Cambridge University Press
978-1-108-77152-8 — Cambridge Lower Secondary Mathematics Learner's Book 8 with Digital Access (1 Year), 2nd ed.
Lynn Byrd, Greg Byrd, Chris Pearce
Excerpt
More Information

1 Integers

Getting started

1 **a** Find all the prime numbers less than 20.
b Show that there are two prime numbers between 20 and 30.

2 **a** Find all the factors of 18.
b Find all the 2-digit multiples of 18.
c Find the highest common factor of 18 and 12.
d Find the lowest common multiple of 18 and 12.

3 Work out

a $-6+3$ **b** $-6-3$ **c** -6×3
d $-6\div3$ **e** $8+-10$ **f** $-5 - -9$

4 Write whether each of these numbers is a square number, a cube number or both.

a 49 **b** 27 **c** 1000
d 64 **e** 121 **f** 225

5 Find

a $\sqrt{100}$ **b** $\sqrt[3]{125}$ **c** $\sqrt{15^2-12^2}$

Prime numbers have exactly two factors, 1 and the number itself.

Some examples of prime numbers are 7, 31, 83, 239 and 953.

The number 39 is the product of two prime numbers (3 and 13).
It is quite easy to find these two numbers.

The number 2573 is also the product of two prime numbers (31 and 83).
It is much harder to find the two numbers in this case.

It is easy to multiply two prime numbers together using a calculator or a computer.

It is much harder to carry out the inverse operation – that is, to find the two prime numbers that multiply to a given product. This fact is the basis of a system used to encode messages sent across the internet.

The **RSA cryptosystem** was invented by Ronald **R**ivest, Adi **S**hamir and Leonard **A**dleman in 1977. It uses two large prime numbers with about 150 digits each. These numbers are kept secret, but anybody can use their product, *N*, which has about 300 digits.

If someone sends their credit card number to a website, their computer does a calculation using *N* to encode their credit card number. The computer that receives the coded number does another calculation to decode it. Anyone who does not know the two factors of *N* will not be able to do this. Your credit card number is protected.

1.1 Factors, multiples and primes

In this section you will …

- write a positive integer as a product of prime factors
- use prime factors to find a highest common factor (HCF) and a lowest common multiple (LCM).

Key words

factor tree
highest common factor (HCF)
index
integer
lowest common multiple (LCM)
prime factor

Any **integer** bigger than 1:

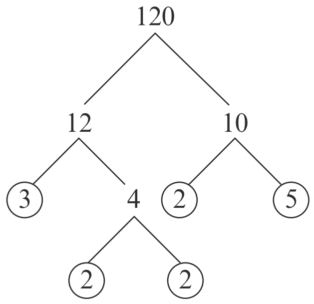
- is a prime number, or
- can be written as a product of prime numbers.

Example:

$46 = 2 \times 23$ 47 is prime $48 = 2 \times 2 \times 2 \times 2 \times 3$
$49 = 7 \times 7$ $50 = 2 \times 5 \times 5$

You can use a **factor tree** to write an integer as a product of its **prime factors**.

This is how to draw a factor tree for 120.



1 Write 120.
2 Draw branches to two numbers that have a product of 120. Do not use 1 as one of the numbers. Here we have chosen 12 and 10.
$120 = 12 \times 10$
3 Do the same with 12 and 10. Here $12 = 3 \times 4$ and $10 = 2 \times 5$
4 3, 2 and 5 are prime numbers, so circle them.
5 Draw two more branches from 4. $4 = 2 \times 2$. Circle the 2s.
6 Now all the end numbers are prime, so stop.
7 120 is the product of all the end numbers: $120 = 2 \times 2 \times 2 \times 3 \times 5$
8 You can check that this is correct using a calculator.

You can also write the result like this: $120 = 2^3 \times 3 \times 5$

2^3 means $2 \times 2 \times 2$ and the small 3 is an **index**.

Now check that $75 = 3 \times 5^2$

You can use products of prime factors to find the **HCF** and **LCM** of two numbers.

CAMBRIDGE
Cambridge University Press
978-1-108-77152-8 — Cambridge Lower Secondary Mathematics Learner's Book 8 with Digital Access (1 Year), 2nd ed.
Lynn Byrd, Greg Byrd, Chris Pearce
Excerpt
More Information

Worked example 1.1

a Find the LCM of 120 and 75.
b Find the HCF of 120 and 75.

Answer

a Write 120 and 75 as products of their prime factors:

$120 = \boxed{2 \times 2 \times 2 \times 3} \times 5$

$75 = 3 \times \boxed{5 \times 5}$

Look at the prime factors of both numbers.

For the LCM, use the **larger** frequency of each prime factor.

- 120 has three 2s and 75 has no 2s. The LCM must have three 2s.
- 120 has one 3 and 75 has one 3. The LCM must have one 3.
- 120 has one 5 and 75 has two 5s. The LCM must have two 5s.

The LCM is $2 \times 2 \times 2 \times 3 \times 5 \times 5 = 2^3 \times 3 \times 5^2 = 8 \times 3 \times 25 = 600$

b For the HCF use the **smaller** frequency of each factor: there are no 2s in 75, and there is one 3 and one 5 in both numbers.

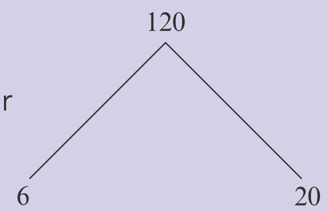
Multiply these factors.

The HCF is $3 \times 5 = 15$

Exercise 1.1

Think like a mathematician

1 The factor tree for 120 in Section 1.1 started with 12 × 10.



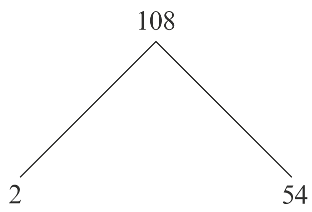
a Draw a factor tree for 120 that starts with 6 × 20.

b Compare your answer to part **a** with a partner's. Are your trees the same or different?

c Draw some different factor trees for 120. Can you say how many different trees are possible?

d Do all factor trees for 120 have the same end points?

2 **a** Complete this factor tree for 108.



b Draw a different factor tree for 108.

c Write 108 as a product of its prime factors.

d Compare your factor trees and your product of prime factors with a partner's. Have you drawn the same trees or different ones? Are your trees correct?

3 **a** Draw a factor tree for 200 that starts with 10×20.
b Write 200 as a product of prime numbers.
c Compare your factor tree with a partner's. Have you drawn the same tree or different ones? Are your trees correct?
d How many different factor trees can you draw for 200 that start with 10×20?

4 **a** Draw a factor tree for 330.
b Write 330 as a product of prime numbers.

5 Match each number to a product of prime factors.
The first one has been done for you: **a** and **i**.

a	20 ———	**i**	$2^2 \times 5$
b	24	**ii**	$2 \times 3 \times 7$
c	42	**iii**	$2^2 \times 3^2 \times 5$
d	50	**iv**	2×5^2
e	180	**v**	$2^3 \times 3$

6 Work out the product of each set of prime factors.
a $3^2 \times 5 \times 7$ **b** $2^3 \times 5^3$ **c** $2^2 \times 3^2 \times 11$
d $2^4 \times 7^2$ **e** 3×17^2

Tip
You can use a factor tree to help you.

7 Write each of these numbers as a product of prime factors.
a 28 **b** 60 **c** 72
d 153 **e** 190 **f** 275

8 **a** Copy the table and write each number as a product of prime numbers.

Number	Product of prime numbers
35	5×7
70	
140	
280	

b Add more rows to the table to continue the pattern.

9 **a** Write 1001 as a product of prime numbers.
b Write 4004 as a product of prime numbers.
c Write 6006 as a product of prime numbers.

10 **a** Use a factor tree to write 132 as a product of prime numbers.
b Write 150 as a product of prime numbers.
c $132 \times 150 = 19\,800$. Use this fact to write 19 800 as a product of prime numbers.

11 **a** Write each of these numbers as a product of prime numbers.

i 15 **ii** 15^2 **iii** 28

iv 28^2 **v** 36 **vi** 36^2

b What do you notice about your answers to **i** and **ii**, **iii** and **iv**, **v** and **vi**?

c If $96 = 2^5 \times 3$, show how to find the prime factors of 96^2. Will your method work for all numbers?

12 $40 = 2 \times 2 \times 2 \times 5$ and $28 = 2 \times 2 \times 7$

Use these facts to find

a the HCF of 40 and 28 **b** the LCM of 40 and 28.

13 $450 = 2 \times 3 \times 3 \times 5 \times 5$ and $60 = 2 \times 2 \times 3 \times 5$

Use these facts to find

a the HCF of 450 and 60 **b** the LCM of 450 and 60.

14 $180 = 2^2 \times 3^2 \times 5$ and $54 = 2 \times 3^3$

Use these facts to find

a the HCF of 180 and 54 **b** the LCM of 180 and 54.

15 **a** Write 45 as a product of prime numbers.

b Write 75 as a product of prime numbers.

c Find the LCM of 45 and 75.

d Find the HCF of 45 and 75.

16 **a** Draw factor trees to find the LCM of 90 and 140.

b Compare your answer with a partner's. Did you draw the same factor trees? Have you both got the same answer?

17 **a** Write 396 as a product of prime numbers.

b Write 168 as a product of prime numbers.

c Find the HCF of 396 and 168.

d Find the LCM of 396 and 168.

18 **a** Find the HCF of 34 and 58.

b Find the LCM of 34 and 58.

19 Show that the HCF of 63 and 110 is 1.

20 37 and 47 are prime numbers.

a What is the HCF of 37 and 47?

b What is the LCM of 37 and 47?

c Write a rule for finding the HCF and LCM of two prime numbers.

d Compare your answer to part **c** with a partner's answer. Check your rules by finding the HCF and LCM of 39 and 83.

Tip

Use a calculator to help you.

Cambridge University Press
978-1-108-77152-8 — Cambridge Lower Secondary Mathematics Learner's Book 8 with Digital Access (1 Year), 2nd ed.
Lynn Byrd, Greg Byrd, Chris Pearce
Excerpt
More Information

In this exercise you have:

- used factor trees to write an integer as a product of prime factors
- found the HCF of two integers by first writing each one as a product of prime numbers
- found the LCM of two integers by first writing each one as a product of prime numbers.

a Which questions have you found the easiest? Explain why.

b Which questions have you found the hardest? Explain why.

Summary checklist

- [ ] I can write an integer as a product of prime numbers.
- [ ] I can find the HCF and LCM of two integers by first writing each one as a product of prime numbers.

1.2 Multiplying and dividing integers

In this section you will …

- multiply and divide integers, in particular when both are negative
- understand that brackets, indices and operations follow a particular order.

Key words

brackets
conjecture
inverse
investigate

You can add and subtract any two integers.

For example:

$2 + -4 = -2$　　$-2 + -4 = -6$　　$-2 - 4 = -6$　　$-2 - -4 = 2$

You can also multiply and divide a negative integer by a positive one.

For example:

$2 \times -9 = -18$　　$-6 \times 3 = -18$　　$-18 \div 3 = -6$　　$20 \div -5 = -4$

In this section you will **investigate** how to multiply or divide any two integers. You will use number patterns to do this.

CAMBRIDGE
Cambridge University Press
978-1-108-77152-8 — Cambridge Lower Secondary Mathematics Learner's Book 8 with Digital Access (1 Year), 2nd ed.
Lynn Byrd, Greg Byrd, Chris Pearce
Excerpt
More Information

Worked example 1.2

Look at this sequence of subtractions.

$3 - 6 = -3$
$3 - 4 = -1$
$3 - 2 =$
$3 - 0 =$
$3 - -2 =$
$3 - -4 =$

a Copy the sequence and fill in the missing answers.
b Write the next three lines in the sequence.
c Describe any patterns in the sequence.

A sequence is a set of numbers or expressions made and written in order, according to some pattern.

Answer

a $3 - 2 = 1$
$3 - 0 = 3$
$3 - -2 = 5$
$3 - -4 = 7$

b $3 - -6 = 9$
$3 - -8 = 11$
$3 - -10 = 13$

c The first number, 3, does not change.
The number being subtracted decreases by 2 each time.
The answer increases by 2 each time.

Exercise 1.2

Think like a mathematician

1 Here is the start of a sequence of multiplications.
$-3 \times 4 = -12$
$-3 \times 3 =$
$-3 \times 2 =$

a Copy the sequence and write six more terms. Use a pattern to fill in the answers.
b Describe the patterns in the sequence.

Continued

c Here is the start of another sequence of multiplications.

$-5 \times 4 =$

$-5 \times 3 =$

$-5 \times 2 =$

Copy the sequence and write six more terms.
Describe any patterns in the sequence.

d In the sequences in **a** and **c**, you have some products of two negative integers. What can you say about the product of two negative integers?

e Make up a sequence of your own like the ones in **a** and **c**.

f Share your answers to parts **d** and **e** with a partner. Are your partner's sequences correct?

2 Work out these multiplications.

a 5×-2 **b** -5×2 **c** -5×-2 **d** -2×-5

3 Work out these multiplications.

a -6×-4 **b** -7×-7 **c** -10×-6 **d** -8×-11

4 Copy and complete this multiplication table.

×	−5	3	−8
4			
−3		−9	
−6	30		

5 Work out

a $(3 + 5) \times -4$ **b** $(-3 + -5) \times -6$

c $-4 \times (5 - 8)$ **d** $-6 \times (-2 - -7)$

Tip

Do the calculation in **brackets** first.

6 Round these numbers to the nearest whole number to estimate the answer.

a 3.9×-6.8 **b** -11.2×2.95

c $(-6.1)^2$ **d** $(-4.88)^2$

7 **a** Put these multiplications into groups based on the answers.

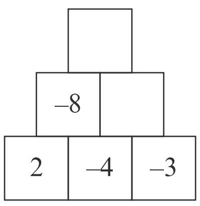
3×-4 -6×-2 12×1

-4×-3 2×-6 -12×-1

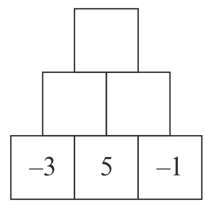
b Find one more product to put in each group.

8 These are multiplication pyramids.

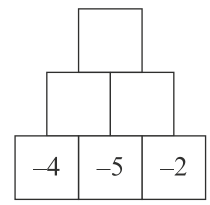
a



b



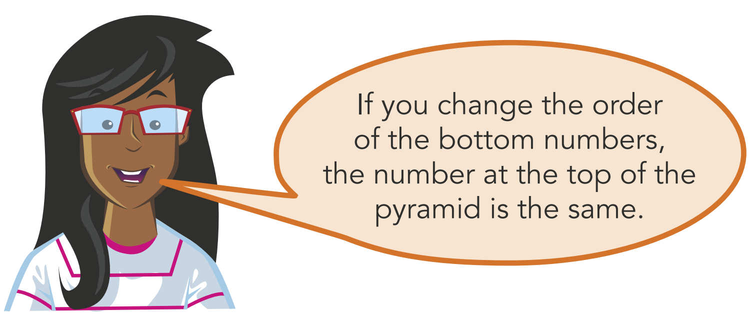
c



Each number is the product of the two numbers below it. For example, in **a**, $2 \times -4 = -8$

Copy and complete the multiplication pyramids.

9 **a** Draw a multiplication pyramid like those in Question 8, with the integers −2, 3 and −5 in the bottom row, in that order. Complete your pyramid.



b Is Zara correct? Test her idea by changing the order of the numbers in the bottom row of your pyramid.

10 Find the missing numbers in these multiplications.

a $-3 \times \square = -12$ **b** $-5 \times \square = 45$

c $\square \times -6 = 24$ **d** $\square \times -10 = 80$

Think like a mathematician

11 A multiplication can be written as a division.
For example, $5 \times 8 = 40$ can be written as $40 \div 8 = 5$ or $40 \div 5 = 8$

a Here is a multiplication: $-4 \times 6 = -24$
Write it as a division in two different ways.

b Write a multiplication of a positive integer and a negative integer.
Then write it as a division in two different ways.

c Here is a multiplication: $-7 \times -2 = 14$
Write it as a division in two different ways.

d Write a multiplication of two negative integers.
Then write it as a division in two different ways.

Tip

A conjecture is a possible value based on what you know.