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Nick Hamshaw
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Chapter 1
Algebra

EXERCISE 1A

1 a $4x - 3 = 7$ or $4x - 3 = -7$

$4x = 10$ $\quad$ $4x = -4$

$x = \frac{5}{2}$ $\quad$ $x = -1$

Check:

$\left|4 \times \frac{5}{2} - 3\right| = |7| = 7$ ✓ $\quad$ $|4 \times (-1) - 3| = |-7| = 7$ ✓

Solutions are:

$x = \frac{5}{2}$ or $x = -1$

c $\frac{3x - 2}{5} = 4$ or $\frac{3x - 2}{5} = -4$

$3x - 2 = 20$ $\quad$ $3x - 2 = -20$

$3x = 22$ $\quad$ $3x = -18$

$x = \frac{22}{3}$ $\quad$ $x = -6$

Check:

$\left|\frac{3 \times \frac{22}{3} - 2}{5}\right| = |4| = 4$ ✓ $\quad$ $\left|\frac{3 \times (-6) - 2}{5}\right| = \left|-\frac{20}{5}\right| = |-4| = 4$ ✓

Solutions are:

$x = \frac{22}{3}$ or $x = -6$

e $\frac{x + 2}{3} - \frac{2x}{5} = 2$ or $\frac{x + 2}{3} - \frac{2x}{5} = -2$

$5(x + 2) - 3(2x) = 30$ $\quad$ $5(x + 2) - 3(2x) = -30$

$5x + 10 - 6x = 30$ $\quad$ $5x + 10 - 6x = -30$

$x = -20$ $\quad$ $x = 40$

Check:

$\left|\frac{-20 + 2}{3} - \frac{2 \times (-20)}{5}\right| = |-6 + 8| = |2| = 2$ ✓

$\left|\frac{40 + 2}{3} - \frac{2 \times 40}{5}\right| = |14 - 16| = |-2| = 2$ ✓

Solutions are:

$x = -20$ or $x = 40$

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f $2x + 7 = 3x$ or $2x + 7 = -3x$

$x = 7$ $\quad$ $5x = -7$

$x = -\frac{7}{5}$

Check:

$|2 \times 7 + 7| = |21| = 21,\ 3 \times 7 = 21$ ✓

$\left|2 \times \left(-\frac{7}{5}\right) + 7\right| = \left|-\frac{14}{5} + \frac{35}{5}\right| = \left|\frac{21}{5}\right| = \frac{21}{5},\ 3 \times \left(-\frac{7}{5}\right) = -\frac{21}{5} \neq \frac{21}{5}$ ✗

The only solution is $x = 7$.

> The original equation in part **f** requires x to be positive, because $3x$ is equal to a modulus function. This is why the false solution $x = -\frac{7}{5}$ was not a correct solution.

2 a

> In the worked solution to part **a** two methods are shown. Unless instructed otherwise, you can choose the method that you are most confident with.

Method 1

$$\left|\frac{2x+1}{x-2}\right| = 5$$

$\frac{2x+1}{x-2} = 5$ or $\frac{2x+1}{x-2} = -5$

$2x + 1 = 5(x - 2)$ $\quad$ $2x + 1 = -5(x - 2)$

$2x + 1 = 5x - 10$ $\quad$ $2x + 1 = -5x + 10$

$3x = 11$ $\quad$ $7x = 9$

$x = \frac{11}{3}$ $\quad$ $x = \frac{9}{7}$

Check:

$$\left|\frac{2 \times \frac{9}{7} + 1}{\frac{9}{7} - 2}\right| = \left|\frac{\left(\frac{25}{7}\right)}{\left(-\frac{5}{7}\right)}\right| = \left|-\frac{25}{7} \times \frac{7}{5}\right| = |-5| = 5 \quad ✓$$

$$\left|\frac{2 \times \frac{11}{3} + 1}{\frac{11}{3} - 2}\right| = \left|\frac{\left(\frac{25}{3}\right)}{\left(\frac{5}{3}\right)}\right| = \left|\frac{25}{3} \times \frac{3}{5}\right| = |5| = 5 \quad ✓$$

Solutions are:

$x = \frac{9}{7}$ or $x = \frac{11}{3}$

Method 2

$$\left|\frac{2x+1}{x-2}\right| = 5 \Rightarrow |2x+1| = 5|x-2|$$

$$(2x+1)^2 = 25(x-2)^2$$
$$4x^2+4x+1 = 25x^2-100x+100$$
$$21x^2-104x+99=0$$
$$(7x-9)(3x-11)=0$$
$$x=\frac{9}{7} \text{ or } x=\frac{11}{3}$$

Solutions are:

$$x=\frac{9}{7} \text{ or } x=\frac{11}{3}$$

c $$\left|2-\frac{x+2}{x-3}\right| = 5$$

$$\left|\frac{2x-6-x-2}{x-3}\right| = 5$$
$$|x-8| = 5|x-3|$$
$$(x-8)^2 = 25(x-3)^2$$
$$x^2-16x+64 = 25x^2-150x+225$$
$$24x^2-134x+161=0$$
$$(4x-7)(6x-23)=0$$
$$x=\frac{7}{4} \text{ or } x=\frac{23}{6}$$

Check:

$$\left|2-\frac{\frac{7}{4}+2}{\frac{7}{4}-3}\right| = \left|2-\frac{\left(\frac{15}{4}\right)}{\left(-\frac{5}{4}\right)}\right| = \left|2+\frac{15}{4}\times\frac{4}{5}\right| = |2+3| = 5 \quad ✓$$

$$\left|2-\frac{\frac{23}{6}+2}{\frac{23}{6}-3}\right| = \left|2-\frac{\left(\frac{35}{6}\right)}{\left(\frac{5}{6}\right)}\right| = \left|2-\frac{35}{6}\times\frac{6}{5}\right| = |2-7| = |-5| = 5 \quad ✓$$

Solutions are:

$$x=\frac{7}{4} \text{ or } x=\frac{23}{6}$$

In part **c** the use of the common denominator allows you to multiply through and solve, as in part **a**.

e $x + |x + 4| = 8$

$|x + 4| = 8 - x$

$x + 4 = 8 - x$ or $x + 4 = x - 8$

$2x = 4$ No solutions.

$x = 2$

Check:

$2 + |2 + 4| = 2 + 6 = 8$ ✓

The only solution is:

$x = 2$

If you arrive at an equation that doesn't make sense then this will mean that there are no solutions, unless you have made a mistake in your working. You should check everything carefully if this happens.

3 a

In the worked solution to part **a** two methods are shown. Unless instructed otherwise, you can choose the method that you are most confident with.

Method 1

$|2x + 1| = |x|$

$2x + 1 = x$ or $2x + 1 = -x$

$x = -1$ $3x = -1$

$x = -\frac{1}{3}$

Check:

$\left|2\left(-\frac{1}{3}\right) + 1\right| = \left|-\frac{2}{3} + 1\right| = \left|\frac{1}{3}\right| = \frac{1}{3}, \left|-\frac{1}{3}\right| = \frac{1}{3}$ ✓

$|2(-1) + 1| = |-2 + 1| = |-1| = 1, |-1| = 1$ ✓

Solutions are:

$x = -\frac{1}{3}$ or $x = -1$

Method 2

$|2x + 1| = |x|$

$(2x + 1)^2 = x^2$

$4x^2 + 4x + 1 = x^2$

$3x^2 + 4x + 1 = 0$

$(3x + 1)(x + 1) = 0$

$x = -\frac{1}{3}$ or $x = -1$

Solutions are:

$x = -\frac{1}{3}$ or $x = -1$

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d $|3x+5| = |1+2x|$

$3x+5 = 1+2x$ or $3x+5 = -(1+2x)$

$x = -4$ $\quad$ $3x+5 = -1-2x$

$5x = -6$

$x = -\frac{6}{5}$

Check:

$\left|3\left(-\frac{6}{5}\right)+5\right| = \left|-\frac{18}{5}+5\right| = \left|\frac{7}{5}\right| = \frac{7}{5}, \left|1+2\left(-\frac{6}{5}\right)\right| = \left|1-\frac{12}{5}\right| = \left|-\frac{7}{5}\right| = \frac{7}{5}$ ✓

$|3(-4)+5| = |-12+5| = |-7| = 7, |1+2(-4)| = |1-8| = |-7| = 7$ ✓

Solutions are:

$x = -\frac{6}{5}$ or $x = -4$

f $3|2x-1| = \left|\frac{1}{2}x-3\right|$

Multiplying both sides by 2:

$$6|2x-1| = |x-6|$$
$$36(2x-1)^2 = (x-6)^2$$
$$36(4x^2-4x+1) = x^2-12x+36$$
$$144x^2-144x+36 = x^2-12x+36$$
$$143x^2-132x = 0$$
$$x(143x-132) = 0$$

$x = 0$ or $x = \frac{132}{143} = \frac{12}{13}$

Check:

$3|2(0)-1| = 3|-1| = 3\times 1 = 3, \left|\frac{1}{2}(0)-3\right| = |-3| = 3$ ✓

$3\left|2\left(\frac{12}{13}\right)-1\right| = 3\left|\frac{11}{13}\right| = \frac{33}{13}, \left|\frac{1}{2}\left(\frac{12}{13}\right)-3\right| = \left|-\frac{33}{13}\right| = \frac{33}{13}$ ✓

Solutions are:

$x = 0$ or $x = \frac{132}{143} = \frac{12}{13}$

Always clear fractions where possible when solving equations.

4 **a** $x^2-2 = 7$ or $x^2-2 = -7$

$x^2 = 9$ $\quad$ $x^2 = -5$

$x = \pm 3$ $\quad$ No real solutions.

Check:

$|3^2-2| = |9-2| = 7$ ✓, $|(-3)^2-2| = |9-2| = 7$ ✓

Solutions are:

$x = \pm 3$

f $|x^2 - 7x + 6| = 6 - x$

$x^2 - 7x + 6 = 6 - x$ or $x^2 - 7x + 6 = x - 6$

$x^2 - 6x = 0$ $\quad$ $x^2 - 8x + 12 = 0$

$x(x - 6) = 0$ $\quad$ $(x - 6)(x - 2) = 0$

$x = 0$ or $x = 6$ $\quad$ $x = 6$ or $x = 2$

Check:

$|0^2 - 7(0) + 6| = |6| = 6, \quad 6 - 0 = 6$ ✓

$|6^2 - 7(6) + 6| = |0| = 0, \quad 6 - 6 = 0$ ✓

$|2^2 - 7(2) + 6| = |-4| = 4, \quad 6 - 2 = 4$ ✓

Solutions are:

$x = 0$ or $x = 6$ or $x = 2$

5 a $x + 2y = 8 \Rightarrow x = 8 - 2y$......... [1]

$|x + 2| = 6 - y$....................... [2]

$x + 2 = 6 - y$ or $x + 2 = y - 6$

Substituting [1] into these separately:

$8 - 2y + 2 = 6 - y$ $\quad$ $8 - 2y + 2 = y - 6$

$y = 4$ $\quad$ $y = \frac{16}{3}$

Substituting the values of y back into [1]:

$x = 8 - 2(4) = 0$ $\quad$ $x = 8 - 2\left(\frac{16}{3}\right) = 8 - \frac{32}{3} = \frac{24}{3} - \frac{32}{3} = -\frac{8}{3}$

Solutions are:

$x = 0$ and $y = 4$ or $x = -\frac{8}{3}$ and $y = \frac{16}{3}$

b $3x + y = 0 \Rightarrow y = -3x$ .... [1]

$y = |2x^2 - 5|$

$2x^2 - 5 = y$ or $2x^2 - 5 = -y$

Substituting [1] into these separately:

$2x^2 - 5 = -3x$ $\quad$ $2x^2 - 5 = 3x$

$2x^2 + 3x - 5 = 0$ $\quad$ $2x^2 - 3x - 5 = 0$

$(2x + 5)(x - 1) = 0$ $\quad$ $(2x - 5)(x + 1) = 0$

$x = -\frac{5}{2}$ or $x = 1$ $\quad$ $x = \frac{5}{2}$ or $x = -1$

Substituting these values back into [1] and remembering that $y \geqslant 0$ because it is equal to the modulus:

$y = \frac{15}{2}$ or $y = -3$ (not a solution) $\quad$ $y = -\frac{15}{2}$ (not a solution) or $y = 3$

Solutions are:

$x = -\frac{5}{2}$ and $y = \frac{15}{2}$

or $x = -1$ and $y = 3$

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6

$$5|x-1|^2 = 2 - 9|x-1|$$
$$5(x-1)^2 = 2 - 9|x-1|$$
$$9|x-1| = 2 - 5(x-1)^2$$

$9x - 9 = 2 - 5(x-1)^2$ or $9x - 9 = 5(x-1)^2 - 2$

$9x - 9 = 2 - 5(x^2 - 2x + 1)$ $\quad$ $9x - 9 = 5(x^2 - 2x + 1) - 2$

$5x^2 - x - 6 = 0$ $\quad$ $5x^2 - 19x + 12 = 0$

$(5x-6)(x+1) = 0$ $\quad$ $(5x-4)(x-3) = 0$

$x = \frac{6}{5}$ or -1 $\quad$ $x = \frac{4}{5}$ or 3

Substitution shows that neither −1 nor 3 are legitimate solutions.

$x = \frac{6}{5}$ or $\frac{4}{5}$

Check:

$$5\left|\frac{6}{5}-1\right|^2 = 5\left|\frac{1}{5}\right|^2 = \frac{5}{25} = \frac{1}{5},\ 2 - 9\left|\frac{6}{5}-1\right| = 2 - 9\left|\frac{1}{5}\right| = 2 - \frac{9}{5} = \frac{1}{5}\ \checkmark$$

$$5\left|\frac{4}{5}-1\right|^2 = 5\left|-\frac{1}{5}\right|^2 = \frac{5}{25} = \frac{1}{5},\ 2 - 9\left|\frac{4}{5}-1\right| = 2 - 9\left|-\frac{1}{5}\right| = 2 - \frac{9}{5} = \frac{1}{5}\ \checkmark$$

Solutions are:

$x = \frac{6}{5}$ or $\frac{4}{5}$

Always check that your solutions work in the original equation. Squaring methods can sometimes generate false solutions that are not actually solutions at all.

7 a $x^2 - 5|x| + 6 = 0$

$x^2 - 5x + 6 = 0$ or $x^2 + 5x + 6 = 0$

$(x-2)(x-3) = 0$ $\quad$ $(x+2)(x+3) = 0$

$x = 2$ or $x = 3$ $\quad$ $x = -2$ or $x = -3$

Check:

$2^2 - 5|2| + 6 = 4 - 10 + 6 = 0$ ✓

$3^2 - 5|3| + 6 = 9 - 15 + 6 = 0$ ✓

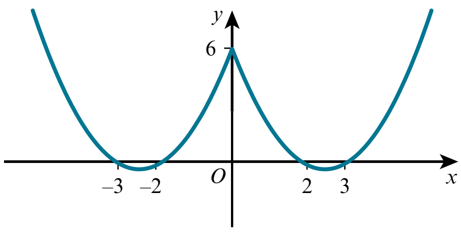
$(-2)^2 - 5|-2| + 6 = 4 - 10 + 6 = 0$ ✓

$(-3)^2 - 5|-3| + 6 = 9 - 15 + 6 = 0$ ✓

Solutions are:

$x = 2$ or $x = 3$ or $x = -2$ or $x = -3$

b



c y-axis or $x = 0$

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8 $|2x+1| + |2x-1| = 3$

$|2x+1| = 3 - |2x-1|$

$2x+1 = 3 - |2x-1|$ .... [1] or $2x+1 = |2x-1| - 3$ .... [2]

Considering [1] first:

$2x+1 = 3 - |2x-1|$

$|2x-1| = 3 - 2x - 1$

$|2x-1| = 2 - 2x$

$2x - 1 = 2 - 2x$ or $2x - 1 = 2x - 2$

$4x = 3$ No solution.

$x = \frac{3}{4}$

Considering [2] next:

$2x+1 = |2x-1| - 3$

$|2x-1| = 2x+4$ or $2x-1 = -2x-4$

$2x-1 = 2x+4$ $4x = -3$

No solutions. $x = -\frac{3}{4}$

Check:

$$\left|2\left(\frac{3}{4}\right)+1\right| + \left|2\left(\frac{3}{4}\right)-1\right| = \left|\frac{10}{4}\right| + \left|\frac{2}{4}\right| = \frac{10}{4} + \frac{2}{4} = \frac{12}{4} = 3 \quad \checkmark$$

$$\left|2\left(-\frac{3}{4}\right)+1\right| + \left|2\left(-\frac{3}{4}\right)-1\right| = \left|-\frac{2}{4}\right| + \left|-\frac{10}{4}\right| = \frac{2}{4} + \frac{10}{4} = \frac{12}{4} = 3 \quad \checkmark$$

Solutions are:

$x = \frac{3}{4}$ or $x = -\frac{3}{4}$

9 $|3x - 2y - 11| = -2\sqrt{31 - 8x + 5y}$

Given that the square root is always positive, the right-hand side is negative, unless it is zero. Given also that the modulus is positive or zero, this equation is only possible if both sides are zero at the same time.

$3x - 2y = 11 \Rightarrow 15x - 10y = 55$ ....[1]

$8x - 5y = 31 \Rightarrow 16x - 10y = 62$ ....[2]

[2] – [1]: $x = 7$

Substituting $x = 7$ into $3x - 2y = 11$:

$21 - 2y = 11$

$2y = 10$

$y = 5$

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Check:

$|3 \times 7 - 2 \times 5 - 11| = |21 - 10 - 11| = 0$

$-2\sqrt{31 - 8 \times 7 + 5 \times 5} = -2\sqrt{31 - 56 + 25} = 0$ ✓

The solution is:

$x = 7$ and $y = 5$

The square root symbol **always** means the positive square root. A common error, here, is to apply the '±' symbol, but that is only used when 'undoing' squares. For example, $x^2 = 9$ gives $x = \pm 3$, whereas the square root of 9 is just 3.

EXERCISE 1B

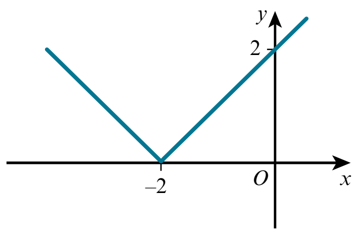
1 a $y = |x + 2|$

$x = 0 \Rightarrow y = |2| = 2$

$y = 0 \Rightarrow x + 2 = 0$

$x = -2$

$$y = \begin{cases} x + 2 & x > -2 \\ -(x + 2) & x \leqslant -2 \end{cases}$$



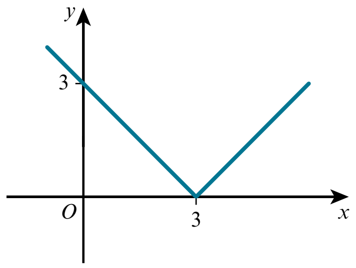
b $y = |3 - x|$

$x = 0 \Rightarrow y = |3| = 3$

$y = 0 \Rightarrow 3 - x = 0$

$x = 3$

$$y = \begin{cases} 3 - x & x < 3 \\ x - 3 & x \geqslant 3 \end{cases}$$



c $y = \left|5 - \frac{1}{2}x\right|$

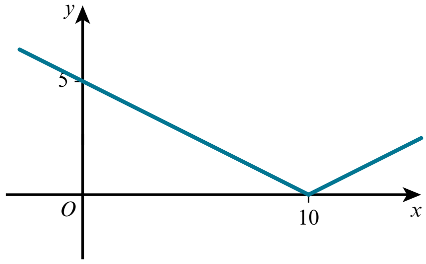
$x = 0 \Rightarrow y = |5| = 5$

$y = 0 \Rightarrow 5 - \frac{1}{2}x = 0$

$\frac{1}{2}x = 5$

$x = 10$

$$y = \begin{cases} 5 - \frac{1}{2}x & x < 10 \\ \frac{1}{2}x - 5 & x \geqslant 10 \end{cases}$$



In these solutions for Question **1**, the point at which the graph crosses the x-axis has been found by seeing where $y = 0$. The point at which the graph crosses the y-axis has been found by seeing where $x = 0$.

2 a $x = 1 \Rightarrow y = |1 - 3| + 2 = 2 + 2 = 4$

$x = 3 \Rightarrow y = |3 - 3| + 2 = 2$

$x = 4 \Rightarrow y = |4 - 3| + 2 = 3$

$x = 5 \Rightarrow y = |5 - 3| + 2 = 4$

$x = 6 \Rightarrow y = |6 - 3| + 2 = 5$

So the completed table is:

x	0	1	2	3	4	5	6
y	5	4	3	2	3	4	5

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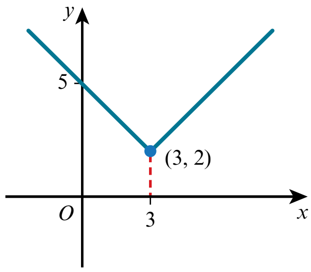
b



c $y = |x|$

$\downarrow$ translation $\begin{pmatrix}3\\0\end{pmatrix}$

$y = |x - 3|$

$\downarrow$ translation $\begin{pmatrix}0\\2\end{pmatrix}$

$y = |x - 3| + 2$

So the overall transformation is a translation $\begin{pmatrix}3\\2\end{pmatrix}$.

3 Sometimes it is easy to see what combination of transformations has taken place, but more complicated cases need to be built step by step. These worked solutions show one possible way of doing this.

a $y = |x|$

$\downarrow$ translation $\begin{pmatrix}-1\\0\end{pmatrix}$

$y = |x + 1|$

$\downarrow$ translation $\begin{pmatrix}0\\2\end{pmatrix}$

$y = |x + 1| + 2$

So the overall transformation is a translation $\begin{pmatrix}-1\\2\end{pmatrix}$

e $y = |x|$

$\downarrow$ translation $\begin{pmatrix}-2\\0\end{pmatrix}$

$y = |x + 2|$

$\downarrow$ reflection in $y = 0$

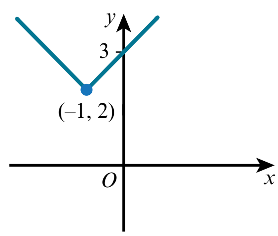
$y = -|x + 2|$

$\downarrow$ translation $\begin{pmatrix}0\\1\end{pmatrix}$

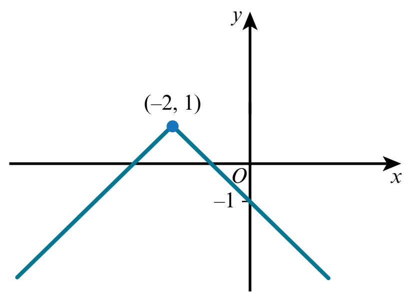
$y = 1 - |x + 2|$

So the overall sequence of transformations is a translation with vector $\begin{pmatrix}-2\\0\end{pmatrix}$ followed by a reflection in the line $y = 0$ followed by a translation with vector $\begin{pmatrix}0\\1\end{pmatrix}$.

4 a



e



5 $f(x) = |5 - 2x| + 3$

The minimum value is when $5 - 2x = 0$ because a modulus can't be negative.

So the minimum is when $x = 2.5$.

$f(0) = 5 + 3 = 8$

So the graph crosses the y-axis at 8.

Draw the graph: