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Analysis and Geometry on Graphs and Manifolds

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Preface

This book brings together contributions for the conference 'Analysis and geometry on graphs and manifolds,' which took place at the University of Potsdam in Potsdam, Germany, from 31st July to 4th August 2017. The aim of the conference was to bring together leading experts in geometric analysis, in both the discrete and continuous settings. This included researchers working on such diverse models as manifolds, graphs, fractals, groups, and metric measure spaces. The goal was for these researchers to share their expertise and to explore common ground. Each day there was also an extensive afternoon session which provided time for young researchers to present partial results and early work.

The overall theme of the conference and of the contributions contained in this volume is the interplay of geometry, spectral theory, and stochastics. This interplay has a long and fruitful history and can be seen as a driving force behind many developments in modern mathematics. The present volume focuses on the global effects of local properties. This can be explored in both the discrete and continuous settings, and there has been a continual interest in contrasting what happens in these two cases. The main goal of this volume is to give an expository overview of these topics. This is achieved by presenting a mixture of survey chapters which examine the landscape of certain subjects and shorter chapters which focus on specific techniques and problems.

We will now briefly comment on the content of the chapters contained in this volume. In doing so, we will also point out some of the connections between the chapters.

Curvature is a natural local quantity arising when studying the geometry of a space. It has been thoroughly investigated in the case of Riemannian manifolds. Recent years have seen an explosion of research aimed at establishing curvature notions in the discrete setting. Here we present several contributions in this direction. Namely, the chapter by Bobo Hua and Yanhui Su gives an overview of results concerning combinatorial curvature in the case of planar tessellations. In particular, they analyse combinatorial, potential, and spectral theoretic consequences of global lower combinatorial curvature bounds. The chapter by David Cushing, Shiping Liu, Florentin Münch and Norbert Peyerimhoff calculates both the Bakry–Émery and Ollivier–Ricci curvatures of a class of graphs called 'antitrees'. Antitrees have recently come of interest as they provide surprising counterexamples to direct analogues of statements from the continuous setting. For general graphs, Bakry–Émery and Ollivier–Ricci are the most commonly appearing curvature notions. As for curvature in the continuous setting, the chapter by Józef Dodziuk describes a procedure for constructing tunnels connecting manifolds of arbitrary dimension and positive scalar curvature while preserving the positivity of the curvature. A general treatment of convergence of operators while glueing and removing subsets in the case of suitable curvature control assumptions for manifolds can be found in the chapter by Colette Anné and Olaf Post.

Global geometry is strongly reflected in the properties of a Markov process exploring the space. An analytic approach to this investigation is provided via the heat equation. The connections between curvature and properties of the heat kernel in the case of unbounded Ricci curvature on manifolds are explored in the chapter by Peter Stollmann and Christian Rose. Another geometric feature explored by the Markov process is that of a boundary at infinity. By construction, this boundary captures global geometric properties of the space. Representations of generalizations of harmonic functions on such boundaries at infinity is the topic of the chapter by Massimo A. Picardello and Wolfgang Woess. Here, the areas of random walks, geometry, and potential theory merge. The trajectories of simple random walks on Sierpinski gasket graphs is the subject of the chapter by Joe P. Chen, Wilfried Huss, Ecaterina Sava-Huss, and Alexander Teplyaev.

Spectral geometry deals with the interplay of spectral theory and both local and global geometric properties. In particular, the bottom of the spectrum and properties of ground states have been investigated in both the discrete and continuous settings. In this book, three chapters deal with these issues. Specifically, the chapter by Daniel Lenz and Peter Stollmann provides lower bounds for eigenvalues on graphs in terms of the inradius of subsets. Furthermore, the chapter by Matthias Keller, Yehuda Pinchover, and Felix Pogorzelski discusses the Hardy inequality, and the chapter by Lior Alon, Michael Bersudsky, Sebastian Egger, and Ram Band deals with Neumann domains of eigenfunctions. These two chapters deal with both graphs and manifolds. The even more fundamental question of the existence and uniqueness of self-adjoint realizations in the discrete setting is detailed in the chapter by Marcel Schmidt.

The bottom of the spectrum is also a prominent topic in models investigated in geometric group theory. In particular, the chapter by Ana Khukhro and Alain Valette discusses expanders in the context of box spaces of Cayley graphs. Furthermore, the article by Ori Parzanchevski deals with Ramanujan digraphs, a counterpart to Ramanujan graphs, which are a special class of expanders. Finally, the chapter by Andrzej Żuk deals with the discretization of partial differential equations and connections to automata groups.

The spectral theory of Schrödinger operators in the case of a simple geometrical setting strongly depends on global features of the potential. This is analogous to how global features for complicated geometries influence the spectral theory of the Laplace–Beltrami operator. The impact of the potential on spectral theory is particularly prominent in the case of potentials generated by random processes or dynamical systems. The chapter by David Damanik and Jake Fillman gives a thorough overview of the spectral theory of Schrödinger operators on the one-dimensional lattice with potentials which are periodic or limit-periodic. The convergence of the integrated density of states in the case of random Schrödinger operators over lattices and amenable groups is investigated in the chapter by Christoph Schumacher, Fabian Schwarzenberger, and Ivan Veselić.

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