

Diarmuid Crowley

Frontmatter

[More Information](#)

LONDON MATHEMATICAL SOCIETY LECTURE NOTE SERIES

Managing Editor: Professor Endre Süli, Mathematical Institute, University of Oxford,
Woodstock Road, Oxford OX2 6GG, United Kingdom

The titles below are available from booksellers, or from Cambridge University Press at
www.cambridge.org/mathematics

- 356 Elliptic curves and big Galois representations, D. DELBOURGO
357 Algebraic theory of differential equations, M.A.H. MACCALLUM & A.V. MIKHAILOV (eds)
358 Geometric and cohomological methods in group theory, M.R. BRIDSON, P.H. KROPHOLLER & I.J. LEARY (eds)
359 Moduli spaces and vector bundles, L. BRAMBILA-PAZ, S.B. BRADLOW, O. GARCÍA-PRADA & S. RAMANAN (eds)
360 Zariski geometries, B. ZILBER
361 Words: Notes on verbal width in groups, D. SEGAL
362 Differential tensor algebras and their module categories, R. BAUTISTA, L. SALMERÓN & R. ZUAZUA
363 Foundations of computational mathematics, Hong Kong 2008, F. CUCKER, A. PINKUS & M.J. TODD (eds)
364 Partial differential equations and fluid mechanics, J.C. ROBINSON & J.L. RODRIGO (eds)
365 Surveys in combinatorics 2009, S. HUCZYNSKA, J.D. MITCHELL & C.M. RONEY-DOUGAL (eds)
366 Highly oscillatory problems, B. ENGQUIST, A. FOKAS, E. HAIRER & A. ISERLES (eds)
367 Random matrices: High dimensional phenomena, G. BLOWER
368 Geometry of Riemann surfaces, F.P. GARDINER, G. GONZÁLEZ-DIEZ & C. KOUROUNIOTIS (eds)
369 Epidemics and rumours in complex networks, M. DRAIEF & L. MASSOULIÉ
370 Theory of p -adic distributions, S. ALBEVERIO, A.YU. KHRENNIKOV & V.M. SHELKOVICH
371 Conformal fractals, F. PRZYTYCKI & M. URBAŃSKI
372 Moonshine: The first quarter century and beyond, J. LEPOWSKY, J. MCKAY & M.P. TUIE (eds)
373 Smoothness, regularity and complete intersection, J. MAJADAS & A. G. RODICIO
374 Geometric analysis of hyperbolic differential equations: An introduction, S. ALINHAC
375 Triangulated categories, T. HOLM, P. JØRGENSEN & R. ROUQUIER (eds)
376 Permutation patterns, S. LINTON, N. RUŠKUC & V. VATTER (eds)
377 An introduction to Galois cohomology and its applications, G. BERTHUY
378 Probability and mathematical genetics, N. H. BINGHAM & C. M. GOLDIE (eds)
379 Finite and algorithmic model theory, J. ESPARZA, C. MICHAUX & C. STEINHORN (eds)
380 Real and complex singularities, M. MANOEL, M.C. ROMERO FUSTER & C.T.C. WALL (eds)
381 Symmetries and integrability of difference equations, D. LEVI, P. OLVER, Z. THOMOVA & P. WINTERNITZ (eds)
382 Forcing with random variables and proof complexity, J. KRAJÍČEK
383 Motivic integration and its interactions with model theory and non-Archimedean geometry I, R. CLUCKERS, J. NICAISE & J. SEBAG (eds)
384 Motivic integration and its interactions with model theory and non-Archimedean geometry II, R. CLUCKERS, J. NICAISE & J. SEBAG (eds)
385 Entropy of hidden Markov processes and connections to dynamical systems, B. MARCUS, K. PETERSEN & T. WEISSMAN (eds)
386 Independence-friendly logic, A.L. MANN, G. SANDU & M. SEVENSTER
387 Groups St Andrews 2009 in Bath I, C.M. CAMPBELL *et al* (eds)
388 Groups St Andrews 2009 in Bath II, C.M. CAMPBELL *et al* (eds)
389 Random fields on the sphere, D. MARINUCCI & G. PECCATI
390 Localization in periodic potentials, D.E. PELINOVSKY
391 Fusion systems in algebra and topology, M. ASCHBACHER, R. KESSAR & B. OLIVER
392 Surveys in combinatorics 2011, R. CHAPMAN (ed)
393 Non-abelian fundamental groups and Iwasawa theory, J. COATES *et al* (eds)
394 Variational problems in differential geometry, R. BIELAWSKI, K. HOUSTON & M. SPEIGHT (eds)
395 How groups grow, A. MANN
396 Arithmetic differential operators over the p -adic integers, C.C. RALPH & S.R. SIMANCA
397 Hyperbolic geometry and applications in quantum chaos and cosmology, J. BOLTE & F. STEINER (eds)
398 Mathematical models in contact mechanics, M. SOFONEA & A. MATEI
399 Circuit double cover of graphs, C.-Q. ZHANG
400 Dense sphere packings: a blueprint for formal proofs, T. HALES
401 A double Hall algebra approach to affine quantum Schur–Weyl theory, B. DENG, J. DU & Q. FU
402 Mathematical aspects of fluid mechanics, J.C. ROBINSON, J.L. RODRIGO & W. SADOWSKI (eds)
403 Foundations of computational mathematics, Budapest 2011, F. CUCKER, T. KRICK, A. PINKUS & A. SZANTO (eds)
404 Operator methods for boundary value problems, S. HASSI, H.S.V. DE SNOO & F.H. SZAFRANIEC (eds)
405 Torsors, étale homotopy and applications to rational points, A.N. SKOROBOGATOV (ed)
406 Appalachian set theory, J. CUMMINGS & E. SCHIMMERLING (eds)
407 The maximal subgroups of the low-dimensional finite classical groups, J.N. BRAY, D.F. HOLT & C.M. RONEY-DOUGAL
408 Complexity science: the Warwick master's course, R. BALL, V. KOLOKOLTSOV & R.S. MACKAY (eds)

Diarmuid Crowley

Frontmatter

[More Information](#)

- 409 Surveys in combinatorics 2013, S.R. BLACKBURN, S. GERKE & M. WILDON (eds)
- 410 Representation theory and harmonic analysis of wreath products of finite groups, T. CECCHERINI-SILBERSTEIN, F. SCARABOTTI & F. TOLLI
- 411 Moduli spaces, L. BRAMBILA-PAZ, O. GARCÍA-PRADA, P. NEWSTEAD & R.P. THOMAS (eds)
- 412 Automorphisms and equivalence relations in topological dynamics, D.B. ELLIS & R. ELLIS
- 413 Optimal transportation, Y. OLLIVIER, H. PAJOT & C. VILLANI (eds)
- 414 Automorphic forms and Galois representations I, F. DIAMOND, P.L. KASSAEI & M. KIM (eds)
- 415 Automorphic forms and Galois representations II, F. DIAMOND, P.L. KASSAEI & M. KIM (eds)
- 416 Reversibility in dynamics and group theory, A.G. O'FARRELL & I. SHORT
- 417 Recent advances in algebraic geometry, C.D. HACON, M. MUSTAŢĂ & M. POPA (eds)
- 418 The Bloch–Kato conjecture for the Riemann zeta function, J. COATES, A. RAGHURAM, A. SAIKIA & R. SUJATHA (eds)
- 419 The Cauchy problem for non-Lipschitz semi-linear parabolic partial differential equations, J.C. MEYER & D.J. NEEDHAM
- 420 Arithmetic and geometry, L. DIEULEFAIT *et al* (eds)
- 421 O-minimality and Diophantine geometry, G.O. JONES & A.J. WILKIE (eds)
- 422 Groups St Andrews 2013, C.M. CAMPBELL *et al* (eds)
- 423 Inequalities for graph eigenvalues, Z. STANIĆ
- 424 Surveys in combinatorics 2015, A. CZUMAJ *et al* (eds)
- 425 Geometry, topology and dynamics in negative curvature, C.S. ARAVINDA, F.T. FARRELL & J.-F. LAFONT (eds)
- 426 Lectures on the theory of water waves, T. BRIDGES, M. GROVES & D. NICHOLLS (eds)
- 427 Recent advances in Hodge theory, M. KERR & G. PEARLSTEIN (eds)
- 428 Geometry in a Fréchet context, C.T.J. DODSON, G. GALANIS & E. VASSILIOU
- 429 Sheaves and functions modulo p , L. TAELEMAN
- 430 Recent progress in the theory of the Euler and Navier–Stokes equations, J.C. ROBINSON, J.L. RODRIGO, W. SADOWSKI & A. VIDAL-LÓPEZ (eds)
- 431 Harmonic and subharmonic function theory on the real hyperbolic ball, M. STOLL
- 432 Topics in graph automorphisms and reconstruction (2nd Edition), J. LAURI & R. SCAPELLATO
- 433 Regular and irregular holonomic D-modules, M. KASHIWARA & P. SCHAPIRA
- 434 Analytic semigroups and semilinear initial boundary value problems (2nd Edition), K. TAIRA
- 435 Graded rings and graded Grothendieck groups, R. HAZRAT
- 436 Groups, graphs and random walks, T. CECCHERINI-SILBERSTEIN, M. SALVATORI & E. SAVA-HUSS (eds)
- 437 Dynamics and analytic number theory, D. BADZIAHIN, A. GORODNIK & N. PEYERIMHOFF (eds)
- 438 Random walks and heat kernels on graphs, M.T. BARLOW
- 439 Evolution equations, K. AMMARI & S. GERBI (eds)
- 440 Surveys in combinatorics 2017, A. CLAESON *et al* (eds)
- 441 Polynomials and the mod 2 Steenrod algebra I, G. WALKER & R.M.W. WOOD
- 442 Polynomials and the mod 2 Steenrod algebra II, G. WALKER & R.M.W. WOOD
- 443 Asymptotic analysis in general relativity, T. DAUDÉ, D. HÄFNER & J.-P. NICOLAS (eds)
- 444 Geometric and cohomological group theory, P.H. KROPHOLLER, I.J. LEARY, C. MARTÍNEZ-PÉREZ & B.E.A. NUCINKIS (eds)
- 445 Introduction to hidden semi-Markov models, J. VAN DER HOEK & R.J. ELLIOTT
- 446 Advances in two-dimensional homotopy and combinatorial group theory, W. METZLER & S. ROSEBROCK (eds)
- 447 New directions in locally compact groups, P.-E. CAPRACE & N. MONOD (eds)
- 448 Synthetic differential topology, M.C. BUNGE, F. GAGO & A.M. SAN LUIS
- 449 Permutation groups and cartesian decompositions, C.E. PRAEGER & C. SCHNEIDER
- 450 Partial differential equations arising from physics and geometry, M. BEN AYED *et al* (eds)
- 451 Topological methods in group theory, N. BROADDUS, M. DAVIS, J.-F. LAFONT & I. ORTIZ (eds)
- 452 Partial differential equations in fluid mechanics, C.L. FEFFERMAN, J.C. ROBINSON & J.L. RODRIGO (eds)
- 453 Stochastic stability of differential equations in abstract spaces, K. LIU
- 454 Beyond hyperbolicity, M. HAGEN, R. WEBB & H. WILTON (eds)
- 455 Groups St Andrews 2017 in Birmingham, C.M. CAMPBELL *et al* (eds)
- 456 Surveys in combinatorics 2019, A. LO, R. MYCROFT, G. PERARNAU & A. TREGLOWN (eds)
- 457 Shimura varieties, T. HAINES & M. HARRIS (eds)
- 458 Integrable systems and algebraic geometry I, R. DONAGI & T. SHASKA (eds)
- 459 Integrable systems and algebraic geometry II, R. DONAGI & T. SHASKA (eds)
- 460 Wigner-type theorems for Hilbert Grassmannians, M. PANKOV
- 461 Analysis and geometry on graphs and manifolds, M. KELLER, D. LENZ & R.K. WOJCIECHOWSKI
- 462 Zeta and L -functions of varieties and motives, B. KAHN
- 463 Differential geometry in the large, O. DEARRICOTT *et al* (eds)
- 464 Lectures on orthogonal polynomials and special functions, H.S. COHL & M.E.H. ISMAIL (eds)

Differential Geometry in the Large

Edited by

OWEN DEARRICOTT
La Trobe University, Australia

WILDERICH TUSCHMANN
Karlsruhe Institute of Technology, Germany

YURI NIKOLAYEVSKY
La Trobe University, Australia

THOMAS LEISTNER
University of Adelaide

DIARMUID CROWLEY
University of Melbourne



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-1-108-81281-8 — Differential Geometry in the Large
Edited by Owen Darricott, Wilderich Tuschmann, Yuri Nikolayevsky, Thomas Leistner,

Diarmuid Crowley
Frontmatter
[More Information](#)

CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India
79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.
It furthers the University's mission by disseminating knowledge in the pursuit of
education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108812818
DOI: 10.1017/9781108884136

© Cambridge University Press 2021

This publication is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without the written
permission of Cambridge University Press.

First published 2021

Printed and bound in Great Britain by Clays Ltd, Elcograf S.p.A.

A catalogue record for this publication is available from the British Library.

ISBN 978-1-108-81281-8 Paperback

Cambridge University Press has no responsibility for the persistence or accuracy
of URLs for external or third-party internet websites referred to in this publication
and does not guarantee that any content on such websites is, or will remain,
accurate or appropriate.

Diarmuid Crowley
 Frontmatter
[More Information](#)

„Bekanntlich setzt die Geometrie sowohl den Begriff des Raumes, als die ersten Grundbegriffe für die Constructionen im Raume als etwas Gegebenes voraus. Sie giebt von ihnen nur Nominaldefinitionen, während die wesentlichen Bestimmungen in Form von Axiomen auftreten. Das Verhältniss dieser Voraussetzungen bleibt dabei im Dunkeln . . .

Diese Dunkelheit wurde auch von Euklid bis auf Legendre . . . weder von den Mathematikern, noch von den Philosophen, welche sich damit beschäftigten, gehoben. Es hatte dies seinen Grund wohl darin, dass der allgemeine Begriff mehrfach ausgedehnter Grössen, unter welchem die Raumgrössen enthalten sind, ganz unbearbeitet blieb.“

From the beginning of Bernhard Riemann's Habilitationsschrift „Ueber die Hypothesen, welche der Geometrie zu Grunde liegen“ (Abhandl. Königl. Ges. Wiss. Göttingen, Vol. 13, 133–150, 1868)

“It is known that geometry assumes, as things given, both the notion of space and the first principles of constructions in space. She gives definitions of them which are merely nominal, while the true determinations appear in the form of axioms. The relation of these assumptions remains consequently in darkness . . .

From Euclid to Legendre . . . this darkness was cleared up neither by mathematicians nor by such philosophers as concerned themselves with it. The reason of this is doubtless that the general notion of multiply extended magnitudes (in which space-magnitudes are included) remained entirely unworked.”

Corresponding passage from William Kingdon Clifford's translation ‘On the Hypotheses which lie at the Bases of Geometry’ (Nature, Vol. 8, 14–17, 1873)

Cambridge University Press

978-1-108-81281-8 — Differential Geometry in the Large

Edited by Owen Darricott , Wilderich Tuschmann , Yuri Nikolayevsky , Thomas Leistner ,

Diarmuid Crowley

Frontmatter

[More Information](#)

Contents

<i>List of Contributors</i>	<i>page</i> xv
<i>Introduction</i>	1
Part One Geometric Evolution Equations and Curvature Flow	9
1 Real Geometric Invariant Theory	
<i>C. Böhm and R.A. Lafuente</i>	11
1.1 Introduction	11
1.2 Examples	16
1.3 Comparison with Complex and Symplectic Case	18
1.4 The Abelian Case	19
1.5 Separation of Closed T-Invariant Sets	23
1.6 The General Case of Real Reductive Groups	25
1.7 Stratification	28
1.8 Properties of Critical Points of the Energy Map	36
1.9 Applications	39
1.10 Appendices	41
1.10.1 Real Reductive Lie Groups	41
1.10.2 The Parabolic Subgroup Q_β	44
<i>References</i>	47
2 Convex Ancient Solutions to Mean Curvature Flow	
<i>T. Bourni, M. Langford and G. Tinaglia</i>	50
2.1 Introduction	50
2.2 Asymptotics for Convex Ancient Solutions	52
2.3 X.-J. Wang's Dichotomy for Convex Ancient Solutions	56

Diarmuid Crowley
 Frontmatter
[More Information](#)

viii

Contents

2.4	Convex Ancient Solutions to Curve Shortening Flow	66
2.5	Rigidity of the Shrinking Sphere	67
2.6	Asymptotics for Convex Translators	68
2.7	X.-J. Wang's Dichotomy for Convex Translators	70
2.8	Rigidity of the Bowl Soliton	71
	<i>References</i>	72
3	Negatively Curved Three-Manifolds, Hyperbolic Metrics, Isometric Embeddings in Minkowski Space and the Cross Curvature Flow	
	<i>P. Bryan, M. Ivaki and J. Scheuer</i>	75
3.1	Introduction	75
3.2	Geometrisation of Three-Manifolds	77
3.3	Embeddability and Hyperbolic Metrics	79
3.4	The Cross Curvature Flow	85
3.4.1	Definition and Basic Properties of the Flow	85
3.4.2	Short Time Existence and Uniqueness	87
3.4.3	Basic Identities and Evolution Equations	90
3.4.4	Towards Hyperbolic Convergence	92
3.4.5	Harnack Inequality and Solitons	94
3.4.6	Monotonicity of Einstein Volume	95
	<i>References</i>	96
4	A Mean Curvature Flow for Conformally Compact Manifolds	
	<i>A.R. Gover and V.-M. Wheeler</i>	98
4.1	Introduction	98
4.2	Conformal Geometry and Hypersurfaces in Conformally Compact Manifolds	101
4.2.1	Conformal Manifolds	102
4.2.2	The Tractor Connection	102
4.2.3	Conformally Compact Manifolds	103
4.2.4	Hypersurfaces	105
4.2.5	A Hypersurface Flow for Conformally Compact Manifolds	106
4.2.6	Boundary Conditions	107
4.2.7	The Flow Problem	107
4.3	The Flow Problem	108
4.3.1	Treating the Flow as a Nonlinear Partial Differential Equation	108
4.3.2	Generalised Mean Curvature Flow in Hyperbolic Space	112
4.3.3	Long Time Existence and Convergence	113

Diarmuid Crowley
 Frontmatter
[More Information](#)

	<i>Contents</i>	
	<i>Contents</i>	ix
4.3.4	Generalised Mean Curvature Flow in Riemannian Manifolds	115
	<i>References</i>	115
5	A Survey on the Ricci Flow on Singular Spaces	
	<i>K. Kröncke and B. Vertman</i>	118
5.1	Introduction and Geometric Preliminaries	118
5.1.1	Isolated Conical Singularities	119
5.1.2	Ricci de Turck Flow and the Lichnerowicz Laplacian	121
5.2	Existence of the Singular Ricci de Turck Flow	122
5.2.1	Tangential Stability	123
5.2.2	The Existence Result	124
5.2.3	Characterizing Tangential Stability	125
5.3	Stability of the Singular Ricci de Turck Flow	127
5.4	Perelman's Entropies on Singular Spaces	128
5.4.1	The λ -Functional	128
5.4.2	The Ricci Shrinker Entropy	129
5.4.3	The Ricci Expander Entropy	129
5.5	Curvature Quantities Along Singular Ricci de Turck Flow	132
5.5.1	Bounded Ricci Curvature Along Singular Ricci de Turck Flow	132
5.5.2	Positive Scalar Curvature Along Singular Ricci de Turck Flow	133
5.6	Open Questions and Further Research Directions	134
5.7	Appendix: Sobolev and Hölder Spaces	134
	<i>References</i>	137
Part Two	Structures on Manifolds and Mathematical Physics	141
6	Some Open Problems in Sasaki Geometry <i>C. Boyer, H. Huang, E. Legendre and C. Tønnesen-Friedman</i>	143
6.1	Introduction	143
6.2	Brief Review of Sasaki Geometry	145
6.2.1	The Sasaki Cone	145
6.2.2	The Transverse Holomorphic Structure	147
6.2.3	The Lie Algebra of Killing Potentials	147
6.3	Extremal Sasaki Geometry	150

Diarmuid Crowley
 Frontmatter
[More Information](#)

x

Contents

6.3.1	Transverse Futaki–Mabuchi	150
6.3.2	The Einstein–Hilbert Functional	153
6.3.3	The Sasaki Energy Functional	155
6.4	The Functionals \mathbf{H}, \mathcal{SE} on Lens Space Bundles Over Riemann Surfaces	161
6.4.1	Explicit Examples	163
	<i>References</i>	166
7	The Prescribed Ricci Curvature Problem for Homogeneous Metrics <i>T. Buttsworth and A. Pulemotov</i>	169
7.1	Introduction	169
7.2	The Prescribed Ricci Curvature Problem	169
7.3	Compact Homogeneous Spaces	171
7.3.1	The Variational Interpretation	172
7.3.2	Maximal Isotropy	173
7.3.3	The Structure Constants	173
7.3.4	The Scalar Curvature Functional and its Extension	174
7.3.5	Non-Maximal Isotropy: The First Existence Theorem	176
7.3.6	Non-Maximal Isotropy: The Second Existence Theorem	178
7.3.7	The Case of Two Isotropy Summands	179
7.3.8	Homogeneous Spheres	181
7.3.9	Further Examples	183
7.3.10	Ricci Iterations	185
7.4	Open Questions and Non-Compact Homogeneous Spaces	185
7.4.1	The Non-Compact Case	186
7.4.2	Unimodular Lie Groups of Dimension 3	187
	<i>References</i>	190
8	Singular Yamabe and Obata Problems <i>A.R. Gover and A.K. Waldron</i>	193
8.1	Introduction	193
8.2	Background and a Singular Obata Problem	195
8.3	Tractor Calculus for Hypersurface Embeddings	201
8.3.1	The Sphere	209
8.4	Singular Yamabe and Obata Problems	211
	<i>References</i>	212

Diarmuid Crowley
 Frontmatter
[More Information](#)

Contents xi

9 Einstein Metrics, Harmonic Forms and Conformally Kähler Geometry	<i>C. LeBrun</i>	215
9.1 Introduction		215
9.2 An Integral Weitzenböck Formula		220
9.3 Some Almost-Kähler Geometry		228
9.4 The Main Theorems		235
<i>References</i>		238
10 Construction of the Supersymmetric Path Integral: A Survey	<i>M. Ludewig</i>	241
10.1 Introduction		241
10.2 First Construction: The Top Degree Functional		244
10.3 Second Construction: The Chern Character		249
10.4 Bismut–Chern Characters, Entire Chains and the Localization Formula		253
<i>References</i>		257
11 Tight Models of de-Rham Algebras of Highly Connected Manifolds	<i>L. Schwachhöfer</i>	260
11.1 Introduction		260
11.2 Rational and Weak Equivalence		263
11.3 Poincaré DGCAs and DGCAs of Hodge Type		265
11.4 Small Algebras of Hodge Type DGCAs		270
11.5 Tight DGCAs of Highly Connected DGCAs		273
11.6 The Bianchi–Massey Tensor		277
<i>References</i>		280
Part Three Recent Developments in Non-Negative Sectional Curvature		283
12 Fake Lens Spaces and Non-Negative Sectional Curvature	<i>S. Goette, M. Kerin and K. Shankar</i>	285
12.1 Introduction		285
12.2 \mathbb{Z}_ℓ Actions on the Family \mathcal{F}		286
<i>References</i>		290
13 Collapsed 3-Dimensional Alexandrov Spaces: A Brief Survey		
<i>F. Galaz-García, L. Guijarro and J. Núñez-Zimbrón</i>	291	
13.1 Introduction		291
13.2 Basic Alexandrov Geometry		293
13.3 Three-Dimensional Alexandrov Spaces		298

Diarmuid Crowley

Frontmatter

[More Information](#)

xii

Contents

13.3.1	Geometric 3-Alexandrov Spaces	301
13.3.2	Geometrization of 3-Alexandrov Spaces	301
13.4	Collapsed Three-Dimensional Alexandrov Spaces	302
13.4.1	General Structure Results	302
13.4.2	Geometrization of Sufficiently Collapsed Three-Dimensional Alexandrov Spaces	307
	<i>References</i>	308
14	Pseudo-Angle Systems and the Simplicial Gauss–Bonnet–Chern Theorem	
	<i>S. Klaus</i>	311
14.1	Introduction	311
14.2	The Simplicial Gauss–Bonnet–Chern Theorem	313
14.3	Systems of Pseudo-Angles	316
14.4	Combinatorial Riemannian Manifolds	319
14.5	Simplicial Sectional Curvature	320
14.6	Simplicial Sectional Curvature and the Hopf Conjecture	322
	<i>References</i>	325
15	Aspects and Examples on Quantitative Stratification with Lower Curvature Bounds	
	<i>N. Li</i>	326
15.1	Introduction	326
15.2	Stratification of Singular Sets	328
15.3	Quantitative Stratification	330
15.3.1	Definitions	330
15.3.2	Results	332
15.4	Key Ingredients and Framework	335
15.4.1	Monotonicity Formula and Bad Scales	335
15.4.2	Splitting Theory	338
15.4.3	Dimension Reduction	340
15.4.4	Good-Scale Annuli Covering	341
15.5	Spaces whose Singular Sets are Cantor Sets	343
	<i>References</i>	350
16	Universal Covers of Ricci Limit and RCD Spaces	
	<i>J. Pan and G. Wei</i>	352
16.1	Introduction	352
16.2	Some Properties of Ricci Limit and RCD Spaces	354
16.3	Universal and δ -Covers	359
16.4	Non-Collapsing Ricci Limit Spaces	364
	<i>References</i>	369

Diarmuid Crowley
Frontmatter
[More Information](#)

Contents xiii

17 Local and Global Homogeneity for Manifolds of Positive Curvature	<i>J.A. Wolf</i>	373
17.1	Introduction	373
17.2	The Classification for Positive Curvature	375
17.3	Positive Curvature and Isotropy Splitting	377
17.4	The Three Remaining Positive Curvature Cases	378
17.5	Dropping Normality in Positive Curvature	380
	<i>References</i>	383

Cambridge University Press

978-1-108-81281-8 — Differential Geometry in the Large

Edited by Owen Darricott , Wilderich Tuschmann , Yuri Nikolayevsky , Thomas Leistner ,

Diarmuid Crowley

Frontmatter

[More Information](#)

Contributors

- Christoph Böhm *University of Münster, Einsteinstraße 62, 48149 Münster, Germany*
- Theodora Bourni *Department of Mathematics, University of Tennessee Knoxville, Knoxville, TN, 37996-1320, USA*
- Charles P. Boyer *Department of Mathematics and Statistics, University of New Mexico, Albuquerque, NM 87131, USA*
- Paul Bryan *Department of Mathematics, Macquarie University NSW 2109, Australia*
- Timothy Buttsworth *Department of Mathematics, Cornell University, Ithaca, NY 14853, USA*
- Fernando Galaz-García *Department of Mathematical Sciences, Durham University, Lower Mountjoy, Stockton Road, Durham DH1 3LE, UK*
- Sebastian Goette *Abteilung für Reine Mathematik, Universität Freiburg, Ernst-Zermelo-Straße 1, D-79104 Freiburg, Germany*
- A. Rod Gover *Department of Mathematics, The University of Auckland, Private Bag 92019, Auckland 1142, New Zealand*
- Luis Guijarro *Departamento de Matemáticas, Facultad de Ciencias, Universidad Autónoma de Madrid, 28049 Cantoblanco, Madrid, Spain*
- Hongnian Huang *Department of Mathematics and Statistics, University of New Mexico, Albuquerque, NM 87131, USA*
- Mohammad N. Ivaki *Department of Mathematics, University of Toronto, Ontario, M5S 2E4, Canada*
- Martin Kerin *School of Mathematics, Statistics and Applied Mathematics, National University of Ireland Galway, University Road, Galway, Ireland, H91 TK33*
- Stephan Klaus *Mathematisches Forschungsinstitut Oberwolfach, Schwarzwaldstrasse 9-11, D-77709 Oberwolfach-Walke, Germany*
- Klaus Kröncke *Fachbereich Mathematik, Bereich AD, Bundesstraße 55, 20146 Hamburg, Germany*

- Ramiro A. Lafuente *School of Mathematics and Physics, The University of Queensland, St Lucia, QLD 4072, Australia*
- Mat Langford *Department of Mathematics, University of Tennessee Knoxville, Knoxville, TN 37996-1320, USA*
- Claude LeBrun *Department of Mathematics, Stony Brook University, Stony Brook, NY 11794-3651, USA*
- Eveline Legendre *Institut de Mathématiques de Toulouse, Université Paul Sabatier, 118 route de Narbonne 31062, Toulouse, France*
- Nan Li *Department of Mathematics, The City University of New York – NYC College of Technology, 300 Jay St., Brooklyn, NY 11201, USA*
- Matthias Ludewig *Fakultät für Mathematik, Universität Regensburg, 93040 Regensburg, Germany*
- Jesús Núñez-Zimbrón *Centro de Ciencias Matemáticas UNAM, Antigua Carretera a Pátzcuaro 8701, Col. Ex Hacienda San José de la Huerta, Morelia, Michoacán, C.P. 58089, México*
- Jiayin Pan *Department of Mathematics, University of California Santa Barbara, Santa Barbara, CA 93106, USA*
- Artem Pulemotov *School of Mathematics and Physics, The University of Queensland, St Lucia, QLD 4072, Australia*
- Julian Scheuer *Department of Mathematics, Columbia University New York, NY 10027, USA*
- Lorenz Schwachhöfer *Fakultät für Mathematik, TU Dortmund University, Vogelpothsweg 87, 44221 Dortmund, Germany*
- Krishnan Shankar *Department of Mathematics, University of Oklahoma, 601 Elm Avenue, Room 423, Norman, OK 73019-3103, USA*
- Giuseppe Tinaglia *Department of Mathematics, King's College London, London WC2R 2LS, UK*
- Christina W. Tønnesen-Friedman *Department of Mathematics, Union College, Schenectady, NY 12308, USA*
- Boris Vertman *Carl von Ossietzky Universität Oldenburg, Institut für Mathematik, Carl-von-Ossietzky-Str. 9-11, D-26129 Oldenburg, Germany*
- Andrew K. Waldron *Center for Quantum Mathematics and Physics, Department of Mathematics, University of California Davis, Davis, CA 95616, USA*
- Guofang Wei *Department of Mathematics, University of California Santa Barbara, Santa Barbara, CA 93106, USA*
- Valentina-Mira Wheeler *School of Mathematics and Applied Statistics, University of Wollongong, Northfields Ave Wollongong, NSW 2522, Australia*
- Joself A. Wolf *Department of Mathematics, University of California, Berkeley, CA 94720-3840, USA*