Integrable Systems

A Celebration of Emma Previato's 65th Birthday

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1 A Word from the Editors

These two volumes celebrate and honor Emma Previato, on the occasion of her 65th birthday. The present volume consists of 16 articles on and around the subject of integrable systems, one of the two main areas where Emma Previato has made many major contributions. The companion volume focuses on Emma's other major research area, algebraic geometry. The articles were contributed by Emma's coauthors, colleagues, students, and other researchers who have been influenced by Emma's work over the years. They present a very attractive mix of expository articles, historical surveys and cutting edge research.

Emma Previato is a mathematical pioneer, working in her two chosen areas, algebraic geometry and integrable systems. She has been among the first women to do research, in both areas. And her work in both areas has been deep and influential. Emma received a Bachelor's degree from the University of Padua in Italy, and a PhD from Harvard University under the direction of David Mumford in 1983. Her thesis was on hyperelliptic curves and solitons. The work on hyperelliptic curves has evolved and expanded into Emma's life-long interest in algebraic geometry. The work on solitons has led to her ongoing research on integrable systems, which is the subject of the present volume.

Emma Previato has been a faculty member at the Department of Mathematics at Boston University since 1983. She has published nearly a hundred research articles, edited six books, and directed seven PhD dissertations. Her broader impact extends through her renowned teaching and her extensive mentoring activities. She runs AFRAMATH, an annual outreach symposium, and works tirelessly on several on- and off-campus mentoring programs. She has also founded and has been leading the activities of the Boston University chapters of MAA and AWM. She serves on numerous advisory boards.

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The subject of integrable systems has a long and rich history. It brings together ideas and techniques from analysis, geometry, algebra, and several branches of physics. Emma Previato's work has made important contributions in all of these directions. We review some of her accomplishments in section 2 below.

We tried to collect in this volume a broad range of articles covering most of these areas. We summarize these contributions in section 3.

The editors and many of the authors have enjoyed years of fruitful inter-actions with Emma Previato. We all join in wishing her many more years of health, productivity, and great mathematics.

2 Emma Previato's Contributions

Emma Previato works in different areas, using methods from algebra, algebraic geometry, mechanics, differential geometry, analysis, and differential equations. The bulk of her research belongs to integrable equations. She is noted for often finding unexpected connections between integrability and many other areas, often including various branches of algebraic geometry.

Early Activity

As an undergraduate at the University of Padua, Italy, Emma wrote a dissertation on group lattices, followed by six journal publications [6, 5, 4, 3, 2, 1]. With methods from algebra, initiated by Dedekind in the 19th century, this area's goal is to relate the group structure to the lattice of subgroups, and provide classifications for certain properties: an excellent overview is the article by Freese [7], a review of the definitive treatise by R. Schmidt, where results from all of Emma's papers are used to give one example, a lattice criterion for a finitely generated group to be solvable.

PhD Thesis and Main Area

Emma's thesis [8], submitted at Harvard in 1983 under the supervision of D.B. Mumford [9], is still her most cited paper. Her thesis advisor was among the pioneers of this beautiful area, integrable equations, which grew and unified disparate parts of mathematics over the next twenty years, and is still very active. Emma's original tool for producing exact solutions to large classes of nonlinear PDEs, the Riemann theta function, remained one of her main interests.

Theta Functions

She later pursued more theoretical aspects of special functions, such as Prym theta functions [14, 12, 11, 10, 13] also surprisingly related to numerical results in conformal field theory, the Schottky problem [15], and Thetanulls [16].

Algebraically Completely Integrable Systems

The area of integrable PDEs is surprisingly related to algebraically completely integrable Hamiltonian systems, or ACIS, in the sense that algebro-geometric (aka finite-gap) solutions of integrable hierarchies linearize on Abelian varieties, which can be organized into angle variables for an ACIS over a suitable base, typically a subset of the moduli space of curves whose Jacobian is the fiber [11, 17]. Thanks to this discovery, the area integrates with classical geometric invariant theory, surface theory, and other traditional studies of algebraic geometry. With the appearance of the moduli spaces of vector bundles and Higgs bundles over a curve, at the hands of N. Hitchin in the 1980s, large families of ACIS were added to the examples, as well as theoretical algebro-geometric techniques. In [10, 18, 20, 19], Emma took up the challenge of generalizing the connection between ACIS and integrable hierarchies to curves beyond hyperelliptic. In [21], the families of curves are organized as divisors in surfaces.

Higher Rank and Higher-Dimensional Spectra

On the PDE side, the challenges were of two types. When the ring of functions on the (affine) spectral curve can be interpreted as differential operators with a higher-dimensional space of common eigenfunctions, the fiber of the integrable system is no longer a Jacobian: it degenerates to a moduli space of higher-rank vector bundles, possibly with some auxiliary structures [22]. Neither the PDEs nor the integrable systems have been made explicit in higher rank in general. Some cases, however, are worked out in [26, 25, 24, 23, 27]. The other challenge is to increase the dimension of the spectral variety, for example from curve to surface. Despite much work, this problem too has arguably no explicit solution in general. An attempt to set up a general theory over a multi-dimensional version of the formal Universal Grassmann Manifold of Sato which hosts all linear flows of solutions of integrable hierarchies, is given in [28], and more concrete special settings are mentioned below, under the heading of "Differential Algebra".

Special Solutions: Coverings of Curves

An important aspect of theta functions is their reducibility, a property whose investigation goes back to Weierstrass and his student S. Kowalevski. Given their special role in integrability, reducible theta functions are invaluable for applied mathematicians to approximate solutions, or even derive exact expressions and periods in terms of elliptic functions. To the algebro-geometric theory of Elliptic Solitons, initiated by I.M. Krichever and developed by A. Treibich and his thesis supervisor J.-L. Verdier, Emma contributed [31, 32, 35, 30, 34, 29, 12, 33], while [37, 36] generalize the reduction to hyperelliptic curves or Abelian subvarieties. More general aspects of elliptic (sub)covers are taken up in [38].

Another type of special solution is the one obtained by self-similarity [39]; the challenge here is to find an explicit relationship between the PDE flows and the deformation in moduli that obeys Painlevé-type equations: this is one reason why Emma's work has turned to a special function which is associated to Riemann's theta function but only exists on Jacobians: the sigma function (cf. the eponymous section below).

Poncelet and Billiards: Generalizing ACIS

Classical theorems of projective geometry can be generalized to ACIS [41, 40], while the challenge of matching them with integrable hierarchies is still ongoing [42].

Generalizing ACIS: Hitchin Systems

Explicit Hamiltonians for the Hitchin system are only available in theory: they are given explicit algebraic expression in [43] (cf. also [44], which led to work on the geometry of the moduli space of bundles [45]). An explicit integration in terms of special functions leads to the problem of non-commutative theta functions [46].

Differential Algebra

Differential Algebra is younger than Algebraic Geometry, but it has many features in common. Mumford gives credit to J.L. Burchnall and T.W. Chaundy for the first spectral curve, the Spectrum of a commutative ring of differential operators [47]. This is arguably the reason behind algebro-geometric solutions to integrable hierarchies. On the differential-algebra setting, Emma published [48, 49], connecting geometric properties of the curve with differential resultants, a major topic of elimination theory which is currently being worked out

[51, 50] and naturally leads to the higher-rank solutions: their Grassmannian aspects are taken up in [54, 53, 55, 56, 52] the higher-dimensional spectral varieties are addressed in [57]. Other aspects of differential algebra are connected to integrability in [58] (the action of an Abelian vector field on the meromorphic functions of an Abelian variety) and [59] (a p-adic analog); in [60], the deformations act on modular forms.

The Sigma Function

Klein extended the definition of the (genus-one) Weierstrass sigma function to hyperelliptic curves and curves of genus three. H.F. Baker developed an in-depth theory of PDEs satisfied by the hyperelliptic sigma function, which plays a key role in recent work on integrable hierarchies (KdV-type, e.g.). Beginning in the 1990s, this theory of Kleinian sigma functions was revisited, originally by V.M. Buchstaber, V.Z. Enolskii, and D.V. Leykin, much extended in scope, eventually to be developed for "telescopic" curves (a condition on the Weierstrass semigroup at a point). Previato goes beyond the telescopic case in [62, 61], while she investigates the higher-genus analog of classical theorems in [69, 68, 70, 67, 66, 63, 64, 65] and their connections with integrability in [71] and [72], which gives the first algebro-geometric solutions to a dispersionless integrable hierarchy. It is not a coincidence that its integrable flow on the Universal Grassmann Manifold 'cut across' the Jacobian flows of traditional hierarchies, and this is where the two variables of the sigma function (the Jacobian, and the modular ones) should unite to explain the mystery of the Painlevé equations.

Algebraic Coding Theory

Emma's primary contribution to this area is through mentoring undergraduate and graduate thesis or funded-research projects. In fact, this research strand began at the prompting of students in computer science who asked her to give a course on curves over fields of prime characteristic, which she ran for years as a vertically-integrated seminar. Together with her PhD student Drue Coles, she published research papers pursuing Trygve Johnsen's innovative idea of error-correction for Goppa codes implemented via vector bundles [74, 75, 73], then she pursued overviews and extensions of Goppa codes to surfaces [76].

Other

Emma edited or co-edited four books [78, 80, 79, 77]. In addition to book and journal publication, Emma published reviews (BAMS, SIAM), entries in mathematical dictionaries or encyclopaedias, teaching manuals and online

research or teaching materials; she also published on the topic of mentoring in the STEAM disciplines.

3　Articles in this Volume

Gesztesy and Nichols consider a particular class of integral operators $T_{\gamma,\delta}$ in $L^2(\mathbb{R}^n)$, $n \in \mathbb{N}$, $n \geq 2$, with integral kernels bounded. These integral operators (and their matrix-valued analogs) naturally arise in the study of multi-dimensional Schrödinger and Dirac-type operators and they describe an application to the case of massless Dirac-type operators.

H. Knörrer is using quaternions to give explicit formulas for the global symmetries of the three dimensional Kepler problem. The regularizations of the Kepler problem that are based on the Hopf map and on stereographic projections, respectively, are interpreted in terms of these symmetries.

Zoladek gives two proofs of the Jacobi identity for the Poisson bracket on a symplectic manifold.

Luen-Chau Li presents an expository account of the work done in the last few years in understanding a matrix Lax equation which arises in the study of scalar hyperbolic conservation laws with spectrally negative pure-jump Markov initial data. He begins with its extension to general $N \times N$ matrices, which is Liouville integrable on generic coadjoint orbits of a matrix Lie group. In the probabilistically interesting case in which the Lax operator is the generator of a pure-jump Markov process, the spectral curve is generically a fully reducible nodal curve. In this case, the equation is not Liouville integrable, but we can show that the flow is still conjugate to a straight line motion, and the equation is exactly solvable. En route, he establishes a dictionary between an open, dense set of lower triangular generator matrices and algebro-geometric data which plays an important role in the analysis.

F. Calogero and F. Payandeh study solvable dynamical systems in the plane with polynomial interactions. They present a few examples of algebraically solvable dynamical systems characterized by 2 coupled Ordinary Differential Equations. These findings are obtained via a new twist of a recent technique to identify dynamical systems solvable by algebraic operations, themselves explicitly identified as corresponding to the time evolutions of the zeros of polynomials the coefficients of which evolve according to algebraically solvable (systems of) evolution equations.

V. Dragovich, M. Radnovic, and R.F. Ranomenjanahary present recent results about double reflection and incircular nets. The building blocks are pencils of quadrics, related billiards, and quad graphs.

Alessandro Arsie and Paolo Lorenzoni present a survey of the work done by the authors in the last few years developing the theory of bi-flat F-manifolds

and exploring their relationships with integrable hierarchies (dispersionless and dispersive), with Painlevé transcendents, and with complex reflection groups.

Pol Vanhaecke studies some algebraic-geometrical aspects of the periodic 6-particle Kac-van Moerbeke system. This system is known to be algebraically integrable, having the affine part of a hyperelliptic Jacobian of a genus two curve as the generic fiber of its momentum map. Particular attention goes to the divisor needed to complete this fiber into an Abelian variety: it consists of six copies of the curve, intersecting according to a pattern which is determined in the paper. The author also compares this divisor to the divisor which appears in some natural singular compactification of the fiber.

T. Brown and N. Ercolani focus on discrete Painlevé equations and connections between combinatorics and integrable systems. Two discrete dynamical systems are discussed and analyzed whose trajectories encode significant explicit information about a number of problems in combinatorial probability, including graphical enumeration on Riemann surfaces and random walks in random environments. The authors show that the two models are integrable and their analysis uncovers the geometric sources of this integrability and uses this to conceptually explain the rigorous existence and structure of elegant closed form expressions for the associated probability distributions. Connections to asymptotic results are also described. The work brings together ideas from a variety of fields including dynamical systems theory, probability theory, classical analogues of quantum spin systems, addition laws on elliptic curves, and links between randomness and symmetry.

T. Kappeler and P. Topalov study the Arnold-Liouville theorem for integrable PDEs. They present an infinite dimensional version of the Arnold-Liouville theorem.

A. Chern, F. Knoeppel, F. Pedit, and U. Pinkall study commuting Hamiltonian flows of curves in real space forms. They provide a geometric point of view of the Hamiltonian flows.

Franco Magri writes on the Kowalewski Top from the viewpoint of bihamiltonian geometry. The paper is a commentary of one section of the celebrated paper by Sophie Kowalewski on the motion of a rigid body with a fixed point. Its purpose is to show that the results of Kowalewski may be recovered by using the separability conditions obtained by Tullio Levi-Civita in 1904.

Steven Rayan and Jacek Szmigielski study Peakons and Hitchin systems. They review the Calogero-Françoise integrable system, which is a generalization of the Camassa-Holm system. We express solutions as (twisted) Higgs bundles, in the sense of Hitchin, over the projective line. We use this point of view to (a) establish a general answer to the question of linearization of isospectral flow and (b) demonstrate, in the case of two particles, the dynamical

meaning of the theta divisor of the spectral curve in terms of mechanical collisions. They also outline the solution to the inverse problem for CF flows using Stieltjes' continued fractions.

Spalding and Veselov study the tropical version of Markov dynamics on the Cayley cubic. They prove that this action is semi-conjugated to the standard action of $SL_2(\mathbb{Z})$ on a torus and therefore ergodic with the Lyapunov exponent and entropy given by the logarithm of the spectral radius of the corresponding matrix.

G.S. Mauleshova and A.E.Mironov focus on one-point commuting difference operators. They study a new class of rank one commuting difference operators containing a shift operator with only positive degrees. We obtain equations which are equivalent to the commutativity conditions in the case of hyperelliptic spectral curves. Using these equations we construct explicit examples of operators with polynomial and trigonometric coefficients.

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