shift spaces	18
$X \times Y$	Product of shifts	9
$\zeta_\phi(t)$	Zeta function of ϕ	193
$\emptyset$	Zero in symbolic adjacency matrix	65

INDEX

Page numbers where an entry is defined are underlined, those where the entry occurs in a theorem, proposition, etc. are in ***bold italic***, and those where the entry occurs in an exercise are in *regular italic*.

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