

Contents

<i>List of Contributors</i>	<i>page</i> xv
<i>Introduction</i>	1
Part One Geometric Evolution Equations and Curvature Flow	9
1 Real Geometric Invariant Theory	
<i>C. Böhm and R.A. Lafuente</i>	11
1.1 Introduction	11
1.2 Examples	16
1.3 Comparison with Complex and Symplectic Case	18
1.4 The Abelian Case	19
1.5 Separation of Closed T-Invariant Sets	23
1.6 The General Case of Real Reductive Groups	25
1.7 Stratification	28
1.8 Properties of Critical Points of the Energy Map	36
1.9 Applications	39
1.10 Appendices	41
1.10.1 Real Reductive Lie Groups	41
1.10.2 The Parabolic Subgroup Q_β	44
<i>References</i>	47
2 Convex Ancient Solutions to Mean Curvature Flow	
<i>T. Bourni, M. Langford and G. Tinaglia</i>	50
2.1 Introduction	50
2.2 Asymptotics for Convex Ancient Solutions	52
2.3 X.-J. Wang's Dichotomy for Convex Ancient Solutions	56

2.4	Convex Ancient Solutions to Curve Shortening Flow	66
2.5	Rigidity of the Shrinking Sphere	67
2.6	Asymptotics for Convex Translators	68
2.7	X.-J. Wang's Dichotomy for Convex Translators	70
2.8	Rigidity of the Bowl Soliton	71
	<i>References</i>	72
3	Negatively Curved Three-Manifolds, Hyperbolic Metrics, Isometric Embeddings in Minkowski Space and the Cross Curvature Flow	
	<i>P. Bryan, M. Ivaki and J. Scheuer</i>	75
3.1	Introduction	75
3.2	Geometrisation of Three-Manifolds	77
3.3	Embeddability and Hyperbolic Metrics	79
3.4	The Cross Curvature Flow	85
3.4.1	Definition and Basic Properties of the Flow	85
3.4.2	Short Time Existence and Uniqueness	87
3.4.3	Basic Identities and Evolution Equations	90
3.4.4	Towards Hyperbolic Convergence	92
3.4.5	Harnack Inequality and Solitons	94
3.4.6	Monotonicity of Einstein Volume	95
	<i>References</i>	96
4	A Mean Curvature Flow for Conformally Compact Manifolds	
	<i>A.R. Gover and V.-M. Wheeler</i>	98
4.1	Introduction	98
4.2	Conformal Geometry and Hypersurfaces in Conformally Compact Manifolds	101
4.2.1	Conformal Manifolds	102
4.2.2	The Tractor Connection	102
4.2.3	Conformally Compact Manifolds	103
4.2.4	Hypersurfaces	105
4.2.5	A Hypersurface Flow for Conformally Compact Manifolds	106
4.2.6	Boundary Conditions	107
4.2.7	The Flow Problem	107
4.3	The Flow Problem	108
4.3.1	Treating the Flow as a Nonlinear Partial Differential Equation	108
4.3.2	Generalised Mean Curvature Flow in Hyperbolic Space	112
4.3.3	Long Time Existence and Convergence	113

Diarmuid Crowley

Table of Contents

[More Information](#)

	<i>Contents</i>	ix
	4.3.4 Generalised Mean Curvature Flow in Riemannian Manifolds	115
	<i>References</i>	115
5	A Survey on the Ricci Flow on Singular Spaces	
	<i>K. Kröncke and B. Vertman</i>	118
	5.1 Introduction and Geometric Preliminaries	118
	5.1.1 Isolated Conical Singularities	119
	5.1.2 Ricci de Turck Flow and the Lichnerowicz Laplacian	121
	5.2 Existence of the Singular Ricci de Turck Flow	122
	5.2.1 Tangential Stability	123
	5.2.2 The Existence Result	124
	5.2.3 Characterizing Tangential Stability	125
	5.3 Stability of the Singular Ricci de Turck Flow	127
	5.4 Perelman's Entropies on Singular Spaces	128
	5.4.1 The λ -Functional	128
	5.4.2 The Ricci Shrinker Entropy	129
	5.4.3 The Ricci Expander Entropy	129
	5.5 Curvature Quantities Along Singular Ricci de Turck Flow	132
	5.5.1 Bounded Ricci Curvature Along Singular Ricci de Turck Flow	132
	5.5.2 Positive Scalar Curvature Along Singular Ricci de Turck Flow	133
	5.6 Open Questions and Further Research Directions	134
	5.7 Appendix: Sobolev and Hölder Spaces	134
	<i>References</i>	137
	Part Two Structures on Manifolds and Mathematical Physics	141
6	Some Open Problems in Sasaki Geometry <i>C. Boyer, H. Huang, E. Legendre and C. Tønnesen-Friedman</i>	143
	6.1 Introduction	143
	6.2 Brief Review of Sasaki Geometry	145
	6.2.1 The Sasaki Cone	145
	6.2.2 The Transverse Holomorphic Structure	147
	6.2.3 The Lie Algebra of Killing Potentials	147
	6.3 Extremal Sasaki Geometry	150

Diarmuid Crowley

Table of Contents

[More Information](#)

x

Contents

6.3.1	Transverse Futaki–Mabuchi	150
6.3.2	The Einstein–Hilbert Functional	153
6.3.3	The Sasaki Energy Functional	155
6.4	The Functionals \mathbf{H} , \mathcal{SE} on Lens Space Bundles Over Riemann Surfaces	161
6.4.1	Explicit Examples	163
	<i>References</i>	166
7	The Prescribed Ricci Curvature Problem for Homogeneous Metrics <i>T. Buttsworth and A. Pulemotov</i>	169
7.1	Introduction	169
7.2	The Prescribed Ricci Curvature Problem	169
7.3	Compact Homogeneous Spaces	171
7.3.1	The Variational Interpretation	172
7.3.2	Maximal Isotropy	173
7.3.3	The Structure Constants	173
7.3.4	The Scalar Curvature Functional and its Extension	174
7.3.5	Non-Maximal Isotropy: The First Existence Theorem	176
7.3.6	Non-Maximal Isotropy: The Second Existence Theorem	178
7.3.7	The Case of Two Isotropy Summands	179
7.3.8	Homogeneous Spheres	181
7.3.9	Further Examples	183
7.3.10	Ricci Iterations	185
7.4	Open Questions and Non-Compact Homogeneous Spaces	185
7.4.1	The Non-Compact Case	186
7.4.2	Unimodular Lie Groups of Dimension 3	187
	<i>References</i>	190
8	Singular Yamabe and Obata Problems <i>A.R. Gover and A.K. Waldron</i>	193
8.1	Introduction	193
8.2	Background and a Singular Obata Problem	195
8.3	Tractor Calculus for Hypersurface Embeddings	201
8.3.1	The Sphere	209
8.4	Singular Yamabe and Obata Problems	211
	<i>References</i>	212

Diarmuid Crowley

Table of Contents

[More Information](#)

Contents

xi

9 Einstein Metrics, Harmonic Forms and Conformally Kähler Geometry	<i>C. LeBrun</i>	215
9.1	Introduction	215
9.2	An Integral Weitzenböck Formula	220
9.3	Some Almost-Kähler Geometry	228
9.4	The Main Theorems	235
	<i>References</i>	238
10 Construction of the Supersymmetric Path Integral:		
A Survey	<i>M. Ludewig</i>	241
10.1	Introduction	241
10.2	First Construction: The Top Degree Functional	244
10.3	Second Construction: The Chern Character	249
10.4	Bismut–Chern Characters, Entire Chains and the Localization Formula	253
	<i>References</i>	257
11 Tight Models of de-Rham Algebras of Highly Connected Manifolds	<i>L. Schwachhöfer</i>	260
11.1	Introduction	260
11.2	Rational and Weak Equivalence	263
11.3	Poincaré DGCAs and DGCAs of Hodge Type	265
11.4	Small Algebras of Hodge Type DGCAs	270
11.5	Tight DGCAs of Highly Connected DGCAs	273
11.6	The Bianchi–Massey Tensor	277
	<i>References</i>	280
Part Three Recent Developments in Non-Negative Sectional Curvature		283
12 Fake Lens Spaces and Non-Negative Sectional Curvature	<i>S. Goette, M. Kerin and K. Shankar</i>	285
12.1	Introduction	285
12.2	\mathbb{Z}_ℓ Actions on the Family \mathcal{F}	286
	<i>References</i>	290
13 Collapsed 3-Dimensional Alexandrov Spaces:		
A Brief Survey	<i>F. Galaz-García, L. Guijarro and J. Núñez-Zimbrón</i>	291
13.1	Introduction	291
13.2	Basic Alexandrov Geometry	293
13.3	Three-Dimensional Alexandrov Spaces	298

Diarmuid Crowley

Table of Contents

[More Information](#)

xii

Contents

13.3.1	Geometric 3-Alexandrov Spaces	301
13.3.2	Geometrization of 3-Alexandrov Spaces	301
13.4	Collapsed Three-Dimensional Alexandrov Spaces	302
13.4.1	General Structure Results	302
13.4.2	Geometrization of Sufficiently Collapsed Three-Dimensional Alexandrov Spaces	307
	<i>References</i>	308
14	Pseudo-Angle Systems and the Simplicial Gauss–Bonnet–Chern Theorem	
	<i>S. Klaus</i>	311
14.1	Introduction	311
14.2	The Simplicial Gauss–Bonnet–Chern Theorem	313
14.3	Systems of Pseudo-Angles	316
14.4	Combinatorial Riemannian Manifolds	319
14.5	Simplicial Sectional Curvature	320
14.6	Simplicial Sectional Curvature and the Hopf Conjecture	322
	<i>References</i>	325
15	Aspects and Examples on Quantitative Stratification with Lower Curvature Bounds	
	<i>N. Li</i>	326
15.1	Introduction	326
15.2	Stratification of Singular Sets	328
15.3	Quantitative Stratification	330
15.3.1	Definitions	330
15.3.2	Results	332
15.4	Key Ingredients and Framework	335
15.4.1	Monotonicity Formula and Bad Scales	335
15.4.2	Splitting Theory	338
15.4.3	Dimension Reduction	340
15.4.4	Good-Scale Annuli Covering	341
15.5	Spaces whose Singular Sets are Cantor Sets	343
	<i>References</i>	350
16	Universal Covers of Ricci Limit and RCD Spaces	
	<i>J. Pan and G. Wei</i>	352
16.1	Introduction	352
16.2	Some Properties of Ricci Limit and RCD Spaces	354
16.3	Universal and δ -Covers	359
16.4	Non-Collapsing Ricci Limit Spaces	364
	<i>References</i>	369

17 Local and Global Homogeneity for Manifolds of Positive Curvature	<i>J.A. Wolf</i>	373
17.1	Introduction	373
17.2	The Classification for Positive Curvature	375
17.3	Positive Curvature and Isotropy Splitting	377
17.4	The Three Remaining Positive Curvature Cases	378
17.5	Dropping Normality in Positive Curvature	380
	<i>References</i>	383