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THEME 1 Numbers and numeration

TERM 1

Topic 1: Number systems

In this topic, we are going to explore different number systems, work with the binary number system and revise basic operations, as well as convert binary numbers to other bases.

Unit 1: Different number bases

Base 10 is the most commonly used base in the modern world, but there are also other **number bases**, for example, base 4, base 5, and so on.

Base 10 number system

A number written in base 10 is a combination of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The two digits 3 and 4 can be combined as 43 or 34, which gives us two different numbers. The value of 4 in 43 is different from the value of 4 in 34. The value of 4 in 43 is four tens, while the value of 4 in 34 is four ones or units.

In the number system that we use today, the position of a digit in any number is very important and determines the value of that digit in the number. Our number system is a place value system.

Do you still remember that, in the base 10 number system, we refer to Th, H, T and U? This means Thousands, Hundreds, Tens and Units. The table below sets out these values.

Thousands	Hundreds	Tens	Units
1 000 10^3	100 10^2	10 10^1	1 10^0

Any base 10 number can be inserted into this system, so that we can understand the meaning (value) of each digit. We will use the number 3 456 in the table below to illustrate this.

1 000 (10^3)	100 (10^2)	10 (10^1)	1 (10^0)
3	4	5	6

This means $3 \times 10^3 + 4 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$.

We can expand this number by writing it this way:

$$3\ 456 = (3 \times 10^3) + (4 \times 10^2) + (5 \times 10^1) + (6 \times 10^0)$$

Other number bases

Counting can be done in bases other than base 10, such as base 3, base 4, base 5, and so on. The number of digits in any base must be the same as the base number and must include 0. For example, the digits in base 3 are 0, 1 and 2. We can write number bases in two ways, for example 43_5 or 43_{five}.

To write a number in any of the bases, we can only use a combination of the digits for that base, taking into account the place value of each digit. We can construct a place value table for any base. To find the value of any number in any other base, we first expand the number in the given base, for example: $43_5 = (4 \times 5) + 3$.

Fives	Units
5^1	5^0
4	3

$\Rightarrow 43_5 = (4 \times 5^1) + (3 \times 5^0)$

Examples

Use a place value table to expand these numbers.

1. 312_5
2. $2\ 463_{\text{seven}}$
3. $1\ 120_8$

Solutions

1.

25	5	1
5^2	5^1	5^0
3	1	2

$312_5 = 3 \times 25 + 1 \times 5 + 2 \times 1 = 82$

2. $2\ 463_{\text{seven}}$ is the same as $2\ 463_7$.

343	49	7	1
7^3	7^2	7^1	7^0
2	4	6	3

$2\ 463_7 = 2 \times 343 + 4 \times 49 + 6 \times 7 + 3 \times 1 = 927$

3.

512	64	8	1
8^3	8^2	8^1	8^0
1	1	2	0

$1\ 120_8 = 1 \times 512 + 1 \times 64 + 2 \times 8 + 0 \times 1 = 592$

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Exercise 1

1. Write down the base of each number.

a) 5_6	b) 15	c) 11	d) 17_8	e) 28_{nine}
f) 49_{12}	g) 14	h) 11_2	i) 46_{seven}	j) 89
k) 211_3	l) $3\,122_4$	m) 434_7	n) 3 006	o) 625_5

2. Expand the numbers in their respective bases.
   a) 11_5 b) 211_3 c) $3\,122_4$ d) $51\,361_7$ e) $1\,111_9$

3. Write down the value of the digit 5 in these numbers.
   a) 105_8 b) 512_7 c) 125   d) 1 520

4. Write $3\,245_6$ in the base 6 place value system. Explain the meaning of each digit.

5. Write $51\,362_7$ in the base 7 place value system. Explain the meaning of each digit.

Conversion from base 10 to other bases

To convert a number in base 10 to a number in any other base, we carry out successive division of the number by the given base, retaining any remainder at each stage. We continue dividing until the result we have is less than the base, since dividing further will give 0.

Example

Convert 13_{10} to a number in base 5.

Solution

Base	Number	Number ÷ base	Remainder
5	13	2	3
5	2	0	2
	0		

Read the number in the direction of the arrow: "2 remainder 3".
⇒ two groups of 5 and a remainder of 3
$= 2 \times 5 + 3$
$\Rightarrow 13_{10} = 23_5$

Exercise 2

1. Change these numbers to base 8 numbers.
   a) 66   b) 54   c) 77   d) 778   e) 999

2. Change each number in Question 1 to a base 6 number.

3. Change each number in Question 1 to a base 5 number.

4. Change each number in Question 1 to a base 7 number.

Do Worksheet 1 in the JSS 3 Workbook.

Unit 2: The binary number system

Historians believe that the **binary number system** was invented by an Indian mathematician called Pingala. The mathematician Gottfried Leibnitz then further developed the system, by explaining how to use the number system in more detail.

The binary number system, also called the base 2 number system, has two digits to represent all values, namely 0 and 1. Binary numbers are used in computers, representing when digital circuits are either on (1) or off (0).

Binary digits are sometimes called bits, which evolved from the first and last two letters of the two words "binary digits".

The table below sets out these values. Note that the place values begin with 1 and are multiplied by 2 as you move to the left. We only show the place value columns up to 32, but they do extend further to the left.

32	16	8	4	2	1
2^5	2^4	2^3	2^2	2^1	2^0

Converting base 10 numbers to binary numbers (and vice versa)

Converting base 10 numbers to binary numbers (and vice versa) is fairly simple. Remember that each digit in the binary number represents a power of two.

Examples

1. Convert 73 to a binary number.
2. Convert 01001 to a base 10 number.

Solutions

1.

64	32	16	8	4	2	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	0	1	0	0	1

First, find the largest power of 2 that is less than 73, which is 64. Then place a 1 in the 64 column.

Subtract 64 from 73, which is 9. The largest power of 2 that is less than 9, is 8. Place a 1 in the 8 column. Place 0 in the 2^5 and 2^4 columns.

Subtract 8 from 9, which is 1. Place 1 in the 2^0 column and 0 in 2^2 and 2^1 columns. Therefore,

$73 = (1 \times 64) + (1 \times 8) + (1 \times 1) = 1001001$

Another way to convert a decimal number to a binary number is by repeated division. To get the binary number for a given decimal number, divide the decimal number by 2 until the quotient is 0. The remainders form the binary number.

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2	73	R
2	36	1
2	18	0
2	9	0
2	4	1
2	2	0
2	1	0
2	0	1

Read the number from bottom to top.

Note: the last quotient must be zero.

2. First, write down the digits of the binary number. Write powers of 2 for each digit in the binary number from left to right.

16	8	4	2	1
2^4	2^3	2^2	2^1	2^0

Write each binary digit below its corresponding power of 2.

16	8	4	2	1
2^4	2^3	2^2	2^1	2^0
0	1	0	0	1

Write down the final value by adding up all the powers of 2 where there is a 1.

$= 0 \times 16 + 1 \times 8 + 0 \times 4 + 0 \times 0 + 1 \times 1$

$= 9$

Converting binary numbers to other bases

We can also convert binary numbers to other number bases.

Examples

1. Convert 325_8 to a binary number.
2. Convert 214_6 to a binary number.

Solutions

1. Numbers to the base 8 (also called octal numbers) are closely related to binary numbers. Start by converting each digit in the base 8 number to its 3-digit binary equivalent.

   3 = 011

   2 = 010

   5 = 101

   Therefore, 325_8 = 011 010 101.

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2. We first convert 214_6 to a number in base 10.

$214_6 = 2 \times 6^2 + 1 \times 6 + 4 \times 1$
$= 82_{10}$

Now, convert 82_{10} to a binary number, using repeated division.

2	82	R
2	41	0
2	20	1
2	10	0
2	5	0
2	2	1
2	1	0
2	0	1

Read the number from bottom to top.
$82_{10} = 1010010$

Exercise 3

1. Convert the following binary numbers to base 10 numbers.

a) 111 **b)** 110101 **c)** 1100001
d) 1001011 **e)** 11000 **f)** 11110
g) 1110 **h)** 100001

2. Convert the following base 10 numbers to binary numbers.

a) 710 **b)** 1 910 **c)** 5 810
d) 12 310 **e)** 19 510 **f)** 10 410
g) 9 110

3. Convert the following numbers to base 2 numbers.

a) 112_9 **b)** 131_6 **c)** 1566_8
d) 22_4 **e)** 102_3 **f)** 64_6

4. Convert 10101010_2, first to a decimal number and then to a number in base 3.

5. A certain binary number has three digits.

a) What is the smallest and the largest numbers possible?
b) Convert these numbers to base 10.

6. Calculate the difference between the base 10 number 11111 and the binary number 11111 and give your answer in base 10.

Do Worksheet 2 in the JSS 3 Workbook.

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THEME 2 Basic operations

TERM 1

Topic 2: Basic operations in the binary system

Unit 1: Adding and subtracting binary numbers

The four basic rules for adding binary numbers are:

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 10$

Examples

One way to add binary numbers is to first convert the binary number to a base 10 number.

1. $11_2 + 11_2 = 110_2$

$3 + 3 = 6$

2. $111_2 + 11_2 = 1010_2$

$7 + 3 = 10$

3. $110_2 + 100_2 = 1010$

$6 + 4 = 10$

Alternatively, we can add binary numbers in columns.

1.
$$\begin{array}{r} 1\ 1_2 \\ +\ \ 1\ 1_2 \\ \hline 1\ 1\ 0_2 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 1\ 1\ 1_2 \\ 1\ 1_2 \\ \hline 1\ 0\ 1\ 0_2 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 1\ 1\ 0 \\ 1\ 0\ 0 \\ \hline 1\ 0\ 1\ 0_2 \\ \hline \end{array}$$

The four basic rules for subtracting binary numbers are:

$0 - 0 = 0$

$1 - 1 = 0$

$1 - 0 = 1$

$10 - 1 = 1$

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Examples

Similar to adding, one way to subtract binary numbers is to first convert the binary numbers to decimal numbers.

1. $11_2 - 01_2 = 10$
$3 - 1 = 2$

2. $11_2 - 10_2 = 01_2$
$3 - 2 = 1$

3. $101_2 - 011_2 = 010_2$
$5 - 3 = 2$

We can also subtract binary numbers in columns, as with addition.

$$\begin{array}{r} 1\,0\,1\,0 \\ -\quad 1\,1\,1 \\ \hline 1\,1 \end{array}$$

Borrow 1 from the first 1 you come across to the left.
All zeroes preceding this 1 changes from 0 to 1.

Exercise 1

1. Add or subtract the given binary numbers.

a) $\begin{array}{r} 1\,0\,1\,1\,1 \\ -\ 1\,0\,0\,1\,1 \\ \hline \end{array}$ **b)** $\begin{array}{r} 1\,0\,1\,1\,1 \\ -\ 0\,1\,0\,0\,0 \\ \hline \end{array}$ **c)** $\begin{array}{r} 1\,1\,1\,1\,1 \\ -\ 0\,1\,0\,0\,0 \\ \hline \end{array}$ **d)** $\begin{array}{r} 1\,1\,0\,0\,0 \\ -\ 1\,1\,1\,1\,1 \\ \hline \end{array}$

e) $\begin{array}{r} 1\,1\,1\,1\,1 \\ +\ 1\,0\,1\,0\,0 \\ \hline \end{array}$ **f)** $\begin{array}{r} 1\,0\,1\,0\,0 \\ +\ 0\,1\,0\,1\,0 \\ \hline \end{array}$ **g)** $\begin{array}{r} 0\,1\,0\,1\,0 \\ +\ 0\,1\,0\,1\,1 \\ \hline \end{array}$ **h)** $\begin{array}{r} 0\,1\,0\,1\,1 \\ +\ 1\,0\,1\,0\,1 \\ \hline \end{array}$

i) $\begin{array}{r} 1\,0\,1\,0\,1 \\ +\ 1\,1\,1\,1\,0 \\ \hline \end{array}$ **j)** $\begin{array}{r} 1\,1\,1\,1\,0 \\ -\ 0\,1\,0\,0\,1 \\ \hline \end{array}$ **k)** $\begin{array}{r} \underline{1\,1\,0\,1} \\ -\quad 1\,1\,1 \end{array}$ **l)** $\begin{array}{r} \underline{1\,0\,0\,0\,1} \\ -\quad 1\,1\,1\,0 \end{array}$

2. Find x in each of the following, where x is a binary number.

a) $x + 111 = 11110$ **b)** $x + 11110 = 10001$
c) $x - 10 = 101$ **d)** $x + 11 = 1101$

3. Given the binary numbers 111012 and 11102

a) Convert the two binary numbers to base 10.
b) Add the two base 10 numbers in the result above together.
c) Add the two binary numbers together.

Unit 2: Multiplying and dividing binary numbers

The four basic rules for multiplying binary numbers are:

$0 \times 0 = 0$
$0 \times 1 = 0$
$1 \times 0 = 0$
$1 \times 1 = 1$

We multiply binary numbers in the same way as decimal numbers.

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Examples

1.

$$\begin{array}{r} 11 \\ \times \quad \underline{11} \\ 11 \\ + \quad \underline{110} \\ 1001 \end{array}$$

2.

$$\begin{array}{r} 101 \\ \times \quad \underline{111} \\ 101 \\ 1010 \\ + \quad \underline{10100} \\ 100011 \end{array}$$

We divide binary numbers in the same way as decimal numbers.

Examples

1.

$$\begin{array}{r} 10 \\ 11 \overline{)\ 110} \\ \underline{11} \\ 000 \end{array}$$

2.

$$\begin{array}{r} 11 \\ 10 \overline{)\ 110} \\ \underline{10} \\ 10 \\ \underline{10} \\ 00 \end{array}$$

Exercise 2

1. Multiply the following binary numbers.

- **a)** 1100 × 10
- **b)** 11111 × 10
- **c)** 1101 × 11
- **d)** 10011 × 101
- **e)** 101101 × 111
- **f)** 10101 × 10001

2. Divide the following binary numbers.

- **a)** 101110 ÷ 100
- **b)** 100110 ÷ 101
- **c)** 10100 ÷ 10
- **d)** 1100 ÷ 100

3. Convert your answers in Question 1 to decimal numbers.

4. Multiply 23 by 35.

- **a)** Convert your answer to a binary number.
- **b)** Convert 23 and 35 to binary numbers.
- **c)** Multiply the binary numbers obtained in Question 4b, and then check your answer.

Do Worksheet 3 in the JSS 3 Workbook.

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Unit 3: Addition and subtraction in other bases

Addition in other bases

Example

Add 21_5 and 13_5.

Solution

As this addition is in base 5, we put 21 objects (base 5) next to 13 objects (in base 5).

21_5 + 13_5 = 3 fives 4 units

$\therefore 21_5 + 13_5 = 34_5$

In the place-value system, the addition is shown in the table below.

Five	Units
2	1
1	3
3	2

+

It can also be done like this:

21_5
13_5
34_5

$\therefore 21_5 + 13_5 = 34_5$

Exercise 3

Copy and complete this table for addition in base 7.

+	0	1	2	3	4	5	6
0							
1							
2							
3			5				
4						12	
5							
6							

Subtraction in other bases

Subtraction in other bases is easy with the place-value system.