

# Index

- absolute error, 13
- absolute, relative and percentage errors, 13
- accelerated Newton–Raphson method, 96, 138
- accelerating convergence, 110
- Adam–Bashforth–Moulton method, 616
  - Adams–Bashforth predictor formula, 617
  - Adams–Moulton corrector formula, 618
- advantages of NDD interpolation over
  - Lagrange interpolation, 347
- Aitken process, 110
- algorithm for conversion of fractions in
  - number systems, 8
- algorithm for conversion of integers in number
  - systems, 6
- alternating direction implicit (ADI) scheme,
  - 714
- applications of linear system of equations, 258
- approximation by rational function of
  - polynomials, 484
  - Padé approximation, 484
- approximation of data and functions
  - Bézier curve, 456
  - B – Spline curve, 462
  - interpolation, 331, 389
  - least squares curve, 467
  - Padé approximation, 484
  - Spline interpolation, 446
- augmented matrix, 192
- average operator, 367
  
- backward difference approximation of
  - first order ordinary derivative, 662, 669
  - first order partial derivatives, 684
  - second order ordinary derivative, 663, 669
  - second order partial derivative, 684
- backward difference interpolation
  - formula, 395
  - error in Newton backward difference
    - formula, 397
  - Gregory–Newton backward difference
    - formula, 395
- backward difference operator, 366
- Bender–Schmidt explicit scheme, 689, 704
- Bernstein polynomials, 457, 461
- Bessel formula, 406, 500
- Bézier curve, 456
- binary numbers, 1
- Birge–Vieta method, 152
- bisection method, 54, 74, 87
- bivariate interpolation
  - Lagrange bivariate interpolation, 431
  - Newton bivariate interpolation for equi-
    - spaced points, 435
- blunder, 16
- Bolzano method, 54
- Boole rule, 514, 519, 531
- boundary value problem (ODE), 658, 664
  - finite difference method, 661, 664
  - shooting method, 658
- bounds on eigenvalues, 277
  - Brauer theorem, 279
  - Gerschgorin theorem, 277
- Brauer theorem, 279
- B-Spline curve, 462
- central difference approximations of, 671
  - first order ordinary derivative, 663, 669
  - first order partial derivatives, 684–685
  - second order ordinary derivative, 664, 669
  - second order partial derivative, 684, 686

- central difference interpolation formulas, 398
  - Bessel formula, 406
  - Everett formula, 408
  - Gauss backward central difference formula, 402, 413, 417, 505
  - Gauss forward central difference formula, 399, 401–402, 413, 417
  - Steffensen formula, 410
  - Stirling formula, 404
- central difference operator, 366
- change of center in Newton form, 336
- characteristic equation, 271
- characteristic polynomial, 271
- Chebyshev
  - Gauss–Chebyshev method, 546
  - method for nonlinear equations, 106
  - polynomials, 478, 546
  - polynomials approximation, 478
- Cholesky method, 190
- chopping error, 19
- classification of second order quasilinear PDEs, 680
  - elliptic equations, 682, 725
  - hyperbolic equations, 681, 750
  - parabolic equations, 681, 688
- comparison of analytical and numerical methods for ODEs, 582
- comparison of analytical and numerical techniques, 779
- comparison of direct methods for systems of linear equations, 206
- compatibility of explicit scheme, 702
- complex roots, 144, 164
- composite Newton–Cotes quadrature rules, 517
  - Boole rule, 514, 519, 531
  - Simpson 1/3 rule, 513, 518, 529
  - Simpson 3/8 rule, 514, 519, 530
  - trapezoidal rule, 512, 517, 529
  - Weddle rule, 515, 520
- condition number, 34
- consistency of explicit scheme, 702
- constant and variable coefficients differential equations, 579
- conversion of fractions in number systems, 8
- conversion of integers in number systems, 2, 6
- convergence and order of a finite difference scheme, 704
- convergence criteria: root finding methods for
  - nonlinear equations, 74
  - bisection method, 75
  - fixed point method, 76
  - Newton–Raphson method, 81
  - Regula–Falsi method, 85
  - secant method, 85
- convergence criteria: root finding methods for systems of nonlinear equations:
  - fixed point method, 127
  - Newton–Raphson method, 144
  - Seidel iteration method, 131
- convergence criteria: iterative methods for linear systems of equations, 237
  - Gauss–Seidel method, 240, 252
  - Jacobi method, 237, 252
  - relaxation methods, 251–253
- convex hull, 457
- C programs [www.cambridge.org/9781108716000](http://www.cambridge.org/9781108716000)
- Cramer rule, 176
- Crank–Nicolson (CN) scheme, 689–690, 707, 709, 712
- Creating own finite difference scheme, 759
- Crout method, 183
- cubic spline, 446–456
- cubic spline for equi-spaced points, 451
- Descartes rule of signs, 147
- diagonal 5-points formula, 727
- diagonalization, 304–305
- diagonally dominant, 239, 241
- differential equation, xxvi, 576, 642, 679
- differential operator ( $D$ ), 368
- discussion on methods for linear systems, 256
- divided differences, 343–344
- Doolittle method, 183
- double integral, 567
  - Simpson 1/3 rule, 569
  - trapezoidal rule, 567
- Du-Fort and Frankel scheme, 692
- eigenvalues and eigenvectors, 268, 299
  - complex eigenvalues, 273
  - matrix with distinct eigenvalues, 274

- matrix with repeated eigenvalues & linearly dependent eigenvectors, 276
- matrix with repeated eigenvalues & linearly independent eigenvectors, 275
- real eigenvalues, 271
- eigenvalues and eigenvectors of real symmetric matrices, 299
  - Givens Method, 312
  - Householder Method, 319
  - Jacobi Method, 307
- elementary row operations, 192
- elliptic equations, 682, 725
  - Laplace equation, 682, 725–726
  - Poisson equation, 682, 740
- epilogue, 34
- error, 34
  - absolute, relative and percentage errors, 13
  - errors in implementation of numerical methods, 17
  - errors in modeling of real world problems, 15
  - floating point arithmetic and error propagation, 23
  - loss of significance: condition & stability, 34
  - machine eps (Epsilon), 33
  - overflow and underflow, 22
  - round-off error, 17
  - some interesting facts about error, 41
  - truncation error, 30
- error analysis, 13
- error in interpolation, 350
- errors in numerical solutions of differential equations, 623
- error in Newton–Cotes quadrature formulas, 528
  - Boole rule, 531
  - Simpson 1/3 rule, 529
  - Simpson 3/8 rule, 530
  - trapezoidal rule, 529
  - Weddle rule, 531
- error propagation in function of more than one variable, 28
  - general error formula, 28
- error propagation in function of single variable, 27
- Euler method, 588–598, 623, 625–634, 648–651
- Euler–Maclaurin formula, 553
- Everett formula, 408
- existence and uniqueness of solutions, 581
- explicit scheme, 689, 703, 711, 751
  - 1-dimensional heat equation, 688, 703
  - 1-dimensional wave equation, 751
  - 2-dimensional heat equation, 711
- exponent, 18
- exponential fit, 472
- extrapolation, 558
- false position method, 68
- finite difference approximations
  - ordinary derivatives, 661
  - partial derivatives, 683
  - derivatives for unequal intervals, 668
- finite difference interpolation formulae
  - Bessel formula, 500
  - Everett formula, 500
  - Gauss backward central difference formula, 499
  - Gauss forward central difference formula, 499
  - Newton backward difference formula, 458
  - Newton forward difference formula, 497
  - Steffensen formula, 501
  - Stirling formula, 500
- finite difference methods for differential equations, 661, 664, 683
  - boundary value problems, 580, 658, 664
- finite difference operators
  - backward difference, 366
  - central difference, 366
  - forward difference, 365
- finite difference table, 379
- finite difference tables and error propagation, 379, 385
- first order ODEs: Initial value problems, 576
- fixed point, 59
- fixed point method, 60, 76, 88, 125
- fixed point method (or) direct iteration method (or) method of successive approximations, 59
- floating point arithmetic, 23
- floating point arithmetic and error propagation, 23
- floating point numbers, 18, 22–24

- forward difference approximations of, 662–663, 668, 684
  - first order ordinary derivative, 662, 668
  - first order partial derivatives, 684–685
  - second order ordinary derivative, 663, 668
  - second order partial derivative, 684, 686
- forward difference interpolation method, 390, 393
- error in Newton forward difference formula, 393
- Gregory–Newton forward difference formula, 390
- forward difference operator, 365, 368
  
- Gauss backward central difference formula, 402
- Gauss elimination method, 192, 207
- Gauss forward central difference formula, 399
- Gauss quadrature formulas, 535
  - Gauss–Chebyshev method, 546
  - Gauss–Hermite method, 551
  - Gauss–Laguerre method, 549
  - Gauss–Legendre method, 535
- Gauss–Jordan method, 203
- Gauss–Seidel iteration method, 222, 240, 246
- general error formula, 28
- general implicit scheme, 691
- general, particular and singular solutions, 580
- Gerschgorin theorem, 277
- Givens method, 312
- Graeffe root squaring method, 161
- graphical methods for roots of nonlinear equations, 49
- graphical representation of
  - bisection method, 55
  - fixed point method, 64, 81
  - Muller method, 101
  - Newton–Raphson method, 67, 82
  - Regula–Falsi method, 69
  - secant method, 72, 85
- graphical representation of
  - Boole method, 515
  - Simpson 1/3 method, 513, 519
  - Simpson 3/8 method, 514
  - trapezoidal method, 512, 518
- graphical representation of Euler method, 591
  
- Gregory–Newton backward difference formula, 395
- Gregory–Newton forward difference formula, 390
  
- heat conduction equation, 688, 711
- Hermite interpolation, 354
- Heun’s method, 592
- hexadecimal numbers, 2
- homogeneous and nonhomogeneous differential equations, 578
- Householder method, 319
- hyperbolic equation, 680–681, 750
- hyperbolic equation (1-dimensional wave equation), 750
  - explicit scheme, 751
  - implicit scheme, 751
  
- implicit scheme
  - 1-dimensional heat equation, 691
  - 1-dimensional wave equation, 751
- initial and boundary value problems, 580
- intermediate value theorem, 52
- interpolating polynomial, 331, 389
- interpolation, xviii, 331, 389
- interpolation for equal intervals, 389
- interrelations of finite operators, 370
- interval halving method, 54
- inverse power method, 285
  
- Jacobi method for linear systems, 218, 237, 245
- Jacobi method (or) method of simultaneous displacement, 218
- Jacobi method for eigenvalues, 307
  
- Lagrange bivariate interpolation, 431
- Lagrange method, 340
- Lagrange polynomial, 340
- Laplace equation, xxviii, 682, 726
- least squares curve, 467
- least squares method, 467
  - linear curve (or) straight line fitting, 468
  - nonlinear curve fitting by linearization of data, 470
  - quadratic curve fitting, 474

- Lin–Bairstow method, 156
- linear and nonlinear, 786
- linear and nonlinear differential equations, 579
- linear systems of equations, 173
  - Cramer rule, 176
  - Gauss elimination method, 192, 207
  - Gauss–Jordan method, 203
  - Gauss–Seidel method, 222, 240, 246
  - Jacobi method, 218, 237, 245
  - LU decomposition method, 182
  - matrix inversion method, 178
  - relaxation method, 227, 247
- linearity and commutative properties of
  - difference operators, 369
- loss of significance, 34
- LU decomposition method, 182
  - Cholesky method, 190
  - Crout method, 183
  - Doolittle method, 183
- machine eps (Epsilon), 33
- Maclaurin series, 784
- Mantissa, 18
- matrix forms of
  - Gauss–Seidel method, 246
  - Jacobi method, 245
  - relaxation methods, 247, 248
- matrix inversion method, 179
- matrix method for stability of CN scheme, 707
- matrix method for stability of explicit scheme, 705
- matrix with distinct eigenvalues, 274
- matrix with repeated eigenvalues and linearly dependent eigenvectors, 276
- matrix with repeated eigenvalues and linearly independent eigenvectors, 275
- mean or average operator, 367, 369
- mean value theorems, 785
- Milne–Simpson method, 608
- modeling error, 16
- modified (or) generalized Newton–Raphson method, 94
- modified (or) improved Euler method, 592
- Muller method, 101
- multiple roots, 92
- nested Newton form, 334
- Neumann boundary conditions, 683
- Neumann method for stability of CN scheme, 709
- Neumann method for stability of explicit scheme, 708
- Newton–Cotes quadrature formulas (using Lagrange method), 510
  - Boole rule, 514, 519, 531
  - Simpson 1/3 rule, 513, 518, 529
  - Simpson 3/8 rule, 514, 519, 530
  - trapezoidal rule, 512, 517, 529
  - Weddle rule, 515
- Newton bivariate interpolation for equi-spaced points, 435
- Newton divided difference (NDD) method, 343
- Newton divided differences and other finite differences, 377
- Newton form, 333–336
- Newton–Raphson method, 65, 81, 90, 135
- nonlinear curve fitting by linearization of data, 470
- nonlinear equations
  - Aitken  $\Delta^2$ -process, 110
  - bisection method, 54, 74, 87
  - Chebyshev method, 106
  - fixed point method, 60, 76, 88, 125
  - Muller method, 101
  - Newton–Raphson method, 65, 81, 90, 135
  - Regula–Falsi method, 68, 85, 99
  - secant method, 71, 85, 97
- nonlinear systems of equations, 124
  - fixed point method, 127
  - Newton–Raphson (NR) method, 135
  - Seidel iterative method, 131
- normal equations, 469, 475
- normalized form of numbers, 18
- number systems, 1
  - representation of fractions, 8
  - representation of integers, 2
- numerical differentiation, 495
- numerical integration, 509

- octal numbers, 2
- order of convergence, 86, 237
  - bisection method, 87
  - fixed point method, 88
  - Newton–Raphson method, 90
  - Regula–Falsi method, 99
  - secant method, 97
- operational counts for Gauss elimination method, 197
- operators on some functions, 374
- optimal relaxation parameter, 253
- order and degree of differential equations, 578
- order and stability of numerical methods, 624
- ordinary and partial differential equations, 577
- ordinary differential equations with initial conditions, 582
  - Adams method, 616
  - Euler method, 588, 623, 625, 648
  - Milne method, 608
  - modified (or) improved Euler method (or) Heun method, 592
  - Picard method of successive approximations, 582, 644
  - Runge–Kutta (RK) methods, 597, 652
  - Taylor series method, 585, 647
- orthogonal transformations, 306
- overflow and underflow, 22
  
- Padé approximation, 484
- parabolic equation, 681, 688, 711
- parabolic equation (1-dimensional heat conduction or diffusion equation), 688
  - Bender–Schmidt explicit scheme, 689, 703, 705, 708, 765
  - Crank–Nicolson (CN) scheme, 690, 707, 709
  - Du–Fort and Frankel scheme, 692
  - general implicit scheme, 691
  - Richardson scheme, 692
- parabolic equation (2-dimensional heat conduction or diffusion equation)
  - alternating direction implicit (ADI) scheme, 714
  - Crank–Nicolson (CN) scheme, 712
  - explicit scheme, 711
- partial differential equations, 679
  - partial differential equations: finite difference methods, 679
  - partial pivoting, 207
  - Picard method of successive approximations, 582, 644
  - piecewise interpolation, 357
  - pivoting strategies for Gauss elimination method, 207
    - complete pivoting, 210
    - partial pivoting, 207
    - scaled partial pivoting, 209
  - plane rotations, 307
  - Poisson equation, 682, 725
  - polynomial, 147, 333
  - polynomial equations
    - Birge–Vieta method, 152
    - Descartes rule of signs, 147
    - Graeffe root squaring method, 161
    - Lin–Bairstow method, 156
    - Strum sequence, 148
  - polynomial forms
    - change of center in Newton form, 336
    - nested Newton form, 334
    - Newton form, 334
    - power form, 333
    - recursive algorithm for the nested Newton form, 335
    - shifted power form, 333
  - polynomial wiggle, 416
  - power form of polynomials, 333
  - power method, 281
    - inverse power method, 285
    - shifted power method, 288
  - predictor-corrector method for first order IVPs
    - Adams method, 616
    - Milne method, 608
  - propagated error in arithmetic operations, 24
  - properties and convergence of methods for roots of nonlinear equations, 116
  - properties and interrelations of finite operators, 369
  - properties of divided differences, 348
- quadratic convergence, 92
- quadratic curve fitting, 474



- Ralston and Rabinowitz method, 600  
 recursive algorithm for the nested Newton form, 335  
 Regula–Falsi method (or) method of false positions, 68, 85, 99  
 relational table for finite operators, 373  
 relative error, 13  
 relaxation method, 227, 247  
 representation of fractions, 8  
 representation of integers, 2  
 Richardson extrapolation, 558  
 Richardson scheme for heat conduction equation, 692  
 Rolle's theorem, 785  
 Romberg integration, 560  
 roots, 47, 124  
 roots of nonlinear equations, 47  
   Aitken  $\Delta^2$ - process, 110  
   bisection method, 54, 74, 87  
   Chebyshev method, 106  
   fixed point method, 60, 76, 88, 125  
   Muller method, 101  
   Newton–Raphson method, 65, 81, 90, 135  
   Regula–Falsi method, 68, 85, 99  
   secant method, 71, 85, 97  
 rounding error, 17  
 round-off, 17  
 row operations, 192  
 Runge–Kutta (RK) methods, 597  
   1<sup>st</sup> order RK method (Euler method), 598  
   2<sup>nd</sup> order RK method (Modified Euler method and Ralston–Rabinowitz method), 598  
   3<sup>rd</sup> order RK method, 601  
   4<sup>th</sup> order RK method (Classical RK method), 602, 652  
 Rutishauser (LR) method, 291  
 scaled partial pivoting, 209  
 secant method, 71, 85, 97  
 Seidel iterative method, 131  
 shift operator, 367  
 shifted power form, 333  
 shifted power method, 288  
 shooting method, 658  
 significant digits, 18  
 similarity transformation, 304  
 Simpson 1/3 rule, 513, 518, 529  
 Simpson 3/8 rule, 514, 519, 530  
 some important classifications and terms for differential equations, 577  
 some interesting facts about error, 41  
 SOR method, 227, 233  
 spline interpolation, 446  
 splines, 446, 462  
 spectral radius, 128, 252  
 stability of process, 34  
 stability analysis of IVP  $y' = Ay$ ,  $y(0) = y_0$ , 626  
 stability of explicit scheme, 705, 708  
 stability of CN scheme, 707, 709  
 standard 5-points formula, 726  
 Steffensen formula, 410  
 stiff equation, 624  
 Stirling formula, 404  
 stopping criteria for iterations, 56  
 Strum sequence for polynomials, 148  
 Strum sequence for tridiagonal matrix, 311  
 Strum theorem, 149  
 successive over relaxation (SOR) method, 227, 247  
 synthetic division, 153  
 systems of first order odes and higher order odes, 642  
   Euler method, 648  
   Picard method, 644  
   Runge–Kutta 4<sup>th</sup> order method, 652  
   Taylor series method, 647  
 systems of linear equations, 173  
   Cramer rule, 176  
   Gauss elimination method, 192  
   Gauss–Jordan method, 203  
   Gauss–Seidel method, 222, 240, 246  
   Jacobi method, 218, 237, 245  
   LU Decomposition method, 182  
   matrix inversion method, 178  
   relaxation method, 227, 247  
 systems of nonlinear equations, 124  
   fixed point method, 127  
   Newton–Raphson (NR) method, 135  
   Seidel iterative method, 131

## Table:

- approximation techniques for a given data set of  $(n+1)$  points, 488
- finite difference methods for 1-dimensional heat conduction equation, 710
- finite difference methods for 2-dimensional heat conduction equation, 717
- formulation of methods for roots of nonlinear equations, 115
- Iterative methods for linear systems, 255
- methods for solutions of nonlinear systems, 169
- methods for solutions of polynomial equations, 170
- Newton–Cotes quadrature formulas, 534
- numerical schemes for IVP, 634, 635
- numerical schemes for system of IVPs, 658
- numerical techniques for integration, 565, 566
- properties and convergence of methods for roots of nonlinear equations, 116
- summary and observations for roots of nonlinear equations, 117
- summary table for finite differences formulas, 412–415
- summary table for numerical differentiation formulas, 498–501
- Taylor polynomial, 783
- Taylor series, 783
- Taylor series method, 585, 647
- Thomas algorithm, 199
- transcendental equations, 48
- trapezoidal rule, 512, 517, 529
- trial and error methods, 51
- tridiagonal systems, 199
- Truncation error, 30
- underflow, 22
- wave equation, 681, 750
- Weddle rule, 515
- Weierstrass approximation theorem, 359