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THEME 1 Numbers and numeration

TERM 1

Topic 1: Whole numbers and decimal numbers

In Topic 1, we will focus on whole numbers in standard forms, decimal numbers in standard forms, as well as factors, multiples and prime numbers.

Unit 1: Index notation

To avoid writing very long multiples, mathematicians use indices (singular "index") as a form of mathematical shorthand.

For example, $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$ and

$u \times u \times u \times u \times u \times u \times u \times u \times u = u^9$.

5^6 and u^9 are called **index forms** of writing the multiplication calculations. In 5^6, 5 is called the **base** and 6 is called the **index** or **power**.

Another example is $a \times a \times a \times a \times a = a^5$, where a is the base and 5 is the index.

Also, $10 \times 10 \times 10 \times 10 = 10^4$

And, $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{10^3} = 10^{-3}$

The index is the power of a number and the base is the number itself.

power

3^5

base

> Generally, in $a^n = \underbrace{a \times a \times a \times \ldots \times a}_{a \text{ multiplied by itself } n \text{ times}}$
>
> a is the base and n is the index or power.
> Therefore, a^n is in **index form**, where $a \neq 0$.

Laws of indices

To do calculations with indices we follow certain rules. These rules are called the **laws of indices**.

Multiplication of numbers in index form

Examples

1. Use your calculator to evaluate $3^2 \times 3^5$. Now do the same for 3^7. What do you notice?
2. Use your calculator to evaluate $7^5 \times 7^6$. Now do the same for 7^{11}. What do you notice?

Solutions

The example demonstrates that $3^2 \times 3^5 = 3^7 = 3^{2+5}$ and $7^5 \times 7^6 = 7^{5+6} = 7^{11}$. From this, we deduce the first law of indices.

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Be careful!
$a^m + a^n \neq a^{m+n}$
Law 1 only applies to multiplication of numbers in index form, and not to addition of such numbers.

Law 1
To multiply any two real numbers in index form with the same base, add the powers or indices of the base. The general statement of this law is:
$a^m \times a^n = a^{m+n}$, where $a \in \mathbb{R}$ and $a \neq 0$.

Examples

Simplify these expressions.

1. $y^3 \times y^4$ **2.** $4x^3 \times 3x^2$ **3.** $5^3 \times 5^6$
4. $x^8 \times x^7$ **5.** $\frac{y^9}{y^2}$ **6.** $x^5 \times y^7$

Solutions

1. $y^3 \times y^4$
$= (y \times y \times y) \times (y \times y \times y \times y)$
$= y \times y \times y \times y \times y \times y \times y$
$= y^7$

But we can use Law 1:
$y^3 \times y^4$
$= y^{3+4}$
$= y^7$

2. $4x^3 \times 3x^2$
$= (4 \times x \times x \times x) \times (3 \times x \times x)$
$= 4 \times x \times x \times x \times 3 \times x \times x$
$= 12x^5$
or
$4x^3 \times 3x^2$
$= 4 \times 3 \times x^3 \times x^2$
$= 12x^{3+2}$
$= 12x^5$

3. $5^3 \times 5^6 = 5^{3+6} = 5^9$ (use Law 1)

4. $x^8 \times x^7 = x^{8+7} = x^{15}$ (use Law 1)

5. $\frac{y^9}{y^2} = \frac{y^{2+7}}{y^2} = y^2 \times \frac{y^7}{y^2} = y^7$ (Law 1 is used to separate y^9 into $y^2 \times y^7$, and then the common factor is cancelled from the numerator and the denominator)

6. $x^5 \times y^7 = x^5y^7$ (this cannot be simplified further, since $x \neq y$, that is, the bases of the two numbers are not the same)

Exercise 1

1. a) $9^5 \times 9^7$ **b)** $8^6 \times 8^{10}$ **c)** $2^8 \times 2^{10}$

2. Simplify, leaving your answer in index form.

a) $v^7 \times v^8$ **b)** $p^3 \times p^5$ **c)** $q^6 \times q^{11}$

3. Simplify, leaving your answer in index form.

a) $\frac{x^{12}}{x^8}$ **b)** $\frac{v^7 \times v^9}{v^6}$ **c)** $\frac{v^{12}}{v^{17}}$

Division of numbers in index form

Examples

1. Use your calculator to work out $3^6 \div 3^4$.
2. Express your answer as a power of 3. What do you notice?
3. Do the same for $7^{10} \div 7^6$ and $9^{11} \div 9^9$. What do you notice?

Solutions

The example shows that $3^6 \div 3^4 = 3^2 = 3^{6-4}$.

Similarly, $7^{10} \div 7^6 = 7^{10-6} = 7^4$ and $9^{11} \div 9^9 = 9^{11-9} = 9^2$.

From this, we deduce the second law of indices.

Law 2

To divide any two real numbers in index form that have the same base, subtract the powers or indices of the denominator from those of the numerator, and raise the base to this difference.

The general statement of this law is:

$a^m \div a^n = a^{m-n}$, where $a \in \mathbb{R}$ and $a \neq 0$.

Be careful!

$a^m - a^n \neq a^{m-n}$

Law 2 only applies to division of numbers in index form, and not to subtraction of such numbers.

Examples

Simplify, leaving your answer in index form.

1. $x^5 \div x^2$
2. $25ab^2 \div 5b$
3. $3^{12} \div 3^7$
4. $3y^{15} \div y^{11}$

Solutions

1. $\frac{x^5}{x^3} = \frac{x \times x \times x \times \cancel{x} \times \cancel{x}}{\cancel{x} \times \cancel{x}}$

$= x \times x \times x$

$= x^3$

So,

$x^5 \div x^2$

$= x^{5-2}$

$= x^3$

2. $\frac{25ab^2}{5b} = \frac{25}{5} ab^2 \div b$

$= 5ab^2 - 1$

$= 5ab$

3. $3^{12} \div 3^7 = 3^{12-7} = 3^5$ (use Law 2)

4. $3y^{15} \div y^{11} = 3y^{15-11} = 3y^4$ (use Law 2)

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Exercise 2

Simplify, leaving your answer in index form.

1. $3^{20} \div 3^{7}$ **2.** $5^{17} \div 5^{14}$ **3.** $8^{40} \div 8^{27}$ **4.** $u^{16} \div u^{13}$ **5.** $r^{21} \div r^{19}$

The zero power

Example

Use your calculator to evaluate $3^6 \div 3^6$. What do you notice?

Solution

When we use expanded notation we see that $3^6 \div 3^6 = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3 \times 3} = 1$.

From Law 2 of indices, the result is $3^6 \div 3^6 = 3^{6-6} = 3^0$.

Since $3^6 \div 3^6 = 3^6 \div 3^6$, then $3^0 = 1$.
From this, we deduce the third law of indices.

Remember that $a \neq 0$, but $a^0 = 1$, except for $0^0 = 0$.

Law 3
When any real number, except zero, is raised to the power of 0, it is equal to 1. The general statement of this law is:
$a^0 = 1$, where $a \in \mathbb{R}$ and $a \neq 0$.

Example

$2^0 = 2^1 \div 2^1 = 2^4 \div 2^4 = 1$
$4^0 = 4^7 \div 4^7 = 4^{13} \div 4^{13} = 1$
$5^0 = 1$
$6^0 = 1$
$100^0 = 1$, and so on.

Negative powers or negative indexes

By using expanded notation, we can simplify an expression such as $3^3 \div 3^6$ as follows:

$$
\begin{aligned}
3^3 \div 3^6 &= \frac{3^3}{3^6} \\
&= \frac{3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3 \times 3} \\
&= \frac{1}{3 \times 3 \times 3} \\
&= \frac{1}{3^3} \\
&= \frac{1}{9}
\end{aligned}
$$

But from Law 2 of indices, we get $3^3 \div 3^6 = 3^{3-6} = 3^{-3}$.

Therefore, since $3^3 \div 3^6 = 3^3 \div 3^6$, $3^{-3} = \frac{1}{3^3}$.

From this, we can deduce the fourth law of indices.

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Law 4
For any real number a, a raised to a negative power $-m$ is equal to the reciprocal of a raised to the positive power $+m$. The general statement of this law is:
$a^{-m} = \frac{1}{a^m}$, where $a \in \mathbb{R}$ and $a \neq 0$.
The converse is also true: $\frac{1}{a^m} = a^{-m}$.

Examples

1. $2^{-5} = \frac{1}{2^5}$
2. $4^{-3} = \frac{1}{4^3}$
3. $\frac{1}{5^6} = 5^{-6}$
4. 3^{-2}
   $= \frac{1}{3^2}$
   $= \frac{1}{9}$
5. 10^{-4}
   $= \frac{1}{10^4}$
   $= \frac{1}{1\,000}$

Exercise 3

1. Write down the value of these numbers.
   a) 10^0 b) 17^0 c) $(-20)^0$ d) $1\,500^0$ e) $3\,000^0$
2. Write these numbers in index form, with negative indices.
   a) $\frac{1}{3^{10}}$ b) $\frac{1}{5^4}$ c) $\frac{1}{1\,000}$ d) $\frac{1}{6^4}$ e) $\frac{1}{34^3}$
3. Write these numbers as reciprocals, with positive indices.
   a) 3^{-7} b) 7^{-4} c) 11^{-4}
4. Simplify the following fractions, leaving your answers in index form with negative indices.
   a) $\frac{y^7}{y^9}$ b) $\frac{x^5y^7}{x^7y^{12}}$

Raising numbers in index form to a power

To simplify a number in index form that is raised to a power, we can apply Law 1:
$(4^2)^3 = 4^2 \times 4^2 \times 4^2 = 4^{2+2+2} = 4^6$.
From this, we see that $(4^2)^3 = 4^6 = 4^{2 \times 3}$.

Therefore, raising a number in index form to a power is the same as multiplying the powers and raising the number to their product.

From this, we can deduce the fifth law of indices.

Law 5
To raise any real number a in index form to a power, multiply the two powers and raise the number to their product. The general statement of this law is:
$(a^m)^n = a^{m \times n} = a^{mn}$ where a, m and $n \in \mathbb{R}$ and $a \neq 0$.

Be careful!
$(a^m)^n \neq a^{m+n}$

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Examples

$(2^3)^4 = 2^{3 \times 4} = 2^{12}$
$(3^5)^3 = 3^{5 \times 3} = 3^{15}$

Exercise 4

Use Law 4 and Law 5 to simplify the following numbers. Leave your answer in index form.

1. $(5^3)^7$ **2.** $(8^6)^5$ **3.** $(2^{-3})^{-6}$ **4.** $(2^{-8})^{\frac{-1}{2}}$ **5.** $(7^3)^{-2}$

Unit 2: Whole numbers in standard form

Standard form, also called scientific notation, is an easy way of writing down very large or very small numbers.

To write a number in standard form, express it as a number between 1 and 10, multiplied by 10, to some power.

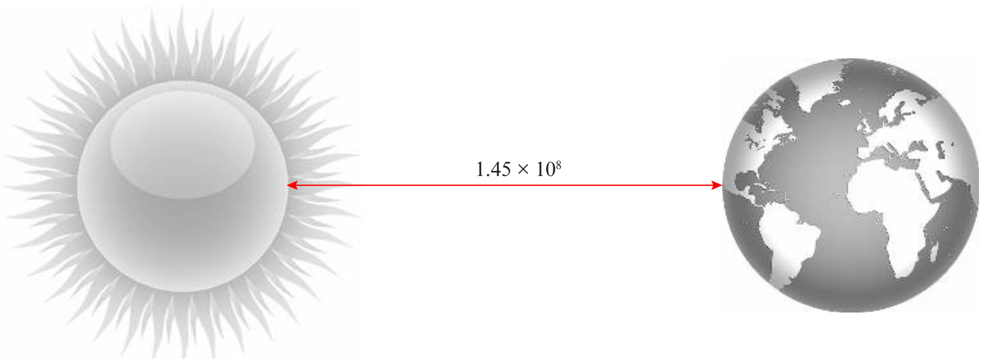
For example, the distance from the Earth to the Sun is 145 000 000 km.
To express this number in a way that is easier to write, we use the exponent laws learnt in the previous section. Table 1.1 below shows how large numbers can be written as powers of 10.

Table 1.1

	In full	Expanded form	Index form
1 million	1 000 000	10 × 10 × 10 × 10 × 10 × 10	10^6
1 billion	1 000 000 000	10 × 10 × 10 × 10 × 10 × 10 × 10 × 10 × 10	10^9
1 trillion	1 000 000 000 000	10 × 10 × 10 × 10 × 10 × 10 × 10 × 10 × 10 × 10 × 10 × 10	10^{12}

The number of zeros indicates how many times 10 is multiplied by itself.
Therefore, the distance from the Earth to the Sun can be written as follows:

$145 \times 1\,000\,000$
$= 145 \times 10^6$



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To write 145 as a number between 1 and 10, we move the decimal point to the left by two places.

1.45×10^8

Notice that the power of 10 has increased from 6 to 8, because we moved the decimal point two places to the left.

Therefore, to convert a number to standard form, create a number between 1 and 10 by moving the decimal point to the left. Count the number of spaces that the decimal point moved, to determine the exponent of 10.

> Standard form is a way of writing down very large or small numbers in the form $A \times 10^n$, where $1 \leq A < 10$ and n = integers (positive or negative integers).

Similarly, numbers in standard form can be converted to a whole number. We simply reverse the process above. For example, to convert 3.46×10^6 to a whole number, we move the decimal point six places to the right, the same number as the exponent of 10:

$3.46 \times 10^6 = 3\,460\,000.$

Decimal numbers in standard form

We can convert a very small number, or decimal number, to standard form. In the same way that we convert a large number to standard form, we also write the decimal number as a number between 1 and 10, multiplied by 10 to a power. The power, however, will be negative.

For example, a grain of sand measures 0.0004 m in diameter. To convert this number to standard form, move the decimal point to the right, past the first non-zero value. In this case, it is 4. Because we make the number larger by moving the decimal point to the right, the exponent will be negative. Therefore:

$0.0004 = 4 \times 10^{-4}$

Decimal numbers that are in standard form, can also be converted back to decimal form. To change it, move the decimal point to the left the same number of times as the exponent. Place zeros as place holders:

$9.31 \times 10^{-4} = 0.000931$

Exercise 5

1. Put these numbers in order of size, starting with the largest number.

a)	6×10^0	0.076	9.2×10^4	4×10^{-3}	67 000
b)	2×10^{-1}	203	47.6	3.1×10^1	0.09
c)	45 000	4×10^5	5.7×10^{-6}	9×10^{-8}	2×10^7
d)	0.004	3×10^{-2}	0.9	8×10^{-4}	6.4×10^{-3}
e)	0.09	5×10^{-3}	0.5	8×10^{-4}	5.4×10^{-2}

2. These numbers are written in standard form. Convert them to ordinary numbers.

a) 1.4×10^2 **b)** 2×10^3 **c)** 6.3×10^1

d) 4.52×10^2 e) 7×10^4 f) 5.6×10^4
g) 4.56×10^4 h) 8.3×10^1 i) 3.5×10^0
j) 4.76×10^6 k) 2×10^5 1) 7.02×10^3

3. Write these numbers in standard form.
a) 23.5 b) 93 400
c) 207 d) 7 210 000
e) 1.32 f) 930 000 000
g) 46 000 h) 9 700 000 000
i) 66 j) 500 000
k) 170 200 l) 4.32

4. These numbers are given in standard form. Write them as ordinary numbers.
a) 2×10^{-1} b) 4.1×10^{-5}
c) 5.63×10^0 d) 3.1×10^{-4}
e) 6×10^{-2} f) 1.8×10^{-3}
g) 4.3×10^{-6} h) 6.7×10^{-1}
i) 4.071×10^{-5} j) 8.78×10^{-3}
k) 3.8×10^{-1} l) 1.67×10^0

5. Write these numbers in standard form.
a) 0.4 b) 0.0023
c) 0.045 d) 0.9
e) 0.83 f) 0.006
g) 0.0056 h) 0.0312
i) 0.204 j) 0.00081
k) 0.3 l) 0.00004

6. A crowd at a football match is estimated to be 46 300 people. Write this number in standard form.

7. It takes light approximately 3.05×10^{-7} seconds to travel 100 m. Write this in normal decimal form.

8. The fastest fighter jet on the planet travels at a speed of Mach 3, approximately 3.675×10^6 km/h. What is Mach 3 as a whole number?

Do Worksheet 1 in the JSS 2 Workbook.

Unit 3: Factors, multiples and prime numbers

A **factor** is a number that can divide exactly into another number without a remainder. For example, the factors of 6 are 1; 2; 3 and 6.

Any whole number is either a prime or a composite number. A **composite number** has more than two factors. A **prime number** is any whole number more than 1, that has only 1 and itself as factors. 2 is the only ever prime number.

The number 1 is the only number that is neither prime nor composite.

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Factors and multiples

Exercise 6

1. List all the factors of the following.
a) 10 **b)** 18 **c)** 37 **d)** 52 **e)** 120

2. The Sieve of Eratosthenes is an ancient method for finding all prime numbers up to a specified number. It was created by Eratosthenes (275–194 B.C.), an ancient Greek mathematician. The numbers from 1 to 100 are listed in the table below.
We can use The Sieve of Eratosthenes to find all prime numbers up to the number 100. Follow the directions.

- Cross out 1 because it is not prime.
- Circle 2 because it is the smallest prime number. Cross out every multiple of 2.
- Circle the next open number, 3. Now, cross out every multiple of 3.
- Circle the next open number, 5. Now, cross out every multiple of 5.
- Circle the next open number, 7. Now, cross out every multiple of 7.
- Continue this process until all numbers in the table have been circled or crossed out.
- When complete, you will have all the prime numbers from 1 to 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

3. List the first seven prime numbers.

4. Say whether the following numbers are prime, composite or neither. Give a reason for your answer.
a) 12 **b)** 19 **c)** 81
d) 54 **e)** 250 **f)** 163

5. From the numbers {1, 2, 3, 4, 7, 9, 12, 18, 19, 24, 27, 48, 80, 84, 92, 96}, select the following.
a) factors of 12 **b)** multiples of 12
c) prime numbers **d)** even prime numbers.

Prime factors

Any number, bigger than 1, can be written as a product of prime numbers in a unique way. For example, $12 = 2 \times 2 \times 3$. There are different ways to write 12 as a product, for example, 2×6 or 4×3, but there is only one way to write it as a product of prime numbers.

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Example

Write 360 as the product of prime factors.

Solution

Use the "ladder" method to find the prime factors of 360.
Start dividing by the smallest prime number.
Keep dividing until you get to 1.

Therefore, $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

We can write this in index form:
$360 = 2^3 \times 3^2 \times 5$

2	360
2	180
2	90
3	45
3	15
5	5
	1

Exercise 7

1. Write the following in index notation.
   a) $2 \times 2 \times 3$
   b) $2 \times 3 \times 3 \times 5$
   c) $3 \times 5 \times 5 \times 7$
   d) $5 \times 5 \times 7 \times 7 \times 13 \times 13$

2. Write the following as the product of prime factors by completing the ladder.

a)

2	120

b)

2	75

c)

2	216

3. Find the prime factors of the following numbers. Leave your answer in index notation.
   a) 50 b) 225 c) 216 d) 486 e) 965

Unit 4: Highest common factors (HCF)

A **common factor** is any number that divides two numbers exactly. Look at the numbers 20 and 30:

Factors of 20: 1, 2, 4, 5, 10, 20

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

We see that 1, 2, 5 and 10 are common factors of 20 and 30.