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Excerpt
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Part I

The Philosophy of Deduction

1 The Trouble with Deduction

1.1 Introduction

From the observation that all dogs are animals, and that Fido is a dog, I may conclude with absolute certainty, without the shadow of a doubt, that Fido is an animal (i.e., provided that all dogs are indeed animals and that Fido is indeed a dog and not a cleverly disguised automaton, for example). Inferences where the truth of the premise(s) guarantees the truth of the conclusion(s)[1] are known as *deductive inferences*;[2] these are usually contrasted with *inductive* and *abductive* inferences, which do not have the property of necessary truth-preservation. In other words, in an inductive or abductive inference, the premises may be true while the conclusion is not (though the truth of the premises should make the conclusion more *likely* to be true), whereas the very definition of a deductive argument rules out this possibility. Another prominent concept used to bring out the contrast between deductive reasoning and other kinds of reasoning is that of (in)defeasibility; defeasible reasoning, as characterized in the seminal work of John Pollock (Pollock, 1974, 1987) and further studied in philosophy, artificial intelligence, and other fields of inquiry, is the kind of reasoning where premises do confer justification and support to a conclusion (prima facie reasons), but the argument in question may be defeated by new incoming information.

[1] Traditionally, a deductive argument is conceived as having one or more premises (though in certain cases, such as with Aristotelian syllogistic, there is a requirement for multiple premises) and one conclusion. The idea of multiple-conclusion arguments has its proponents (Restall, 2005), but is not unanimously accepted (Caret & Hjortland, 2015). At this point, it makes sense to keep things as general as possible, and thus not to make any restrictions on the number of either premises or conclusions involved in a given argument. However, more often than not, I will speak of premises in the plural and conclusion in the singular, as this reflects the more traditional understanding of a deductive argument throughout history (e.g. Aristotelian syllogistic).

[2] Initially, I will use 'inference' and 'argument' interchangeably, but later on it will be important to discuss the differences between the two notions. In particular, 'inference' is usually associated with mono-agent situations of mental, epistemic acts, whereas 'argument' is often (though not always) used in multi-agent contexts of argumentation. Thus arguments are typically viewed as linguistic entities, whereas inferences are also used in the sense of mental entities.

Simple examples such as Fido's above may seem to suggest that deduction is not a particularly powerful or interesting reasoning tool, one that only allows for the derivation of rather trivial conclusions. But this is not what the history of mathematics, science, and philosophy suggests: for millennia, deductive argumentation has occupied a crucial role in various areas of intellectual inquiry. Originally, the two canonical presentations of what could be described as the 'deductive method,'[3] that is, the method of inquiry where deductive argumentation occupies pride of place, are Euclid's *Elements* and Aristotle's *Posterior Analytics*. In both cases, one begins with a few purportedly self-evident truths – axioms – and then derives further truths from them in a stepwise manner by means of deductive inferences, in what is also known as the *axiomatic-deductive method*. These two models, the Euclidean model for mathematics and the Aristotelian model for the (empirical) sciences, remained influential for millennia, and still represent what could be described as the classical conception of mathematics and science (de Jong & Betti, 2010). In philosophy, influential authors adopted the Euclidean *more geometrico* for the development and presentation of their philosophical systems, most notably Baruch Spinoza (Spinoza, 1985). More recently, Carl Hempel and Paul Oppenheim's deductive-nomological model of scientific inquiry (Hempel & Oppenheim, 1948) is another example of deduction presented as a quintessential component of scientific inquiry.

Indeed, it is not surprising that philosophers, mathematicians, and scientists would be impressed by the deductive method, with its allure of certainty and its promise of unshakable foundations. But doubts concerning the reliability and applicability of deductive reasoning as a method of inquiry have also been raised, including ancient Skeptic criticism, distaste for 'the logic of the schools' (Descartes, 1985), and, more recently, worries concerning the non-ampliative, non-informative nature of deductive reasoning (Hintikka, 1973). In fact, the very notion of deduction raises a number of issues that, despite having received sustained attention from philosophers, remain puzzling.

In this chapter, I bring to the fore and further clarify these issues. Before addressing them, I present the three key features of deductive reasoning that will act as the cornerstones for the analysis throughout the book.

1.2 What Is a Deductive Argument?

Despite considerable variation in its numerous manifestations, three core features of deductive reasoning seem to stand out. They are aptly captured in the following definition of a mathematical proof, taken from the 1989 guideline of the National Council of Teachers of Mathematics (and quoted in Balacheff,

[3] Though this is probably more of a cluster of methods than a unified method as such.

1991, p. 177): a mathematical proof is “a careful sequence of steps with each step following logically from an assumed or previously proved statement and from previous steps.” So, a deductive argument is (i) a stepwise process, (ii) where each step ‘follows logically’ (iii) from assumed or previously established statements. In the remainder of this book, it will be further argued that focusing on these three aspects offers an adequate vantage point to investigate deduction in its many facets. Here, they are presented in decreasing order of general recognition of their centrality for the notion of deduction.[4]

1.2.1 Necessary Truth-Preservation

Recall the example above: if Fido is a dog and all dogs are animals, then it is *necessarily* the case that Fido is an animal; things just couldn’t possibly be any other way, if the premises are true. This property is typically referred to as the property of *necessary truth-preservation*, and is usually thought to be what distinguishes deductive arguments from inductive and abductive arguments (Douven, 2011), or deductive from defeasible arguments (Pollock, 1987). Indeed, this is what distinguishes deductive reasoning from other modes of reasoning – a necessary, constitutive property for any argument to count as deductively valid (though it may not be *sufficient* for deductive validity).

Another property that is closely related to necessary truth-preservation is the property of *monotonicity*: if an inference from A and B to C is valid, then adding any arbitrary premise D will not block the inference to the conclusion C from A, B, and D. Monotonicity follows quite straightforwardly from necessary truth-preservation in the following way: what necessary truth-preservation ensures is that in all situations where A and B are the case, C will also be the case. Now, this includes all situations where A, B, and D are the case, for any arbitrary D, since these constitute a subclass (proper or not) of the class of situations where A and B are the case. And thus, the addition of a premise will only restrict (or keep unchanged) the class of situations under consideration, which will then still satisfy C. In effect, inductive, abductive, and more generally defeasible inferences, which lack the property of necessary truth-preservation, also lack the property of monotonicity (Koons, 2013).

[4] It might be thought that necessary truth-preservation alone constitutes the true core of deduction, as a necessary as well as sufficient condition for a reasoning to count as a deduction, and that the other two requirements, especially perspicuity, in fact define what counts as a ‘good’ deduction rather than deduction tout court. To some extent, this is merely a terminological matter; but my choice to include these two other features of deduction as what constitutes its core reflects the functionalist commitment that underpins this investigation. I am not only interested in what a deduction is in some abstract, freestanding sense; I am mostly interested in what deduction can do for us, and thus in those instances that fulfill the roles attributed to a deductive argument in, for example, mathematical practice.

Notice, however, that there are examples of deductive systems that lack the property of monotonicity, in particular, relevant logics, which require that there be a relation of relevance between premises and conclusion and thus restrict the addition of arbitrary premises to a given (relevantly valid) argument. These systems will have necessary truth-preservation as a necessary but not as a sufficient condition for deductive validity. (Classical logic is what is obtained if necessary truth-preservation is viewed as both necessary *and* sufficient for validity, at least if we restrict ourselves to bivalent systems.)

Once deductive validity is defined as having necessary truth-preservation as a necessary condition, what it takes to show that an argument or inference is deductively *in*valid is to show that it is possible for the premises to be true while the conclusion(s) is (are) not, usually by describing a situation where this is the case. These situations are typically referred to as *counterexamples*. What a counterexample shows is that the truth of the premise(s) does not necessitate the truth of the conclusion, and instead is compatible with the non-truth of the conclusion (it may in fact be compatible both with its truth and with its non-truth).

This may all seem quite straightforward at first sight, but the nature of the necessity relating premises to conclusions in deductively valid inferences/arguments is perhaps one of the most mysterious features of deduction. What kind of necessity is this? Is it metaphysical? Semantic/linguistic? Logical? This question will be discussed in more detail in Section 1.3.2 below.

1.2.2 Stepwise Structure: Perspicuity

However, necessary truth-preservation is not the whole story. Take for example Fermat's last theorem, which was proved in the 1990s by Andrew Wiles after having defied mathematicians for centuries. (Wiles' proof is exceedingly complex and long.) Now imagine that I state the axioms of Peano Arithmetic and then in one step, with no intermediaries, conclude Fermat's last theorem. This 'argument' is truth-preserving, and indeed necessarily so (as we now know); no counterexample can be provided. And yet, such a one-step 'argument' will not be deemed satisfactory by anyone minimally acquainted with the deductive method. This is because something else is required of a good deductive argument other than necessary truth-preservation: it must somehow make clear what the connection is between premises and conclusion such that the truth of the premise(s) guarantees the truth of the conclusion(s). In other words, a deductive inference/argument, especially when formulated publicly (i.e. not 'mentally' by a given individual), must fulfill an *epistemic function* (more on which soon), and so each step must be individually perspicuous and the whole still comprehensible.[5]

Notice that for individual inferential steps to be properly chained, obtaining the desired effect of leading from premises to conclusion(s) in a deductive argument, the property of *transitivity* must be in place. That is, if A implies B and B implies C, this entails that A implies C. Transitivity is usually taken to be a rather straightforward principle, as indeed the very possibility of a multiple-step deductive argument seems to hinge on it. But it has been contested in recent work, especially with respect to the sorites paradox (Zardini, 2008; Fjellstad, 2016).

Indeed, a deductive argument, say a mathematical proof, will typically contain numerous steps, each of which may be individually simple and thus individually not very informative, but by chaining such steps in a suitable way we may derive non-trivial conclusions from the given premises. And thus, the interesting, informative deductive arguments are typically those with a fair number of steps, precisely because it is a desirable feature of a deductive inference that it be compelling – that each of its steps be at least to some extent self-evident, or in any case that they be suitably *justified*. Importantly, the level of granularity required for a deductive argument to be considered adequate will vary according to context; for example, a mathematical proof presented in a journal for professional mathematicians will typically be more 'dense,' i.e. less detailed, than a proof presented in an introductory textbook for students (Schiller, 2013).

Consider for instance proofs in Euclid's *Elements*, where the steps are often justified in terms of the postulates presented at the very beginning of the work, or else by other theorems previously proved. The balance between perspicuity and informativeness is thus achieved by chaining a significant number of individually evident one-step inferences. (Notice, though, that a very long proof, with a very large number of steps, which can no longer be easily surveyed by a human at a glance, is often thought to lack perspicuity [Bassler, 2006]. Indeed, mathematics educators observe that there is often a tension between local and global understanding of proofs [Alibert & Thomas, 1991].)

1.2.3 Bracketing Belief

Perhaps the least recognized of the three key components of deductive reasoning as described here is what we might refer to as the *bracketing belief* requirement. In its basic form, the game of deduction requires the reasoner to take the premises at face value, no questions asked: the focus is exclusively on

5 Wittgenstein speaks of the *Übersichtlichkeit* or 'surveyability' of proofs (Marion, 2011), which is sometimes interpreted as an objection to very long mathematical proofs, but more generally simply amounts to a recognition of the essential epistemic import of proofs.

the *connection* between premises and conclusions, not on the nature or plausibility of the premises or conclusions. (However, this is not the case for Aristotle's theory of science in the *Posterior Analytics* and the axiomatic method more generally, where a number of requirements are placed on acceptable premises.)

For those suitably 'indoctrinated' in the game of deduction, this feature may appear to be unproblematic or even trivial, but this is in fact not the case. Try to explain to a group of uninitiated, logically naïve interlocutors (say, high-school students) that 'All cows are blue, and all blue things are made of stone, so all cows are made of stone' is a perfectly fine deductive inference. In most cases, the reaction will be of mild indignation that such a strange argument can be deemed 'good' in any sense whatsoever, given the absurdity of the sentences involved. (In more technical terms: the distinction between the soundness and the validity of an argument is usually not grasped by those not having received some kind of training in logic, mathematics, or philosophy.)

What is required of the reasoner is that she put her own beliefs about premises and conclusions aside in order to focus exclusively on the *connection* between premises and conclusions. As it happens, this is a cognitively demanding task, as the extensive literature on the so-called belief bias effect illustrates: despite being told to focus solely on the validity of arguments, participants often let their judgments of validity be influenced by the (un)believability of the sentences involved (Markovits & Nantel, 1989; Evans, 2016) (see Chapter 8). Typically, arguments with believable conclusions will be deemed valid, whereas arguments with unbelievable conclusions will be deemed invalid, regardless of their actual validity (though validity also has an effect, as, within each of these two believability classes, valid arguments are more often deemed valid than invalid ones).

In fact, it has been observed that, in reasoning experiments, participants with little or no formal schooling often resist the very idea of reasoning on the basis of premises that they have no knowledge of. In the 1930s, the Russian psychologist Alexander Luria conducted reasoning experiments with unschooled peasants in the then-Soviet republic of Uzbekistan, which showed that the unschooled participants did not spontaneously dissociate their beliefs in the premises from the reasoning itself (see Chapter 9). Here is a description of one of his experiments:

> "In the Far North, where there is snow, all bears are white. Novaya Zemlya is in the Far North. What colour are bears there?" In response to this problem, [a given participant] protested: "You've seen them – you know. I haven't seen them, so how could I say!?" . . . the interviewer encouraged him to focus on the wording of the problem: "But on the basis of what I said, what do you think?" and re-stated the problem. This repetition met with the same refusal: "But I never saw them, so how could I say?" (Harris, 2000, p. 96)

Thus, it seems that inferring conclusions from premises while disregarding one's own doxastic attitudes toward premises and conclusions may require specific training. Yet, it is an integral component of deductive reasoning. Indeed, in mathematics it is very common to produce conditional proofs: 'If A and B are true, then so is C.' For example, the ABC conjecture, which, pending wide acceptance of the correctness of Shinichi Mochizuki's purported proof by the mathematical community, is still a conjecture (see Chapter 11), has been proved to imply a number of other interesting conjectures, such as Catalan's conjecture (Glivický & Kala, 2017). In such cases, the mathematician takes a conjecture as her starting point and goes on to investigate what follows from it even if she does not (yet) have a definite position on the conjecture itself.

These three features of deductive reasoning will provide the cornerstones for the analysis throughout the book. At this point, I have only presented each of them superficially, but we will see that each raises a number of puzzles and issues; none of them is either cognitively or philosophically straightforward. In Section 1.3, I present an overview of the main difficulties and issues pertaining to the concept of deductive reasoning as they have been discussed in the literature. We will see that the property of necessary truth-preservation has been quite extensively discussed and problematized, but the other two properties less so.

Before we move on, a few remarks are in order on a property that is typically associated with deduction, and yet is conspicuously absent from my list: the property of *formality*. According to a familiar story, deductive validity is a matter of *logical form*; an argument is valid if and only if it suitably instantiates one of the logical forms recognized as ensuring validity. Elsewhere, however, I have argued extensively against accounts of the nature of deductive validity in terms of logical form (Dutilh Novaes, 2012a, 2012b). Rather than being that in virtue of which an argument is deductively valid, logical forms/schemata are in fact convenient devices that allow us to track deductive validity with less effort (though for a limited range of arguments). Philosophically, however, the doctrine of logical form fails to deliver a satisfactory account of validity (as also argued by authors such as John Etchemendy [1983] and Stephen Read [1994] before me). For this reason, the property of formality will not be considered among the key features of the notion of deduction in the present investigation, despite the widespread (but to my mind erroneous) belief that it is indeed one of its key features.

1.3 Issues

In this section, I present three philosophical questions pertaining to deductive reasoning which remain by and large unresolved. The point of this section is mainly to show that the puzzle of deduction has not yet been cracked, and thus that an extensive investigation on deductive reasoning and argumentation is still very much needed. These open questions will then be further addressed in later chapters.

1.3.1 Where Is Deduction to Be Found?

In much of the literature in philosophy of logic and on deductive reasoning more generally, it is often assumed that deduction is a widespread phenomenon. For example, according to Stewart Shapiro in his recent book *Varieties of Logic*, “logic is ubiquitous” (Shapiro, 2015, p. 209). In these discussions, it is customary to adduce armchair arguments on what ‘people’ do or do not conclude deductively in a number of scenarios; the presupposition seems to be that something like deductive competence (akin to Chomskian linguistic competence) is a given in humans (and perhaps even in some non-human animals).

But is it really so? The experimental literature in the psychology of reasoning seems to suggest that things are not so simple. (See Dutilh Novaes, 2012a, chapter 4 for a systematic survey of these findings, and Chapter 8 of this book.) Initially conducted against the background of a Piagetian (neo-Kantian) paradigm where the traditional canons of logic were thought to correspond to the basic building blocks of human cognition, since the 1960s experiments have shown time and again that participants typically perform ‘poorly’ in deductive reasoning tasks (at least in experimental settings) (Johnson-Laird, 2008). Deviations from the normative responses as dictated by the canons of deductive reasoning were robust, consistent, and systematic, but traditional logic (in particular, syllogistic and classical propositional logic) continued to provide the theoretical background for the formulation and interpretation of experiments for decades. It was only in the 1980s that some researchers (in particular Mike Oaksford and Nick Chater) began to question the adequacy of traditional logic as a theoretical framework for the investigation of human reasoning, and only in the 2000s that the idea that traditional deductive logic is not in any way an adequate descriptive model of human reasoning became more or less a consensus among psychologists of reasoning (Elqayam, 2018). Tellingly, a survey article by one of the leading researchers in the field, Jonathan Evans (2002), is informally known among psychologists as the ‘death of deduction.’

Other than questioning the adequacy of logic as the right normative theoretical framework, a number of responses to the discrepancy between the deductive canons and these experimental findings have been formulated (Elio, 2002). One may, for example, maintain that the deductive canons do indeed define the ideal of rationality, and that the realization that human reasoners do not conform to these canons forces upon us the bitter conclusion that humans are irrational. One may also discount these discrepancies as a competence/

performance gap, similar to how generative grammar explains the fact that competent speakers regularly make grammatical mistakes.[6] Another response is to discredit the importance of these experimental findings in view of their presumed lack of ecological validity: what human reasoners do in the artificial experimental setting of these experiments is not illustrative of how they in fact reason in more realistic situations (Gigerenzer, 1996).

A view that has garnered much prominence in recent decades (though probably still not unanimously accepted) is that human reasoning has a very strong component of defeasibility, both in everyday life and in specific, specialized contexts such as legal argumentation and scientific reasoning (Pollock, 1987; Stenning & van Lambalgen, 2008; Oaksford & Chater, 2002; Koons, 2013). Defeasible reasoning can be defined in the following way: from premises A and B, the agent may reasonably conclude C. However, upon receiving new information, say D, C is no longer plausible to the agent, even if A and B still stand. In other words, the agent takes A and B to imply C, but not A, B, *and* D to imply C. Naturally, given that it has the properties of necessary truth-preservation and monotonicity as described above, deductive reasoning is fundamentally at odds with the principles of defeasible reasoning; indeed, it is *indefeasible*. How best to model defeasible reasoning formally (Bayesian probabilities, non-monotonic logics, etc.) is still a matter of contention, but it is clear that monotonic, indefeasible deductive logics are utterly inadequate for this job.

In sum, whether deductive reasoning is ubiquitous is by and large an empirical question, and the empirical data currently available suggest that it is not. So where is deduction to be found? Are we theorizing about a non-existent phenomenon, like seventeenth-century scientists who theorized about phlogiston? I submit that deductive reasoning is *not* like phlogiston; it may not be as widespread and ubiquitous as Shapiro and many others still surmise, but it *is* a thing. Indeed, as hinted at the beginning of this chapter, deductive reasoning is a central component of mathematical practice, both historically and in its current state, and is thus also present in fields where mathematics is widely used. Deduction is also present in a number of other areas of inquiry, in particular in philosophy.

Notice that not all reasoning in mathematics is deductive, strictly speaking (Aberdein, 2009), but much of it is: what counts as a mathematical proof is still by and large dictated by the canons established by Euclidean mathematics (even if there have been significant transformations across time – in particular, but not exclusively, pertaining to standards of rigor). Indeed, Euclid-style proofs are still a fundamental part of the mathematics curriculum, and any

[6] This response is from the start not very convincing, because the magnitude of deviation between competence and performance in the case of deductive reasoning appears to be much larger than in the case of language use.