

Proof and Falsity

This book argues that the meaning of negation, perhaps the most important logical constant, cannot be defined within the framework of the most comprehensive theory of proof-theoretic semantics, as formulated in the influential work of Michael Dummett and Dag Prawitz. Nils Kürbis examines three approaches that have attempted to solve the problem – defining negation in terms of metaphysical incompatibility, treating negation as an undefinable primitive, and defining negation in terms of a speech act of denial – and concludes that they cannot adequately do so. He argues that whereas proof-theoretic semantics usually only appeals to a notion of truth, it also needs to appeal to a notion of falsity, and proposes a system of natural deduction in which both are incorporated. Offering new perspectives on negation, denial and falsity, this book will be important for readers working on logic, metaphysics and the philosophy of language.

NILS KÜRBIŠ is a research fellow in the Department of Philosophy at University College London. He has published articles in the *Journal of Philosophical Logic*, *Logique et Analyse* and *Grazer Philosophische Studien*.

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A Logical Investigation

NILS KÜRBIŠ

University College London



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(2015): 713–727) were welded into various sections of Chapters 1 and 2. Chapter 3 is an adapted and slightly expanded version of ‘What is Wrong with Classical Negation?’ (*Grazer Philosophische Studien* 91/1 (2015): 51–85). Chapter 6 contains modifications of ‘Some Comments on Ian Rumfitt’s Bilateralism’ (*The Journal of Philosophical Logic* 45/6 (2016): 623–644) and of ‘Bilateralist Detours. From Intuitionist to Classical Logic and Back’ (*Logique et Analyse* 60/239 (2017): 301–316). The editors of these journals, Martina Fürst, Ángel García Rodríguez, John F. Horty and Jean Paul Van Bendegem, kindly granted permission to recycle the material in this book.

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