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Differential Geometry in the Large

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University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

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Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108812818
DOI: 10.1017/9781108884136

First published 2021

Printed and bound in Great Britain by Clays Ltd, Elcograf S.p.A.

A catalogue record for this publication is available from the British Library.

ISBN 978-1-108-81281-8 Paperback

„Bekanntlich setzt die Geometrie sowohl den Begriff des Raumes, als die ersten Grundbegriffe für die Constructionen im Raume als etwas Gegebenes voraus. Sie giebt von ihnen nur Nominaldefinitionen, während die wesentlichen Bestimmungen in Form von Axiomen auftreten. Das Verhältniss dieser Voraussetzungen bleibt dabei im Dunkeln . . .
Diese Dunkelheit wurde auch von Euklid bis auf Legendre . . . weder von den Mathematikern, noch von den Philosophen, welche sich damit beschäftigten, gehoben. Es hatte dies seinen Grund wohl darin, dass der allgemeine Begriff mehrfach ausgedehnter Grössen, unter welchem die Raumgrössen enthalten sind, ganz unbearbeitet blieb.“

From the beginning of Bernhard Riemann's Habilitationsschrift
„Ueber die Hypothesen, welche der Geometrie zu Grunde liegen“
(Abhandl. Königl. Ges. Wiss. Göttingen, Vol. 13, 133–150, 1868)

“It is known that geometry assumes, as things given, both the notion of space and the first principles of constructions in space. She gives definitions of them which are merely nominal, while the true determinations appear in the form of axioms. The relation of these assumptions remains consequently in darkness . . .
From Euclid to Legendre . . . this darkness was cleared up neither by mathematicians nor by such philosophers as concerned themselves with it. The reason of this is doubtless that the general notion of multiply extended magnitudes (in which space-magnitudes are included) remained entirely unworked.”

Corresponding passage from William Kingdon Clifford's translation
'On the Hypotheses which lie at the Bases of Geometry'
(Nature, Vol. 8, 14–17, 1873)

Contents

Contributors

Christoph Böhm *University of Münster, Einsteinstraße 62, 48149 Münster, Germany*

Theodora Bourni *Department of Mathematics, University of Tennessee Knoxville, Knoxville, TN, 37996-1320, USA*

Charles P. Boyer *Department of Mathematics and Statistics, University of New Mexico, Albuquerque, NM 87131, USA*

Paul Bryan *Department of Mathematics, Macquarie University NSW 2109, Australia*

Timothy Buttsworth *Department of Mathematics, Cornell University, Ithaca, NY 14853, USA*

Fernando Galaz-García *Department of Mathematical Sciences, Durham University, Lower Mountjoy, Stockton Road, Durham DH1 3LE, UK*

Sebastian Goette *Abteilung für Reine Mathematik, Universität Freiburg, Ernst-Zermelo-Straße 1, D-79104 Freiburg, Germany*

A. Rod Gover *Department of Mathematics, The University of Auckland, Private Bag 92019, Auckland 1142, New Zealand*

Luis Guijarro *Departamento de Matemáticas, Facultad de Ciencias, Universidad Autónoma de Madrid, 28049 Cantoblanco, Madrid, Spain*

Hongnian Huang *Department of Mathematics and Statistics, University of New Mexico, Albuquerque, NM 87131, USA*

Mohammad N. Ivaki *Department of Mathematics, University of Toronto, Ontario, M5S 2E4, Canada*

Martin Kerin *School of Mathematics, Statistics and Applied Mathematics, National University of Ireland Galway, University Road, Galway, Ireland, H91 TK33*

Stephan Klaus *Mathematisches Forschungsinstitut Oberwolfach, Schwarzwaldstrasse 9-11, D-77709 Oberwolfach-Walke, Germany*

Klaus Kröncke *Fachbereich Mathematik, Bereich AD, Bundesstraße 55, 20146 Hamburg, Germany*

Ramiro A. Lafuente *School of Mathematics and Physics, The University of Queensland, St Lucia, QLD 4072, Australia*

Mat Langford *Department of Mathematics, University of Tennessee Knoxville, Knoxville, TN 37996-1320, USA*

Claude LeBrun *Department of Mathematics, Stony Brook University, Stony Brook, NY 11794-3651, USA*

Eveline Legendre *Institut de Mathématiques de Toulouse, Université Paul Sabatier, 118 route de Narbonne 31062, Toulouse, France*

Nan Li *Department of Mathematics, The City University of New York – NYC College of Technology, 300 Jay St., Brooklyn, NY 11201, USA*

Matthias Ludewig *Fakultät für Mathematik, Universität Regensburg, 93040 Regensburg, Germany*

Jesús Núñez-Zimbrón *Centro de Ciencias Matemáticas UNAM, Antigua Carretera a Pátzcuaro 8701, Col. Ex Hacienda San José de la Huerta, Morelia, Michoacán, C.P. 58089, México*

Jiayin Pan *Department of Mathematics, University of California Santa Barbara, Santa Barbara, CA 93106, USA*

Artem Pulemotov *School of Mathematics and Physics, The University of Queensland, St Lucia, QLD 4072, Australia*

Julian Scheuer *Department of Mathematics, Columbia University New York, NY 10027, USA*

Lorenz Schwachhöfer *Fakultät für Mathematik, TU Dortmund University, Vogelpothsweg 87, 44221 Dortmund, Germany*

Krishnan Shankar *Department of Mathematics, University of Oklahoma, 601 Elm Avenue, Room 423, Norman, OK 73019-3103, USA*

Giuseppe Tinaglia *Department of Mathematics, King's College London, London WC2R 2LS, UK*

Christina W. Tønnesen-Friedman *Department of Mathematics, Union College, Schenectady, NY 12308, USA*

Boris Vertman *Carl von Ossietzky Universität Oldenburg, Institut für Mathematik, Carl-von-Ossietzky-Str. 9-11, D-26129 Oldenburg, Germany*

Andrew K. Waldron *Center for Quantum Mathematics and Physics, Department of Mathematics, University of California Davis, Davis, CA 95616, USA*

Guofang Wei *Department of Mathematics, University of California Santa Barbara, Santa Barbara, CA 93106, USA*

Valentina-Mira Wheeler *School of Mathematics and Applied Statistics, University of Wollongong, Northfields Ave Wollongong, NSW 2522, Australia*

Joself A. Wolf *Department of Mathematics, University of California, Berkeley, CA 94720-3840, USA*