

## Index

- absolutely monotonic function, 127
- Basic Composition formula, 38
- Bernstein's theorem, 127
- Boas–Widder theorem, 125, 176
- Cauchy mean-value theorem for divided differences, 125
- Cauchy–Binet formula, 34
- Cayley–Menger matrix, 192
- matrix, alternate form, 102, 192
- Chebyshev polynomials, 109, 110
- chordal graph, 70
- column-strict Young tableau, 220
- complete homogeneous symmetric polynomials  $h_k(\mathbf{u})$ , 246, 256
- completely monotone functions, 199
- conditionally positive semidefinite matrix, 103, 104, 205
- cone, 12
- contiguous minor, 26
- correlation matrix, 9, 56, 90, 109
- covariance matrix, 56
- critical exponent for:
- a graph  $G$ , 64
  - Loewner convexity, 79
  - Loewner monotonicity, 66
  - Loewner positivity, 41
  - Loewner super-additivity, 68
  - positive definiteness, 44
- Daletskii–Krein formula, 18
- Descartes' rule of signs, 33, 73
- Dirac measure  $\delta_x$ , 10
- divided differences, 124
- dominated convergence theorem (Lebesgue), 122
- elementary symmetric polynomials  $e_k(\mathbf{u})$ , 256
- entrywise
- map/function/transform  $f[A]$ , 40
  - map/function/transform  $f_G[A]$ , 59
  - polynomial positivity preserver, 216
  - power  $A^{\circ\alpha}$ , 40
  - powers preserving positivity  $\mathcal{H}_G$ , 64
- Fekete–Schoenberg lemma, 29
- forward difference  $(\Delta_h^k f)(x)$ , 124, 176
- Further questions, 71, 82, 250
- Gaussian kernel  $T_{G_\sigma}$ , 37, 105
- gcd matrix, 86
- Gegenbauer polynomials, 109, 110
- geometric mean  $A\#B$ , 87
- Gram matrix, 8, 107, 109, 191
- Hadamard composition of polynomials/power series, 19
- Hamburger's theorem, 132
- Hankel
- moment matrix  $H_\mu$ , 10
  - $TN$   $n \times n$  matrices  $\text{HTN}_n$ , 53
- Hilbert sphere  $S^\infty$ , 106
- Historical notes, 11, 18, 19, 26, 97, 99, 101, 104, 121, 130, 138, 193, 200
- holomorphic function, 151
- homogeneous space,  $n$ -point, 191, 205
- homotopy argument, 34, 74
- Horn–Loewner theorem, stronger form, 114

Identity theorem, 145  
 integration trick, 42, 66, 68, 80, 81  
 integration trick/extension principle, 45, 241, 250, 253

Jacobi formula, 7  
 Jacobi–Trudi identity (dual), 257

Kronecker product  $A \otimes B$ , 19  
 Kuratowski embedding, 203

Laguerre–Poulain–Rolle trick, 33, 73

Loewner  
   convex function, 79, 159, 167  
   matrix  $L_f(x_1, \dots, x_k)$ , 18, 101  
   monotone function, 66, 159, 167  
   monotone function theorem, 100  
   order, 66, 167  
   positive function, 40, 155  
   super-additive function, 66

majorization (weak), 234

metric  
   geometry, 101, 190, 200, 203  
   space, 101

mid-convex  
   additive function, 46  
   multiplicative function, 49  
   Ostrowski’s theorem, 46  
   Sierpiński’s theorem, 48

mollifier, 121

moment problem:  
   Hamburger, 11  
   Stieltjes, 26

moment-sequence  $\mathbf{s}(\mu)$ , 10, 131

monomial positive polynomial, 236

Montel’s theorem, 152

Moore–Penrose inverse  $A^\dagger$ , 14, 87, 265

Morera’s theorem, 152

Motzkin, Theodore, 138

PMP matrix, 91, 268

Pólya frequency function, 37

positive definite  
   function, 104, 108, 110  
   kernel, 131, 155  
   matrix, 4

positive semidefinite  
   kernel, 155  
   matrices  $\mathbb{P}_n, \mathbb{P}_n(I)$ , 3, 40  
   matrices with zero pattern/sparsity  $\mathbb{P}_G$ , 12, 58

positivity certificate, 139

positivity-certificate trick, 142

Principal specialization formula, 222, 241

probability, connections, 20

Rayleigh quotient, generalized, 265, 266

Rayleigh–Ritz theorem, 170

real analytic function  $f \in C^\omega(I)$ , 144

Schoenberg’s theorem:  
   original form, 98  
   stronger form, 130, 132

Schur  
   complement, 13  
   polynomial  $s_n(\mathbf{u})$ , 221  
   polynomial, leading term of, 228  
   positive polynomial, 236  
   product of matrices  $A \circ B$ , 18  
   product theorem, 19, 88  
   sign changes  $S^-(\cdot)$ , 25, 73  
   singular value decomposition, 4  
   spectral theorem, 4  
   spherical distance  $\angle$ , 107  
   sum-of-squares polynomial, 138  
   Sylvester’s criterion, 6, 89

Toeplitz cosine matrix, 9, 25, 86, 92

totally nonnegative  $(TN, TN_p)$   
   kernel, 37  
   kernel of order  $p$ , 37  
   matrix, 23  
   matrix of order  $p$ , 23  
   totally positive  $(TP, TP_p)$   
   kernel, 37  
   kernel of order  $p$ , 37  
   matrix, 23  
   matrix of order  $p$ , 23

triangulation/trilateration, 193

truncation  $A^{(1)}$  of a matrix  $A$ , 26

Vandermonde determinant  $V(\mathbf{u})$ , 115, 222

Vandermonde matrix, generalized, 32

variation diminishing property: sign regular matrices, 24

Vasudeva’s theorem:  
   original form, 98  
   stronger form, 127

Weyl dimension formula, 222

Weyl’s inequality, 170

Wiener norm, 139, 207