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Nick Hamshaw , Elaine Dorsett
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Chapter 1
Velocity and acceleration

EXERCISE 1A

1 $s = vt$

$120 = 15v$

$v = \frac{120}{15} = 8 \text{ m s}^{-1}$

2 $s = vt$

$s = 9 \times 7 = 63 \text{ m}$

3 a $s = vt$

$150 = 25t$

$t = \frac{150}{25} = 6 \text{ s}$

b The gazelle does not move and the cheetah reaches the appropriate speed instantaneously.

> Many of the examples and questions you complete at this level assume constant speed or instantaneous change of speed. Neither of these are quite realistic so it is important you always question the validity of the model being used.

4 150 million km $= 150 \times 10^6 = 1.5 \times 10^8$ km

$= 1.5 \times 10^8 \times 1000 \text{ m} = 1.5 \times 10^{11} \text{ m}$

$s = vt$

$1.5 \times 10^{11} = 3 \times 10^8 \times t$

$t = \frac{1.5 \times 10^{11}}{3 \times 10^8} = 0.5 \times 10^3 = 5 \times 10^2 \text{ s}$

$= 500 \text{ s} = 8$ mins 20 seconds

5 $s = vt$

$1 = 1223.657 \times t$

$t = \frac{1}{1223.657}$ hours

$t = \frac{1}{1223.657} \times 60 \times 60$ seconds

$= 2.94$ seconds (3 significant figures)

6 $s_1 = vt = 5 \times 7 = 35 \text{ m}$

$s_2 = vt = 7 \times 13 = 91 \text{ m}$

Total distance $= 35 + 91 = 126$ m

Total time $= 7 + 13 = 20$ s

a average speed $= \frac{\text{total distance}}{\text{total time}} = \frac{126}{20} = 6.3 \text{ m s}^{-1}$

b The change of speed is instantaneous. The speed is constant for each section.

7 $s = vt$

$s_1 = 6 \times 10 = 60$ m forwards

$s_2 = 3 \times 5 = 15$ m backwards

a $s_1 - s_2 = 60 - 15 = 45 \text{ m}$

b $\frac{\text{total displacement}}{\text{total time}} = \frac{45}{15} = 3 \text{ m s}^{-1}$

c $\frac{\text{total distance}}{\text{total time}} = \frac{75}{15} = 5 \text{ m s}^{-1}$

8 $s = vt = 11 \times 5 = 55$ m travelled

Total time $= 15$ s

$s = vt = 12 \times 15 = 180 \text{ m}$

Travels $180 - 55 = 125$ m in 10 seconds

Average speed over next 10 seconds $= \frac{125}{10}$

$= 12.5 \text{ m s}^{-1}$

9 Air:

$s = vt$

$33 = 330t$

$t = \frac{33}{330} = 0.1 \text{ s}$

Wood:

$s = vt$

$33 = 3300t$

$t = \frac{33}{3300} = 0.01 \text{ s}$

Difference = 0.1 – 0.01 = 0.09 s

When you need to carry out similar calculations for two or more situations, always separate the working for each situation and label which case you are considering.

10 It is usually easier to convert minutes into seconds; speeds are rarely given in units per minute.

3 minutes and 10 seconds = 180 + 10 = 190 seconds

Time jogging = t seconds

Time sprinting = 190 – t seconds

$$s = v_1 t + v_2(190 - t)$$
$$1000 = 4t + 7(190 - t)$$
$$4t + 1330 - 7t = 1000$$
$$3t = 330$$
$$t = 110 \text{ seconds}$$

So, time spent sprinting = 190 – 110 = 80 s

11 Distance = d

Faster:

$s = vt$

$d = 45t$

Slower:

$s = vt$

$d = 44(t + 8)$

$45t = 44(t + 8)$

$45t = 44t + 352$

$t = 352$ seconds

$d = 45 \times 352 = 15800$ m

Notice that we usually avoid including units until the end of a particular section of calculation. This can avoid confusion, especially when there are algebraic terms to work with.

12 First puck travels for t seconds

Second puck travels for $t - 0.2$ seconds

Total distance travelled by pucks = 2 m

If one puck travels d m before the collision, the second puck must travel the remainder of the original 2 m distance, i.e. 2 – dm.

$$1.3t + 1.7(t - 0.2) = 2$$
$$1.3t + 1.7t - 0.34 = 2$$
$$3t = 2.34$$
$$t = 0.78 \text{ s}$$

$s = vt = 1.3 \times 0.78 = 1.01$ m

13 In this question, you are asked to show that the average speeds over AC and AB are the same. It is worth noting that, if the average over AB is the same as the average over BC, then this is also the same as the average over AC.

a $t_1 = t_2 = t$

Total time = $2t$

Total distance = $s_1 + s_2$

$$\text{Average speed} = \frac{s_1 + s_2}{2t} = \frac{1}{2}\left(\frac{s_1}{t} + \frac{s_2}{t}\right)$$
= average of speeds

b $s_1 = s_2 = s$

$$\text{Average speed for AC} = \frac{s + s}{t_1 + t_2} = \frac{2s}{t_1 + t_2}$$

$$\text{Average speed for AB} = \frac{s}{t_1}$$

$$\text{Equal if and only if } \frac{2s}{t_1 + t_2} = \frac{s}{t_1}$$

$$\Leftrightarrow \frac{2}{t_1 + t_2} = \frac{1}{t_1}$$
$$\Leftrightarrow t_1 + t_2 = 2t_1$$
$$\Leftrightarrow t_1 = t_2$$

The symbol ⇔ is specifically used to mean ‘if and only if’. This symbol indicates that the two mathematical statements on either side of it are exactly equivalent.

14 a Right:

$s = v_1 t_1$

Left:

$s = v_2 t_2$

$$v = \frac{\text{Total distance}}{\text{Total time}} = \frac{s + s}{t_1 + t_2} = \frac{2s}{t_1 + t_2}$$

$$= \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}}$$

$$= \frac{2s}{\frac{sv_2 + sv_1}{v_1 v_2}}$$

$$= \frac{2sv_1 v_2}{sv_2 + sv_1}$$

$$= \frac{2v_1 v_2}{v_1 + v_2}$$

b If

$$\frac{2v_1 v_2}{v_1 + v_2} = 2v_1$$

then

$$\frac{v_2}{v_1 + v_2} = 1$$

$$v_2 = v_1 + v_2$$

$$v_1 = 0$$

Which would then mean that the motion never actually takes place!

> Don't expect to 'spot' how to do questions like this immediately, at least not to start with. The more you experience difficult algebraic challenges, the better you will get. Initial failure should not put you off because it is all part of the learning process.

EXERCISE 1B

1 $a = \frac{v - u}{t}$

$a = \frac{10 - 4}{3} = 2 \text{ m s}^{-2}$

> Remember that the formula $a = \frac{v - u}{t}$ is the same as $v = u + at$, just rearranged.

2 $a = \frac{v - u}{t}$

$a = \frac{10 - 0}{4} = 2.5 \text{ m s}^{-2}$

3 $a = \frac{v - u}{t}$

$6 = \frac{12 - 3}{t}$

$6t = 9$

$t = 1.5 \text{ s}$

4 $a = \frac{v - u}{t}$

$3 = \frac{v - 4}{5}$

$15 = v - 4$

$v = 19 \text{ m s}^{-1}$

5 $a = \frac{v - u}{t}$

$1.5 = \frac{9 - u}{4}$

$6 = 9 - u$

$u = 3 \text{ m s}^{-1}$

6 $a = \frac{v - u}{t}$

$-2 = \frac{8 - u}{3}$

$-6 = 8 - u$

$u = 14 \text{ m s}^{-1}$

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7 $s = \frac{1}{2}(u+v)t$

To find t using $a = (v-u)/t$

$0.5 = (8-4)/t$

$0.5t = 4$

$t = 8\ s$

To find s:

$= \frac{1}{2}(4+8) \times 8$

$= 48$ m

8 a $s = \frac{1}{2}(u+v)t$

$60 = \frac{1}{2}(u+9) \times 10$

$60 = 5u + 45$

$5u = 15$

$u = 3\ \text{m s}^{-1}$

$a = \frac{v-u}{t} = \frac{9-3}{10} = 0.6\ \text{m s}^{-2}$

> Note that the formula $s = \frac{1}{2}(u+v)t$ is the same as writing that 'distance travelled is average speed multiplied by time'.

b We assume that the sprinter has a specific position. In reality sprinters have size and shape and so are spread over a distance range at any one time. We also assume that the sprinter can keep a constant acceleration.

9 $a = \frac{v-u}{t}$

First part: $a = \frac{1-0}{t}$

$at = 1$ .... [1]

Second part: $a = \frac{5-1}{t+1}$

$a(t+1) = 4$

$at + a = 4$ .... [2]

Substitute [1] into [2]

$1 + a = 4$

$a = 3\ \text{m s}^{-2}$

10 $s = \frac{1}{2}(u+v)t$

$100 = \frac{1}{2}(u+v) \times 10$

$u + v = 20$ .... [1]

$a = \frac{v-u}{t}$

$-0.4 = \frac{v-u}{10}$

$v - u = -4$ .... [2]

[1] + [2]

$2v = 16$

$v = 8\ \text{m s}^{-1}$

11 If the cyclist does nothing:

$u = 10$

$s = 80$

$a = 0.1$

$v^2 = u^2 + 2as$

$v^2 = 100 + 2 \times 0.1 \times 80$

$= 116$

$v = \sqrt{116} = 10.8\ \text{m s}^{-1}$

This is a safe speed but the cyclist could pedal just a little.

EXERCISE 1C

1 a $s = ut + \frac{1}{2}at^2$

$= 2 \times 4 + \frac{1}{2} \times 3 \times 4^2$

$= 8 + 24$

$= 32$ m

> If you find it difficult to see which equation to use, start by writing down the letters you know the values for and the letter you are trying to find. Then match these to the four variables in one of the equations.

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b $s = vt - \frac{1}{2}at^2$

$= 17 \times 8 - \frac{1}{2} \times 2 \times 8^2$

$= 136 - 64$

$= 72$ m

c $s = ut + \frac{1}{2}at^2$

$40 = 3 \times 5 + \frac{1}{2}a(5)^2$

$40 = 15 + \frac{25}{2}a$

$\frac{25}{2}a = 25$

$a = 2$ m s^{-2}

d $s = vt - \frac{1}{2}at^2$

$28 = 13 \times 4 - \frac{1}{2}a(4)^2$

$28 = 52 - 8a$

$8a = 24$

$a = 3$ m s^{-2}

e $v^2 = u^2 + 2as$

$14^2 = 2^2 + 2 \times a \times 24$

$196 - 4 = 48a$

$192 = 48a$

$a = 4$ m s^{-2}

f $s = ut + \frac{1}{2}at^2$

$45 = 6u + \frac{1}{2} \times 1.5 \times 6^2$

$45 = 6u + 18 \times 1.5$

$6u = 45 - 27$

$6u = 18$

$u = 3$ m s^{-1}

g $s = vt - \frac{1}{2}at^2$

$24 = 4v - \frac{1}{2} \times -2.5 \times 4^2$

$24 = 4v + 20$

$4v = 4$

$v = 1$ m s^{-1}

h $v^2 = u^2 + 2as$

$25 = 4 + 2 \times 0.75s$

$1.5s = 21$

$s = 14$ m

2 a $s = ut + \frac{1}{2}at^2$

$24 = 10t + \frac{1}{2} \times -2 \times t^2$

$24 = 10t - t^2$

$t^2 - 10t + 24 = 0$

$(t - 6)(t - 4) = 0$

$t = 4, 6$

First time is after 4 seconds

b $s = vt - \frac{1}{2}at^2$

$21 = 5t - \frac{1}{2} \times 0.5 \times t^2$

$21 = 5t - \frac{1}{4}t^2$

$84 = 20t - t^2$

$t^2 - 20t + 84 = 0$

$(t - 14)(t - 6) = 0$

$t = 6, 14$

First time is after 6 seconds

c $s = ut + \frac{1}{2}at^2$

$20 = 3t + \frac{1}{2} \times 1 \times t^2$

$t^2 + 6t - 40 = 0$

$(t + 10)(t - 4) = 0$

$t = 4, -10$

$t > 0$

$t = 4$ seconds

3 $v^2 = u^2 + 2as$

$v^2 = 25 + 2 \times -2 \times 6$

$v^2 = 25 - 24 = 1$

$v = \pm\sqrt{1} = \pm 1$

Initial velocity > 0

Change in direction so velocity now < 0

$v = -1$ m s^{-1}

4 $v^2 = u^2 + 2as$

$169 = u^2 + 2 \times 1 \times 60$

$u^2 = 169 - 120 = 49$

$v = \pm\sqrt{49} = \pm 7$

$v > 0$ so $u > 0$ as direction not changed

$u = 7$ m s^{-1}

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5 a $v^2 = u^2 + 2as$

$v^2 = 3^2 + 2 \times 2 \times 18$

$= 9 + 72$

$= 81$

$v = \pm\sqrt{81} = \pm 9$

$v > 0$

$v = 9 \text{ m s}^{-1}$

b $v > 0$ because $u > 0$ and $a > 0$

> The reason for part b is to explain why $v > 0$ in part a. Whenever you start with a positive velocity and continue with a positive acceleration, you will also have a final velocity that is positive. If both u and a were negative, then v would also be negative. Things are much more complicated when u and a have different signs, because v may be positive or negative depending on how long the motion lasts. You have to think carefully about every stage of the motion to decide.

6 $v^2 = u^2 + 2as$

$0 = 400 + 2 \times -4 \times s$

$400 = 8s$

$s = \frac{400}{8} = 50 \text{ m}$

7 $v^2 = u^2 + 2as$

$3600 = 0 + 2 \times a \times 400$

$800a = 3600$

$a = \frac{3600}{800} = 4.5 \text{ m s}^{-2}$

8 $v^2 = u^2 + 2as$

$6400 = 0 + 2 \times 4 \times s$

$8s = 6400$

$s = 800 \text{ m}$

9 $v^2 = u^2 + 2as$

$0 = 20^2 + 2 \times a \times 40$

$80a = -400$

$a = -5 \text{ m s}^{-2}$

Deceleration $= 5 \text{ m s}^{-2}$

> Note that negative acceleration is the same as positive deceleration. It is also true that positive acceleration is negative deceleration!

10 a $s = \frac{1}{2}(u + v)t$

$240 = \frac{1}{2}(30 + v) \times 12$

$240 = 180 + 6v$

$6v = 60$

$v = 10 \text{ m s}^{-1}$

b Deceleration is constant

11 $v^2 = u^2 + 2as$

$0 = 4.8^2 - 2 \times 0.3s$

$s = \frac{4.8^2}{0.6} = 8 \times 4.8 = 38.4 \text{ m}$

$38.4 - 38 = 0.4 \text{ m}$

12 $v^2 = u^2 + 2as$

$0 = 2.4^2 - 2 \times 0.3 \times s$

$0.6s = 2.4^2$

$s = 9.6 \text{ m}$

Stops before 10 m covered so it does not reach the hole.

13 If the car chooses to brake:

$v = u + at$

$= 17 - 8 \times 2$

$= 1 \text{ m s}^{-1}$

At this rate of braking the car still has positive velocity after 2 seconds and so will overrun the lights. The car cannot brake.

If the car accelerated instead:

$s = ut + \frac{1}{2}at^2$

$= 17 \times 2 + \frac{1}{2} \times 4 \times 2^2$

$= 34 + 8 = 42 \text{ m}$

At this rate of acceleration, the car will travel 2 m past the lights at the point of changing, so it is safe to accelerate.

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14 a $s = \frac{1}{2}(u+v)t$ and $v = u + at$

Replace v with $u + at$

$s = \frac{1}{2}(u+u+at)t$

$= \frac{1}{2}(2ut + at^2)$

$= ut + \frac{1}{2}at^2$

Although you will not usually be asked to prove these equations, it is still very sensible for you to work through how it is done. Really, this is just another example of solving simultaneous equations by substitution.

b $s = \frac{1}{2}(u+v)t$ and $v = u + at$

Rearranging the second equation $u = v - at$

$s = \frac{1}{2}(u+v)t$ Replace u with $v - at$

$= \frac{1}{2}(v - at + v)t$

$= vt - \frac{1}{2}at^2$

Rearranging $v = u + at$:

$t = \frac{v-u}{a}$

Substitute into $s = \frac{1}{2}(u+v)t$

$s = \frac{1}{2}(u+v)\left(\frac{v-u}{a}\right)$

$s = \frac{(v+u)(v-u)}{2a}$

$2as = v^2 + uv - uv - u^2$

$v^2 = u^2 + 2as$

15 Velocity at time $\frac{1}{2}t = u + a\left(\frac{1}{2}t\right) = u + \frac{1}{2}at$ ...... [1]

But $v = u + at$

$t = \frac{v-u}{a}$

Substituting into [1]

Velocity at time $\frac{1}{2}t = u + \frac{1}{2}a\left(\frac{v-u}{a}\right)$

$= u + \frac{1}{2}(v-u) = \frac{2u+v-u}{2} = \frac{u+v}{2}$

16 $v^2 = u^2 + 2as$ .... [1]

$\left(\text{speed at distance } \frac{1}{2}s\right)^2 = u^2 + 2a\left(\frac{1}{2}s\right) = u^2 + as$ [2]

[1] gives

$v^2 - u^2 = 2as$

$as = \frac{v^2 - u^2}{2}$

Substitute into [2]

$\left(\text{speed at distance } \frac{1}{2}s\right)^2 = u^2 + \frac{v^2-u^2}{2}$

$= \frac{2u^2 + v^2 - u^2}{2} = \frac{u^2+v^2}{2}$

speed at distance $\frac{1}{2}s = \sqrt{\frac{u^2+v^2}{2}}$

Half the distance happens after half the time because the object is slower in the first part of the motion and faster in the second part.

Speed at half the distance happens after speed at half the time, therefore, and so must be larger.

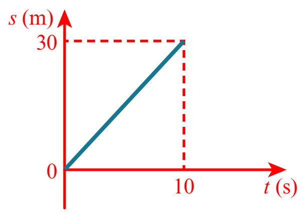
But speed at half the time is the mean of the initial and final speeds.

So speed at half the distance is greater than the mean of the initial and final speeds.

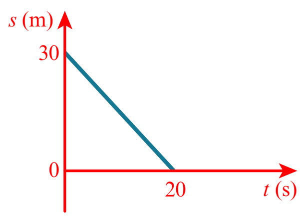
EXERCISE 1D

1 When sketching displacement–time graphs, it is essential that you label the axes correctly and include the units. You don't need to draw the graphs accurately, but you should try to show them roughly to scale and include all the key values.

a $s = vt = 3 \times 10 = 30$ m



b $t = \frac{s}{v} = \frac{30}{1.5} = 20\ s$



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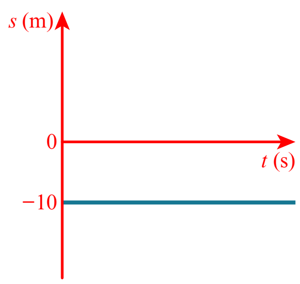
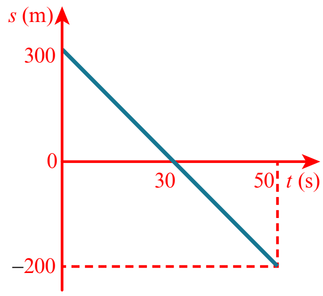


d First part:

$t = \frac{s}{v} = \frac{300}{10} = 30$ s

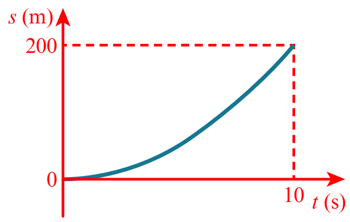
Second part:

$t = \frac{s}{v} = \frac{200}{10} = 20$ s



2 **a** $s = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times 4 \times 10^2 = 200$ m



b $v = u + at$

Remember that when an object rises vertically, it stops instantaneously before it starts to fall.

Speed is zero at maximum point

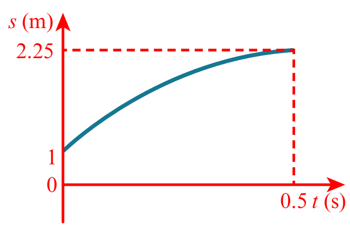
$0 = 5 - 10t$

$t = 0.5$

$s = ut + \frac{1}{2}at^2$

$= 5 \times 0.5 + 0.5 \times -10 \times 0.5^2 = 1.25$

Height $= 1 + 1.25 = 2.25$ m



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c At the lowest point:

$v = u + at$

$0 = -10 + 2t$

$t = 5s$

$s = ut + \frac{1}{2}at^2$

$= -10 \times 5 + \frac{1}{2} \times 2 \times 5^2$

$= -50 + 25 = -25$

$100 - 25 = 75$ m above the ground

At 175 m:

$s = ut + \frac{1}{2}at^2$

$75 = -10t + \frac{1}{2} \times 2 \times t^2$

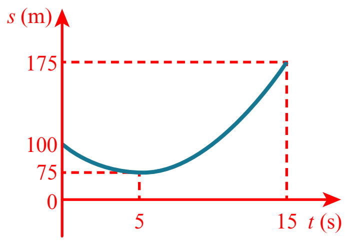
$t^2 - 10t - 75 = 0$

$(t - 15)(t + 5) = 0$

$t = -5, 15$

$t > 0$

$t = 15$



Although we have presented the graph at the end of the calculation, you should draw the graph as you go along. Always start by sketching the overall shape. This will show you the main values that you need to calculate, such as minimum points, total distances at certain times, etc.

d Highest point:

$v = u + at$

$0 = 5 - 10t$

$t = 0.5$

$s = ut + \frac{1}{2}at^2$

$= 5 \times \frac{1}{2} - \frac{1}{2} \times 10 \times 0.5^2$

$= 1.25$ m

At sea level:

$s = ut + \frac{1}{2}at^2$

$-18.75 = 5t - \frac{1}{2} \times 10 \times t^2$

$5t^2 - 5t - 18.75 = 0$

$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 5 \times -18.75}}{2 \times 5}$

$= -1.5$ or 2.5

$t > 0$

$t = 2.5$ s

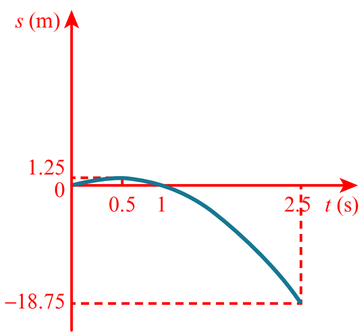
Level with top of cliff:

$s = ut + \frac{1}{2}at^2$

$0 = 5t - \frac{1}{2} \times 10 \times t^2$

$5 = 5t$

$t = 1$ s



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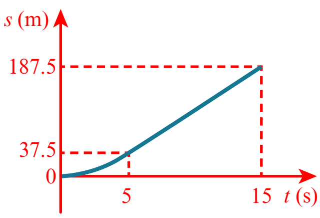
3 a In this question note that the speed at the end of the first stage (v in the first equation) becomes the speed at the start of the second stage (u in the second part).

After 5 seconds:

$$s = ut + \frac{1}{2}at^2$$
$$= 0 + 0.5 \times 3 \times 5^2$$
$$= 37.5$$
$$v = u + at$$
$$= 0 + 3 \times 5$$
$$= 15 \text{ m s}^{-1}$$

After 15 seconds:

$$s = ut + \frac{1}{2}at^2$$
$$= 15 \times 10 + 0$$
$$= 150 \text{ m}$$
$$37.5 + 150 = 187.5$$

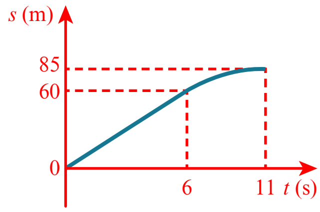


b At 6 seconds:

$$s = ut + \frac{1}{2}at^2$$
$$= 10 \times 6 + 0$$
$$= 60 \text{ m}$$

When it comes to rest:

$$v = u + at$$
$$0 = 10 - 2t$$
$$t = 5 \text{ s}$$
$$6 + 5 = 11 \text{ s}$$
$$s = ut + \frac{1}{2}at^2$$
$$= 10 \times 5 + \frac{1}{2} \times -2 \times 5^2$$
$$= 50 - 25 = 25 \text{ m}$$
$$60 + 25 = 85 \text{ m}$$

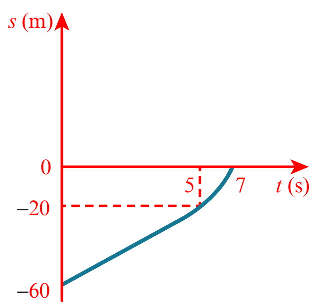


c At 20 m away:

$$s = ut + \frac{1}{2}at^2$$
$$40 = 8t$$
$$t = 5 \text{ s}$$

At the lights:

$$s = ut + \frac{1}{2}at^2$$
$$20 = 8t + \frac{1}{2} \times 2 \times t^2$$
$$t^2 + 8t - 20 = 0$$
$$(t + 10)(t - 2) = 0$$
$$t = 2, -10$$
$$t > 0 \text{ so } t = 2$$
$$5 + 2 = 7$$



Remember that the traffic lights are at distance 0, so this is why the positive distances become negative displacements on the graph. We calculated the *distance* that remained to be covered, but on the graph we show the *position* (or displacement) in relation to the traffic lights.