CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-71600-0 — Numerical Methods
Rajesh Kumar Gupta
Table of Contents
More Information

Contents

Preface *xvii*
Acknowledgments *xxix*

Chapter 1 Number Systems **1**
1.1 Introduction 1
Table 1.1 Binary, Octal, Decimal and Hexadecimal Numbers 2
1.2 Representation of Integers 2
1.2.1 Conversion from Any Number System to the Decimal Number System 3
1.2.2 Conversion between Binary, Octal and Hexadecimal Number Systems 4
1.2.3 Conversion from Decimal Number System to Any Other Number System 4
1.2.4 Conversion from One Number System to Any Other Number System 6
1.3 Representation of Fractions 8
Exercise 1 11

Chapter 2 Error Analysis **13**
2.1 Absolute, Relative and Percentage Errors 13
2.2 Errors in Modeling of Real World Problems 16
2.2.1 Modeling Error 16
2.2.2 Error in Original Data (Inherent Error) 16
2.2.3 Blunder 16
2.3 Errors in Implementation of Numerical Methods 17
2.3.1 Round-off Error 17
2.3.2 Overflow and Underflow 22
2.3.3 Floating Point Arithmetic and Error Propagation 23
2.3.3.1 Propagated Error in Arithmetic Operations 24
2.3.3.2 Error Propagation in Function of Single Variable 27
2.3.3.3 Error Propagation in Function of More than One Variable 28
2.3.4 Truncation Error 30
2.3.5 Machine eps (Epsilon) 33
2.3.6 Epilogue 34
2.3.7 Loss of Significance: Condition and Stability 34
2.4 Some Interesting Facts about Error 41
Exercise 2 42

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-71600-0 — Numerical Methods
Rajesh Kumar Gupta
Table of Contents
More Information

Chapter 3 Nonlinear Equations 47
3.1 Introduction 47
3.1.1 Polynomial Equations 48
3.1.2 Transcendental Equations 48
3.2 Methods for Solutions of the Equation $f(x) = 0$ 48
3.2.1 Direct Analytical Methods 49
3.2.2 Graphical Methods 49
3.2.3 Trial and Error Methods 51
3.2.4 Iterative Methods 52
3.3 Bisection (or) Bolzano (or) Interval-Halving Method 54
3.4 Fixed-Point Method (or) Direct-Iteration Method (or) Method of Successive-Approximations (or) Iterative Method (or) One-Point-Iteration Method 59
3.5 Newton–Raphson (NR) Method 65
3.6 Regula Falsi Method (or) Method of False Position 68
3.7 Secant Method 71
3.8 Convergence Criteria 74
3.8.1 Convergence of Bisection Method 75
3.8.2 Convergence of Fixed-Point Method 76
3.8.3 Convergence of Newton–Raphson Method 81
3.8.4 Convergence of Regula Falsi Method 85
3.8.5 Convergence of Secant Method 85
3.9 Order of Convergence 86
3.9.1 Order of Convergence for Bisection Method 87
3.9.2 Order of Convergence for Fixed-Point Method 88
3.9.3 Order of Convergence for Newton–Raphson Method 90
3.9.4 Order of Convergence for Secant Method 97
3.9.5 Order of Convergence for Regula Falsi Method 99
3.10 Muller Method 101
3.11 Chebyshev Method 106
3.12 Aitken Δ^2 Process: Acceleration of Convergence of Fixed-Point Method 110
Table 3.3 Formulation of Methods 115
Table 3.4 Properties and Convergence of Methods 116
3.13 Summary and Observations 117
Exercise 3 118

Chapter 4 Nonlinear Systems and Polynomial Equations 124
4.1 Fixed-Point Method 125
4.2 Seidel Iteration Method 131
4.3 Newton–Raphson (NR) Method 135
4.4 Complex Roots 144
4.5 Polynomial Equations 147
4.5.1 Descartes Rule of Signs 147
4.5.2 Strum Sequence 148

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-71600-0 — Numerical Methods
Rajesh Kumar Gupta
Table of Contents
More Information

4.6	Birge–Vieta (or) Horner Method	152
4.7	Lin–Bairstow Method	156
4.8	Graeffe Root Squaring Method	161
	Table 4.2 Methods for Solutions of the Systems of Nonlinear Equations	169
	Table 4.3 Methods for the Solutions of the Polynomial Equations	170
	Exercise 4	171
Chapter 5	**Systems of Linear Equations**	**173**
5.1	Introduction	173
5.2	Cramer Rule	176
5.3	Matrix Inversion Method	178
5.4	LU Decomposition (or) Factorization (or) Triangularization Method	182
	5.4.1 Doolittle Method	183
	5.4.2 Crout Method	183
	5.4.3 Cholesky Method	190
5.5	Gauss Elimination Method	192
	5.5.1 Operational Counts for Gauss Elimination Method	197
	5.5.2 Thomas Algorithm (Tridiagonal Matrix Algorithm)	199
5.6	Gauss–Jordan Method	203
5.7	Comparison of Direct Methods	206
5.8	Pivoting Strategies for Gauss Elimination Method	207
5.9	Iterative Methods	217
5.10	Jacobi Method (or) Method of Simultaneous Displacement	218
5.11	Gauss–Seidel Method (or) Method of Successive Displacement (or) Liebmann Method	222
5.12	Relaxation Method	227
5.13	Convergence Criteria for Iterative Methods	237
5.14	Matrix Forms and Convergence of Iterative Methods	245
	Table 5.2 Formulae for Iterative Methods	255
5.15	Discussion	256
5.16	Applications	258
	Exercise 5	261
Chapter 6	**Eigenvalues and Eigenvectors**	**268**
6.1	Introduction	268
6.2	Eigenvalues and Eigenvectors	270
	6.2.1 Real Eigenvalues	271
	6.2.2 Complex Eigenvalues	273
	6.2.3 Matrix with Real and Distinct Eigenvalues	274
	6.2.4 Matrix with Real and Repeated Eigenvalues	275
	6.2.4.1 Linearly Independent Eigenvectors	275
	6.2.4.2 Linearly Dependent Eigenvectors	276
6.3	Bounds on Eigenvalues	277
	6.3.1 Gerschgorin Theorem	277
	6.3.2 Brauer Theorem	279

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-71600-0 — Numerical Methods
Rajesh Kumar Gupta
Table of Contents
More Information

6.4	Rayleigh Power Method	281
	6.4.1 Inverse Power Method	285
	6.4.2 Shifted Power Method	288
6.5	Rutishauser (or) LU Decomposition Method	291
	Exercise 6	295
Chapter 7	**Eigenvalues and Eigenvectors of Real Symmetric Matrices**	**299**
7.1	Introduction	299
	7.1.1 Similarity Transformations	304
	7.1.2 Orthogonal Transformations	306
7.2	Jacobi Method	307
7.3	Strum Sequence for Real Symmetric Tridiagonal Matrix	311
7.4	Givens Method	312
7.5	Householder Method	319
	Exercise 7	326
Chapter 8	**Interpolation**	**331**
8.1	Introduction	331
8.2	Polynomial Forms	333
	8.2.1 Power Form	333
	8.2.2 Shifted Power Form	333
	8.2.3 Newton Form	334
	8.2.4 Nested Newton Form	334
	8.2.5 Recursive Algorithm for the Nested Newton Form	335
	8.2.6 Change of Center in Newton Form	336
8.3	Lagrange Method	340
8.4	Newton Divided Difference (NDD) Method	343
	8.4.1 Proof for Higher Order Divided Differences	346
	8.4.2 Advantages of NDD Interpolation over Lagrange Interpolation	347
	8.4.3 Properties of Divided Differences	348
8.5	Error in Interpolating Polynomial	350
8.6	Discussion	353
8.7	Hermite Interpolation	354
8.8	Piecewise Interpolation	357
8.9	Weierstrass Approximation Theorem	359
	Exercise 8	359
Chapter 9	**Finite Operators**	**364**
9.1	Introduction	364
9.2	Finite Difference Operators	365
	9.2.1 Forward Difference Operator (Δ)	365
	9.2.2 Backward Difference Operator (∇)	366
	9.2.3 Central Difference Operator (δ)	366

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-71600-0 — Numerical Methods
Rajesh Kumar Gupta
Table of Contents
More Information

9.3	Average, Shift and Differential Operators	367
	9.3.1 Mean or Average Operator (μ)	367
	9.3.2 Shift Operator (E)	367
	9.3.3 Differential Operator (D)	368
	Table 9.1 Finite Differences and Other Operators	368
9.4	Properties and Interrelations of Finite Operators	369
	9.4.1 Linearity and Commutative Properties	369
	9.4.2 Interrelations of Finite Operators	370
	Table 9.2 Relations between the Operators	373
9.5	Operators on Some Functions	374
9.6	Newton Divided Differences and Other Finite Differences	377
9.7	Finite Difference Tables and Error Propagation	379
	Table 9.3 Forward Differences	380
	Table 9.4 Backward Differences	380
	Table 9.5 Central Differences	381
	Exercise 9	386
Chapter 10	**Interpolation for Equal Intervals and Bivariate Interpolation**	**389**
10.1	Gregory–Newton Forward Difference Formula	390
	10.1.1 Error in Newton Forward Difference Formula	393
10.2	Gregory–Newton Backward Difference Formula	395
	10.2.1 Error in Newton Backward Difference Formula	397
10.3	Central Difference Formulas	398
10.4	Gauss Forward Central Difference Formula	399
10.5	Gauss Backward Central Difference Formula	402
10.6	Stirling Formula	404
10.7	Bessel Formula	406
10.8	Everett Formula	408
10.9	Steffensen Formula	410
	Table 10.1 Finite Differences Formulas	412
10.10	Bivariate Interpolation	431
	10.10.1 Lagrange Bivariate Interpolation	431
	10.10.2 Newton Bivariate Interpolation for Equi-spaced Points	435
	Exercise 10	442
Chapter 11	**Splines, Curve Fitting, and Other Approximating Curves**	**445**
11.1	Introduction	445
11.2	Spline Interpolation	446
	11.2.1 Cubic Spline Interpolation	448
	11.2.2 Cubic Spline for Equi-spaced Points	451
11.3	Bězier Curve	456
11.4	B-Spline Curve	462
11.5	Least Squares Curve	467
	11.5.1 Linear Curve (or) Straight Line Fitting	468
	11.5.2 Nonlinear Curve Fitting by Linearization of Data	470

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-71600-0 — Numerical Methods
Rajesh Kumar Gupta
Table of Contents
More Information

	Table 11.1 Linearization of Nonlinear Curves	471
	11.5.3 Quadratic Curve Fitting	474
11.6	Chebyshev Polynomials Approximation	478
11.7	Approximation by Rational Function of Polynomials (Padé Approximation)	484
	Table 11.2 Summary and Comparison	488
	Exercise 11	489
Chapter 12	**Numerical Differentiation**	**495**
12.1	Introduction	495
12.2	Numerical Differentiation Formulas	497
	Table 12.1 Summary Table for Numerical Differentiation Formulas	498
	Exercise 12	507
Chapter 13	**Numerical Integration**	**509**
13.1	Newton–Cotes Quadrature Formulas (Using Lagrange Method)	510
	13.1.1 Trapezoidal Rule ($n = 1$)	512
	13.1.2 Simpson 1/3 Rule ($n = 2$)	513
	13.1.3 Simpson 3/8 Rule ($n = 3$)	514
	13.1.4 Boole Rule ($n = 4$)	514
	13.1.5 Weddle Rule ($n = 6$)	515
13.2	Composite Newton–Cotes Quadrature Rules	517
	13.2.1 Composite Trapezoidal Rule	517
	13.2.2 Composite Simpson 1/3 Rule	518
	13.2.3 Composite Simpson 3/8 Rule	519
	13.2.4 Composite Boole Rule	519
13.3	Errors in Newton–Cotes Quadrature Formulas	528
	13.3.1 Error in Trapezoidal Rule ($n = 1$)	529
	13.3.2 Error in Simpson 1/3 Rule ($n = 2$)	529
	13.3.3 Error in Simpson 3/8 Rule ($n = 3$)	530
	13.3.4 Error in Boole Rule ($n = 4$)	531
	13.3.5 Error in Weddle Rule ($n = 6$)	531
	Table 13.1 Newton–Cotes Quadrature Formulas	534
13.4	Gauss Quadrature Formulas	535
	13.4.1 Gauss–Legendre Formula	535
	13.4.2 Gauss–Chebyshev Formula	546
	13.4.3 Gauss–Laguerre Formula	549
	13.4.4 Gauss–Hermite Formula	551
13.5	Euler–Maclaurin Formula	553
13.6	Richardson Extrapolation	558
13.7	Romberg Integration	560
	Table 13.2 Numerical Techniques for Integration	565
13.8	Double Integrals	567
	13.8.1 Trapezoidal Rule	567
	13.8.2 Simpson 1/3 Rule	569
	Exercise 13	571

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-71600-0 — Numerical Methods
Rajesh Kumar Gupta
Table of Contents
More Information

Chapter 14 First Order Ordinary Differential Equations: Initial Value Problems 576
14.1 Some Important Classifications and Terms 577
14.1.1 Ordinary and Partial Differential Equations 577
14.1.2 Order and Degree of Differential Equations 578
14.1.3 Homogeneous and Non-homogeneous Differential Equations 578
14.1.4 Constant and Variable Coefficient Differential Equations 579
14.1.5 Linear and Nonlinear Differential Equations 579
14.1.6 General, Particular and Singular Solutions 580
14.1.7 Initial Value Problem (IVP) and Boundary Value Problem (BVP) 580
14.1.8 Existence and Uniqueness of Solutions 581
14.1.9 Comparison of Analytical and Numerical Methods 582
14.2 Picard Method of Successive Approximations 582
14.3 Taylor Series Method 585
14.4 Euler Method 589
14.5 Modified (or) Improved Euler Method (or) Heun Method 592
14.6 Runge–Kutta (RK) Methods 597
14.7 Milne Method (Milne Simpson Method) 608
14.8 Adams Method (Adams–Bashforth Predictor and Adams–Moulton Corrector Formulas) 616
14.9 Errors in Numerical Methods 623
14.10 Order and Stability of Numerical Methods 624
14.11 Stability Analysis of IVP $y' = Ay$, $y(0) = y_0$ 626
14.12 Backward Euler Method 628
Table 14.1 Numerical Schemes for IVP 634
Exercise 14 636

Chapter 15 Systems of First Order ODEs and Higher Order ODEs: Initial and Boundary Value Problems 642
15.1 Picard Method 644
15.2 Taylor Series Method 647
15.3 Euler Method 648
15.4 Runge–Kutta Fourth Order Method 652
Table 15.1 Formulations for Solutions of IVPs 658
15.5 Boundary Value Problem: Shooting Method 658
15.6 Finite Difference Approximations for Derivatives 661
15.6.1 First Order Derivatives 662
15.6.2 Second Order Derivatives 663
15.7 Boundary Value Problem: Finite Difference Method 664
15.8 Finite Difference Approximations for Unequal Intervals 668
15.9 Discussion 671
Exercise 15 672

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-71600-0 — Numerical Methods
Rajesh Kumar Gupta
Table of Contents
More Information

Chapter 16 Partial Differential Equations: Finite Difference Methods 679
16.1 Classification of Second-Order Quasi-Linear PDEs 680
16.2 Initial and Boundary Conditions 682
16.3 Finite Difference Approximations for Partial Derivatives 683
16.4 Parabolic Equation (1-dimensional Heat Conduction Equation) 688
16.4.1 Bender–Schmidt Explicit Scheme 689
16.4.2 Crank–Nicolson (CN) Scheme 690
16.4.3 General Implicit Scheme 691
16.4.4 Richardson Scheme 692
16.4.5 Du-Fort and Frankel Scheme 692
16.5 Consistency, Convergence and Stability of Explicit and Crank–Nicolson Schemes 701
16.5.1 Consistency 702
16.5.2 Consistency of Explicit Scheme 703
16.5.3 Convergence and Order 704
16.5.4 Stability 705
16.5.5 Matrix Method for Stability of Explicit Scheme 705
16.5.6 Matrix Method for Stability of CN Scheme 707
16.5.7 Neumann Method for Stability of Explicit Scheme 708
16.5.8 Neumann Method for Stability of CN Scheme 709
Table 16.1 Summary Table of Finite Difference Methods for 1-Dimensional Heat Conduction Equation 710
16.6 2-Dimensional Heat Conduction Equation 711
16.6.1 Explicit Scheme 711
16.6.2 Crank–Nicolson (CN) Scheme 712
16.6.3 Alternating Direction Implicit (ADI) Scheme 714
Table 16.2 Summary Table of Finite Difference Methods for 2-Dimensional Heat Conduction Equation 717
16.7 Elliptic Equations (Laplace and Poisson Equations) 725
16.7.1 Laplace Equation 726
16.7.2 Poisson Equation 740
16.8 Hyperbolic Equation (Wave Equation) 750
16.8.1 Explicit Scheme 751
16.8.2 Implicit Scheme 751
16.9 Creating Own Scheme for a Problem 759
Exercise 16.1 Parabolic Equation (Heat Conduction (or) Diffusion Equation) 761
Exercise 16.2 Elliptic Equation (Laplace and Poisson Equations) 770
Exercise 16.3 Hyperbolic Equation (Wave Equation) 773

Appendix A Comparison of Analytical and Numerical Techniques 779

Appendix B Numerical Techniques and Computer 781

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-71600-0 — Numerical Methods
Rajesh Kumar Gupta
Table of Contents
More Information

Appendix C Taylor Series **783**
Taylor Series for the Functions of More than One Variable 785
Lagrange Mean Value (LMV) Theorem 785
Rolle Theorem 785

Appendix D Linear and Nonlinear **786**

Appendix E Graphs of Standard Functions **788**
Algebraic Functions 788
Transcendental Functions 789

Appendix F Greek Letters **790**

Index **791**

© in this web service Cambridge University Press & Assessment www.cambridge.org