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Elements in the Structure and Dynamics of Complex Networks
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MODULARITY AND DYNAMICS ON COMPLEX NETWORKS

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Abstract: Complex networks are typically not homogeneous, as they tend to display an array of structures at different scales. A feature that has attracted a lot of research is their modular organisation (i.e., networks may often be considered as being composed of certain building blocks, or modules). In this Element, the authors discuss a number of ways in which this idea of modularity can be conceptualised, focusing specifically on the interplay between modular network structure and dynamics taking place on a network. They discuss, in particular, how modular structure and symmetries may impact on network dynamics and, vice versa, how observations of such dynamics may be used to infer the modular structure. They also revisit several other notions of modularity that have been proposed for complex networks and show how these can be related to and interpreted from the point of view of dynamical processes on networks.

Keywords: modularity, networks, time scale, dynamics, block models

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