CHAPTER 1

Logical Pluralism Introduced

Logical pluralists argue that there is more than one correct logic. Initially, this may look like an overwhelmingly plausible claim. Given that even basic logic textbooks introduce a number of different logical systems like, for instance, propositional logic and first-order predicate logic, it seems rather bold to deny that there is a plurality of logics. What is more, at least prima facie all these systems can be taken to be correct in some sense or the other. They may be sound and complete with respect to a suitable semantics, for example.

It is important to point out, therefore, that in its recent formulations, logical pluralism is supposed to be a much more exciting idea. The systems that logical pluralists usually take to be correct – classical logic, intuitionistic logic, and some relevant logic, among others – are typically considered to be *rival* accounts of logical consequence. Partisans of the logics in question tend to think that there is something wrong with the other accounts. Intuitionists oppose the idea that the "law" of excluded middle, $\vDash A \vee \neg A$, is in fact a law of logic. Classical logicians insist that it is. Relevant logicians reject classical validity on the basis that in classically valid arguments, the conclusions do not always follow from their premises. Classical logicians reject relevance constraints as overly restrictive. The pluralist proposal that all those, supposedly, rivalling accounts can be correct is therefore quite surprising.

These initial observations suggest two perspectives on logical pluralism. First, it can be seen as a conciliatory approach to the debate between advocates of different logical systems like the ones just mentioned. Conciliatory pluralism can be motivated by the diversity of formal systems and proof patterns actually used in everyday logical or mathematical practice. If different notions of validity can be extracted from natural language use and if a variety of formal structures relying on different background logics are legitimate parts of mathematics, then this seems to put pressure on the traditional view that there is only one correct answer to the question of

whether or not a specific argument is valid. After all, the rejection of pluralistic approaches to these matters seems to suggest that a considerable part of logical and mathematical practice is misguided. On this reading, pluralism provides a philosophical justification for the status quo in the deductive disciplines.

But, secondly, pluralism can also be characterized as a revisionary view. Revisionist pluralism challenges the philosophical orthodoxy according to which an argument is either deductively valid or invalid, period. The claim that there is more than one correct logic is in clear conflict with the monist assumption that there is exactly one correct logic. In line with these perspectives, the pluralist position is sometimes painted as a defence of practising logicians and mathematicians against philosophers' dogmatism. On this reading, pluralism amounts to a thesis about the nature of logical consequence and to the rejection of competing philosophical positions. Pluralists reject logical monism – understood as the view that there is exactly one correct logical theory – as well as logical nihilism – understood as the view that there is no correct logical theory at all.

There are quite a number of potential upshots of pluralism in this revisionary reading. The claim that there is no unequivocal answer to whether or not a given argument form is deductively valid – and this does not mean valid in some system but valid per se – runs against large parts of the philosophical tradition. Logicians typically considered the existence of counterexamples to be sufficient to establish the invalidity of an argument form, but on the pluralist approach this is no longer the case. According to the pluralist, some instances of a given argument form may be such that they provide a genuine counterexample to one logic but not to another, even if those logics share the same logical vocabulary.

Also, at least some of the logics that are considered to all be correct by the pluralist were taken to deliver rivalling accounts of logical consequence. If the pluralist is right, then these logics are, in fact, perfectly compatible and can live together as one happy family. But if that is the case, why were the partisans of these logics confused about the apparent conflict between their theories?

Finally, to name just one further prospect, logic was thought to at least aspire to provide a safe basis for reasoning. The underlying idea is that whenever we are confronted with a deductively valid argument, we know that a false conclusion will never follow from true premises, come what may. Logical pluralism puts pressure on that view. If arguments can be valid in one sense but not in another or if arguments may be deductively valid in one domain of discourse but not in another, it is no longer obvious

that we can rely on logics in this way – even on the assumption that they are correct.

These cursory considerations suggest that logical pluralism in its revisionary reading is an exciting thesis that is worth our attention. But what should we make of it? One of the main claims to be defended in this book is that logical pluralism in the revisionary philosophical reading ought to be resisted. I argue that pluralism in this sense is an unstable position that has quite a number of problematic consequences. All things considered, the view is considerably less attractive than pluralists suggest. Now, if there is no plurality of correct logics, why maintain that there is any correct logic at all? I argue that even though some of the diagnoses offered by logical nihilists are quite convincing, the view is ultimately implausible since it relies on questionable assumptions about what logical theories ought to achieve. This leaves logical monism as the most plausible view when it comes to logics understood as theories of the deductive validity of arguments – or so I argue.

Another central claim is that, contrary to the pluralist's contention, monism about logical consequence has no detrimental consequences for the practice of logical or mathematical theory building. On a suitable understanding of the terms *logic* and *correctness,* monism can be reconciled with the plurality of logical systems being employed in logical and mathematical practice. The upshot is that monists can account for the motivations of conciliatory pluralism even if revisionist pluralism is rejected. There can be more than one correct logic in a sense that is still consistent with logical monism. The result is a monist account of logical consequence that comes with the appeal of pluralism while avoiding its problems.

1.1 Early Pluralistic Accounts of Logic

Pluralistic views about logic are many and varied. Accordingly, the first challenge for a fruitful discussion of logical pluralism is to get a clear understanding of the family of theories at issue. Pluralistically minded approaches to logic have been ascribed to logicians like Hugh McColl and, more prominently, Rudolf Carnap (see Rahman & Redmond 2008, Restall 2002, Russell 2019b), but the mother of modern logical pluralism is, arguably, Susan Haack. In her monograph *Philosophy of Logics* (1978), she not only defends the first explicitly pluralistic account of logic, she also develops important terminological and methodological distinctions that helped to shape the current debate.

Haack discriminates positions in terms of their answers to three basic questions concerning the metaphysical status of logic (see Haack 1978, 225). First, can logical systems be correct in the sense that arguments that are valid within the formal system correspond to informal arguments that are valid in some extra-systematic sense? According to Haack, monists and pluralists about logic both give an affirmative answer to the first question. A negative answer amounts to what Haack calls *instrumentalism* about logic. But while monists and pluralists agree about the first question, they disagree about the second: is there exactly one such logic? The monist's answer is affirmative, the pluralist, however, insists that there are at least two correct logics. Haack goes on to distinguish two kinds of pluralism in terms of their answer to the third question: are logical principles completely general? According to local logical pluralists, the answer is no – different logics are applicable to different areas of discourse. On that view, the plurality of logics is a result of a plurality of domains that require deductive reasoning. Global logical pluralists, on the other hand, maintain that logic is, in fact, completely general. On their view, the plurality of logics results primarily from a plurality of meanings of central logical concepts (see Haack 1978, 223).

Haack tentatively defends a version of the latter, global version of logical pluralism. According to her, informal arguments can be adequately represented by more than one logical system if those systems assign different but similar meanings to the logical vocabulary. Even though I argue that this view is ultimately not convincing, Haack's remarks on logical pluralism lay the groundwork for many of the conceptual distinctions made in this book. Two of the three dimensions of plurality discussed in Chapter 3 – namely, the applicational dimension and the sematic dimension – are directly inspired by the distinctions sketched above. The discussion in Chapter 4 relies essentially on Haack's terminology.

1.2 The Generalized Tarski Thesis

The more recent discussion of logical pluralism was initiated by Jc Beall and Greg Restall in a series of papers (2000, 2001, 2002) and a book (2006). Like Haack, they defend a version of global logical pluralism. At the core of their pluralism is a conception of validity as necessary truth-preservation. It is based on the Tarskian model-theoretic view, according to which a conclusion follows logically from the premises just in case every model of the premises is also a model of the conclusion (Tarski 1936). Beall and Restall

generalize that thought by replacing the *models* in Tarski's definition with *cases*, resulting in their *Generalized Tarski Thesis* (GTT) (Beall & Restall 2006, 29):

GTT An argument is valid$_x$, if and only if, in every case$_x$ in which the premises are true, so is the conclusion.

A case is any entity in which claims may be true (see Beall & Restall 2006, 89), so Tarskian models remain as a special case of GTT. Crucially, however, different consequence relations may emerge depending on the type of cases under consideration. In combination with the acceptance of more than one type of case, GTT yields logical pluralism. As far as Beall and Restall are concerned, a number of types of cases qualify as admissible instances of GTT. Tarskian models yield classical logic, situations yield relevant logic, and stages yield intuitionistic logic.

In principle, many other types of cases would satisfy GTT as well. One might, for instance, restrict the class of cases to only the actual case, @. This would yield a notion of validity$_@$ according to which only arguments with an actually true conclusion or at least one actually false premise would count as valid$_@$. To most, this outcome would not be very attractive. Beall and Restall avoid this and other unorthodox results by imposing additional constraints on consequence relations. In order for a consequence relation to count as an admissible instance of GTT, its judgements about consequence need to be *necessary, normative,* and *formal* (cf. Beall & Restall 2006, 35).

Beall and Restall rely on two argument types to support their pluralistic approach. The first is the argument from appearance, which departs from the observation that there appear to be at least two different senses of *validity.* For instance, there seems to be a sense in which the argument from $A \wedge \neg A$ to arbitrary B is *valid:* there is no consistent case in which $A \wedge \neg A$ is true and B not true. But there is also a sense in which the argument is *invalid: A* does not *follow from* the premises since the conclusion B is not related in any way to the premise $A \wedge \neg A$. According to Beall and Restall the upshot is as follows: "provided that each of the noted senses of 'validity' corresponds to an admissible instance of GTT, there are at least two relations of logical consequence (in English), and so logical pluralism follows." (2006, 31). So Beall and Restall's pluralism is directly linked to conceptual considerations involving the natural language concept of *validity.* The general strategy of their book is to show that at least two senses of that concept satisfy the desiderata on logicality sketched above, that is, they are instances of GTT that are necessary, formal, and normative. So there is a direct part of the argument of appearance that takes broadly conceptual

intuitions at face value and there is a defensive part of the arguments that provides a justification for why those conceptual intuitions are to be taken seriously.

The second general argument highlights the advantages of logical pluralism when compared with competing accounts – Russell (2019b) calls it *the argument from virtue*. Beall and Restall contend, first, that their view comes "at little or no cost" and, second, that pluralism offers a more charitable interpretation of debates in the philosophy of logic than competing views like logical monism (2006, 31).

The two argument types reflect the two readings of pluralism mentioned earlier. The argument from appearance follows the conciliatory line, and the argument from virtue is part of the revisionist strategy. Accordingly, I address them in different ways. First, I submit that the argument from appearance does not commit us to a substantial form of logical pluralism (see Chapters 2 and 4). Secondly, I argue that the virtues of logical pluralism are overstated: the view has quite substantial costs (see Chapter 8), and it is not more charitable than competing views (see Chapters 6 and 7).

1.3 Relativity to Legitimate Mathematical Structures

The second book length defence of logical pluralism is given by Stewart Shapiro. In his monograph *Varieties of Logic* (2014b), he argues that "there are different, mutually incompatible, but equally legitimate ways to sharpen or further articulate the intuitive notion(s) of logical consequence and validity" (Shapiro 2014b, 1–2). The central elements of his view are a *folk-relativism* about logic and a *Hilbertian approach* to the legitimacy of mathematical structures (Shapiro 2014b, 63–67). Combined, these ingredients amount to what I take to be the most permissive pluralism that is currently available.

The folk-relativistic part of the view relativizes validity and logical consequence to mathematical structures. Shapiro gives three examples of intuitionistic mathematical theories – Heyting arithmetic augmented with all instances of Church's thesis, intuitionistic analysis, and smooth infinitesimal analysis – that are inconsistent with full-strength classical logic (cf. Shapiro 2014b, 67–75). In order to avoid triviality when working with these mathematical theories, a logic weaker than classical logic, more specifically, a logic that does not have the law of excluded middle as one of its theorems is required. Classical logic, however, is perfectly consistent with other, more standard mathematical structures. In fact, classical principles may very well be needed to establish some central results of the theory

in question. According to Shapiro, these observations support folk relativism about logic. Different mathematical structures may require different logics.

The Hilbertian approach completes the pluralistic nature of the view. Recall that pluralism about a given subject X is the view that different accounts of X are equally correct – or equally good or equally legitimate. On the Hilbertian view, consistency is *the only* formal criterion for legitimacy. So if a proposed theory is consistent, "then there is no further metaphysical hoop the proposed theory must jump through before being legitimate mathematics" (Shapiro 2014b, 67). Now, if there are consistent structures that require different logics, then the combination of folk relativism and the Hilbertian approach yields logical pluralism. As Shapiro points out, accounting for paraconsistent logics may require even less demanding criteria like non-triviality. The result is the same in both cases: there is more than one logical theory that is correct in virtue of being legitimate.

This accounts for one of the main motivations of Shapiro's proposal, which is to "show how a wide variety of theories, studied by mathematicians whose credentials can hardly be challenged, are legitimate" (2014b, 38). The way I understand him, the primary focus of Shapiro's pluralism is, thus, on the conciliatory reading of logical pluralism. The revisionary, anti-monistic parts of his view seem to be based on the assumptions that monism about classical logic would amount to the claim that important parts of intuitionistic mathematics are illegitimate and that monism about weaker logics would result in hopelessly weak metatheories. I argue that these worries are unwarranted (see, e.g., Chapters 2 and 5). To a certain extent, a conciliatory perspective on seemingly incompatible logics is perfectly consistent with monism about logical consequence.

1.4 The Plurality Thesis

There is an increasing number of further formulations of logical pluralism, but despite differences in focus and in the analysis of logical consequence they share most of the motivations just sketched.[1] Crucially, all versions of logical pluralism share a common theme or core tenet: the thesis that there is more than one correct logic. I call this generic claim *the plurality thesis*, and I take it to be the characterizing feature of logical pluralism. Different

[1] See, among others, Bueno and Shalkowski 2009, Caret 2017, Cook 2010, Field 2009, Hjortland 2013, Kouri Kissel 2018, Pedersen 2014, Restall 2014, Russell 2008, Terrés Villalonga 2020, 2019, Varzi 2002.

kinds of pluralism emerge with different implementations of that thesis and, most notably, of its key components *logic* and *correctness*.

As I suggested in the opening paragraphs, some readings of the plurality thesis are completely uncontroversial. If *logic* refers to any well-defined formal system consisting of a (formal) language and a consequence relation and if *correctness* means that the system in question meets some formal requirements like soundness and completeness with respect to its semantics, then it seems very hard indeed not to be a pluralist. If, on the other hand, a *logic* is taken to be a theory of a specific subject matter – for instance, of deductive validity – and if such a theory is *correct* if it gives the right account of its subject matter, then logical pluralism may turn out to be a more controversial position.

Despite its generality, I propose that endorsement of the plurality thesis is sufficient for logical pluralism. This may seem exceedingly liberal at first sight, but it does give us the flexibility to account for all pluralist positions found in the current literature. It saves us from having to regiment the use of the term "logical pluralism" from the outset, but allows us to impose further conditions on more interesting variants of the view as we go along.

1.5 Outline of the Book

Given the relatively recent development of logical pluralism, it may not come as a surprise that – apart from endorsement of the plurality thesis – it is still an open question what exactly logical pluralism amounts to. While quite a bit of helpful terminological distinctions have been drawn already (see, e.g., Beall & Restall 2006, Caret 2021, Cook 2010, Haack 1978, Kouri Kissel 2018, Pedersen 2014, Shapiro 2014b), there still remains work to be done on the theoretical and the methodological foundations of logical pluralism and its competitors. This is the central aim of Chapter 2. I defend a conception of logic as concerned with the validity of arguments. I then distinguish three senses of *logic:* (i) the purely formal machinery, (ii) interpreted logical theories, and (iii) the subject matter of those theories. The plurality thesis can be understood in either way but not all resulting forms of pluralism are equally interesting.

Apart from drawing the conceptual distinctions that form the basis of the discussion throughout the book, the chapter also argues that the idea of a system-independent notion of logical consequence in itself does not involve substantial metaphysical assumptions. I show that it is, in fact, compatible with a broad range of views about the metaphysical status of logical consequence.

With the notion of extra-systematic logical consequence in place, correctness conditions for a logical theory can then be construed in terms of a relation between the arguments classified as valid in the theory, on the one hand, and the extra-systematic arguments they are supposed to capture, on the other. Depending on the constraints imposed on that relation, different conceptions of the correctness of a logic emerge: a weak and a strong version of the correspondence view of correctness (Cook 2010, Haack 1978) and the logic-as-modelling view (Cook 2010, Shapiro 2006, 2014b).

The distinctions drawn in the chapter allow me to considerably narrow down the implementations of the plurality thesis that are of interest in this book. I focus on a reading according to which there is more than one logical theory that gives a correct account of which arguments are deductively valid. This is the only reading that qualifies for the revisionist approach to logical consequence.

Even this restriction still leaves surprisingly many options for logical pluralism. In Chapter 3, I show that plurality may arise on three dimensions. The applicational, the semantic, and the metaphysical dimensions enable pluralism about domains of applications, about the meaning of logical vocabulary, or about extra-systematic validity, respectively. To a certain extent, those dimensions are independent. One may be a pluralist about one dimension but not about another. The following three chapters (Chapters 4 through 6) constitute separate arguments to the effect that, as far as deductively valid arguments are concerned, the most plausible position on all three dimensions is monism.

I argue for the view that there is exactly one notion of extra-systematic consequence that constitutes the genuinely logical application of logical theories and, further, for the claim that there is exactly one logical theory with an invariant semantic meaning that correctly captures this extra-systematic notion. Given the distinctions drawn in earlier chapters, this still leaves room for a plurality of fruitful applications of logical systems – as long as they are not understood as theories of logical consequence.

Chapter 4 shows that pluralism about logical theories is compatible with different views about the cardinality of extra-systematic logical consequence. For instance, depending on the notion of correctness invoked, pluralism is available even on the assumption that there is exactly one extra-systematic notion of logical consequence. This emphasizes the importance of the underlying notions of correctness (introduced in Chapter 2) in the formulation of the plurality thesis. I discuss possible combinations of views about the nature of logical consequence and correctness conditions imposed on logical theories. I then identify the main targets of the

remaining discussion in the book: (i) *genuine plurality,* that is, the view that there is more than one extra-systematic notion of logical consequence and (ii) a more modest methodological view that acknowledges monism about extra-systematic consequence but argues that there cannot be a single precise theory that captures this relation, for principled reasons. I also provide arguments against logical nihilism, which will be expanded upon in Chapter 5. I close the chapter by outlining an interpretation of the logic-as-modelling view that commits to monism about extra-systematic logical consequence as well as to monism about logical theories.

The remaining chapters constitute arguments against revisionist logical pluralism which are, at the same time, arguments in favour of the outlined monist perspective on logical consequence. Chapter 5 is concerned with the applicational dimension of plurality. I defend the universal applicability of logic against arguments to the effect that logic is domain dependent. I first consider direct arguments for domain-dependence and show that they are inconclusive. In particular, I focus on the argument from mathematical practice sketched above and on the argument from alethic pluralism as defended by Crispin Wright (1994), Michael Lynch (2009) and others. I then discuss indirect arguments for domain-dependence in the form of arguments against the universal applicability of logic and argue that they fail as well. I close by highlighting open problems for domain-dependent pluralism.

The discussion leaves an important interpretation of logical pluralism untouched, namely pluralism about the meaning of logical vocabulary. In Chapter 6, I argue that the revisionist pluralist views typically come with a commitment to a plurality of meanings within the extra-systematic language. The central aim of the chapter is to show that the linguistic mechanisms postulated by logical pluralists – ambiguity, polysemy, vagueness, or context sensitivity – give highly implausible accounts of both the meaning of the connectives and the meaning of the metalogical vocabulary like *valid* or *is a consequence of*. I conclude that an invariantist semantics for the logical vocabulary is the most plausible alternative.

Addressing those pluralists willing to bite the bullet on those linguistic arguments, Chapter 7 discusses the phenomenon of disagreement about logic and the pluralists' inability to account for it both semantically and pragmatically. I argue that no available pluralistic theory can account for both plurality and disagreement on any plausible theory of meaning. The chapter highlights some well-known and some less well-known problems for pluralists and dismisses the defensive strategies proposed by some authors. The moral to draw is that the charity-based strategy underlying the