## Contents

*Foreword*  
Mihai Putinar  
*Preface*  

**PART ONE  PRELIMINARIES: ENTRYWISE POWERS PRESERVING POSITIVITY IN A FIXED DIMENSION**

1. The Cone of Positive Semidefinite Matrices  
   Page 3

2. The Schur Product Theorem and Nonzero Lower Bounds  
   Page 16

3. Totally Positive (TP) and Totally Nonnegative (TN) Matrices  
   Page 23

4. Totally Positive Matrices – Generalized Vandermonde and Hankel Moment Matrices  
   Page 32

5. Entrywise Powers Preserving Positivity in a Fixed Dimension  
   Page 40

6. Midconvex Implies Continuity, and 2 × 2 Preservers  
   Page 46

7. Entrywise Preservers of Positivity on Matrices with Zero Patterns  
   Page 55

8. Entrywise Powers Preserving Positivity, Monotonicity, and Superadditivity  
   Page 64

9. Loewner Convexity and Single Matrix Encoders of Preservers  
   Page 72

10. Exercises  
   Page 84
Table of Contents

PART TWO  ENTRYWISE FUNCTIONS PRESERVING
POSITIVITY IN ALL DIMENSIONS

11  History – Schoenberg, Rudin, Vasudeva, and Metric
    Geometry 97

12  Loewner’s Determinant Calculation in Horn’s Thesis 114

13  The Stronger Horn–Loewner Theorem via Mollifiers 120

14  Stronger Vasudeva and Schoenberg Theorems via
    Bernstein’s Theorem 127

15  Proof of the Stronger Schoenberg Theorem (Part I) –
    Positivity Certificates 135

16  Proof of the Stronger Schoenberg Theorem (Part II) –
    Real Analyticity 144

17  Proof of the Stronger Schoenberg Theorem (Part III) –
    Complex Analysis 151

18  Preservers of Loewner Positivity on Kernels 155

19  Preservers of Loewner Monotonicity and Convexity
    on Kernels 159

20  Functions Acting Outside Forbidden Diagonal Blocks 168

21  The Boas–Widder Theorem on Functions with Positive
    Differences 176

22  Menger’s Results and Euclidean Distance Geometry 190

23  Exercises 203

PART THREE  ENTRYWISE POLYNOMIALS
 PRESERVING POSITIVITY IN A FIXED DIMENSION

24  Entrywise Polynomial Preservers and Horn–Loewner-Type
    Conditions 213

25  Polynomial Preservers for Rank-1 Matrices, via Schur
    Polynomials 220

26  First-Order Approximation and the Leading Term of
    Schur Polynomials 228
## Contents

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>Exact Quantitative Bound – Monotonicity of Schur Ratios</td>
<td>233</td>
</tr>
<tr>
<td>28</td>
<td>Polynomial Preservers on Matrices with Real or Complex Entries</td>
<td>245</td>
</tr>
<tr>
<td>29</td>
<td>Cauchy and Littlewood’s Definitions of Schur Polynomials</td>
<td>256</td>
</tr>
<tr>
<td>30</td>
<td>Exercises</td>
<td>264</td>
</tr>
<tr>
<td></td>
<td>Part I: Bibliographic Notes and References</td>
<td>270</td>
</tr>
<tr>
<td></td>
<td>Part II: Bibliographic Notes and References</td>
<td>273</td>
</tr>
<tr>
<td></td>
<td>Part III: Bibliographic Notes and References</td>
<td>276</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>277</td>
</tr>
<tr>
<td></td>
<td>Index</td>
<td>291</td>
</tr>
</tbody>
</table>