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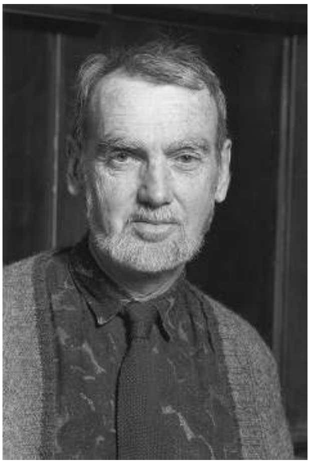
"It is exciting to read this important book. It is a marvelous panorama of the life, work, and inspiration of Robert Langlands, as he discovered and developed his grand ideas, and as he guided his students. These ideas now largely shape the broad architecture of representation theory and automorphic forms, creating a grand bridge between analysis and arithmetic – with connections to mathematical physics. The contributors to this volume offer us – and will surely offer future historians of our mathematical age – a splendid introduction to, and overview of, the early days of Langlands' program."

– Barry Mazur, Harvard University

Robert Langlands was born in 1936, in New Westminster, British Columbia. He received his bachelor's and master's degrees in mathematics at the University of British Columbia in 1958, and his Ph.D. from Yale University in 1960. He not only passed his oral examination but also submitted his Ph.D. thesis at the end of his first year at Yale.

The inception of the Langlands Program can be dated to 1967. In 1972, he was appointed professor at the Institute for Advanced Study in Princeton, New Jersey, USA, where he has been a Professor Emeritus.

Professor Langlands has received numerous prestigious awards, such as the 1996 Wolf Prize (with Andrew Wiles) and the 2007 Shaw Prize (with Richard Taylor). He is also the recipient of the 2018 Abel Prize, which has often been regarded as the equivalent of the "Nobel Prize" in mathematics.

Robert Langlands and his wife, Charlotte, have been living in Canada since March 2020.

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More Information

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Preface

Robert Langlands' contributions to modern number theory, which is generally referred to as the "Langlands Program," has been visionary and ground breaking. The present volume is intended as homage to his early work in which the seeds of the program were planted. This volume covers the period from 1960 to 1967, during which Langlands defined many of the concepts that are basic to the program; at this time, he introduced some of his revolutionary ideas including the functoriality principle and his foundational work on Eisenstein series and Artin's L-functions. The period concludes with Langlands' famous letter to André Weil.

The book is divided into five parts. Parts I, II, and III are a gathering of personal reflections written by a number of his friends and former students; Julia Mueller's *Bulletin of the American Mathematical Society* interview with Langlands is also included. Part IV is a collection of surveys on different aspects of Langlands' work that took place over the period, including different concepts. Part V consists of two articles on Langlands' work in physics that took place later in his career.

The scope and depth of the Langlands Program will keep mathematicians busy for many exciting years, and we hope that this volume will be a useful introduction to it.

James Arthur
Steve Gelbart
Freydoon Shahidi

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Introduction

This volume of contributed chapters is assembled as a tribute to Robert Langlands' prodigious accomplishments over the years from 1960 to 1967, from the age of 24 to 31. Surveys of his work during that period are included in Part I and Part IV of this volume. The two nontechnical chapters in Part I are contributed by Julia Mueller and Steve Gelbart. A few reflections from some of his students and friends are contained in Part II and Part III. The chapters in Part IV are survey articles contributed mostly by experts of the Langlands program, and Part V contains two survey articles of Langlands' work in physics.

The Langlands program was launched soon after Langlands' discovery of his automorphic L-functions. This important discovery together with his principle of functoriality can be regarded as pillars of the Langlands program. This far-reaching principle was first proposed in his manuscript "Problems in the theory of automorphic forms." It is a well-formulated conjecture and can be explained using the basic materials of the program. We have an excellent survey article by James Arthur in Part IV, where his idea is to motivate functoriality the way Langlands originally presented it, with Artin L-functions and principal (Godement–Jacquet) L-functions as the background.

The story of Langlands' discovery of his automorphic L-functions started with his foundational work on the most general Eisenstein series (1962–1964). He had to overcome many obstacles faced by others in order to develop the theory in its most general form over number fields. Roughly speaking, his new ideas in this piece of work were rooted in the works of Selberg, Harish-Chandra, and Gelfand, and most importantly, in harmonic analysis and representation theory.

It was pointed out by Langlands on several occasions (see Sections 14.1 and 14.6 of Shahidi's chapter in Part IV, and Mueller's chapter in Part I) that it was his calculations of the constant terms of those most general Eisenstein series that led him to the discovery of a number of central objects in the Langlands program such as the L-groups and his automorphic

L-functions. Without those objects, the formulation of many of his conjectures would not have been possible. The calculations appeared as a monograph "Euler Products," which he presented in his James Whitmore lectures at Yale. Moreover, his calculations, which were also the centerpiece of his famous letter to André Weil in 1967, explained the duality he discovered and many concepts that were involved with the duality. In short, it is his calculations that allowed him to have a formulation of his functoriality principle. His paper "Problems in the theory of automorphic forms," which appeared later, extended his ideas from the Euler products manuscript to the general setting.

It is well known that L-groups played a central rule in Langlands' discovery of his automorphic L-functions. One of their features is in connecting the arithmetic and analytic properties of Langlands' L-functions via Langlands' isomorphism. Since L-groups have no direct connection with the "Satake parameter," it is clear that "Langlands' isomorphism" is a significant and conceptually challenging extension of the "Satake parameter." More discussions on this topic can be found in the chapters of Mueller and Shahidi in this volume. Shahidi's treatment of this topic is both axiomatical and through examples, while Mueller's presentation is less technical. It should be noted here as well that Mueller's chapter in Part I originally appeared in the *Bulletin of the AMS* (Vol. 55, No. 4, October 2018, pages 493–528).

The theory of Eisenstein series is briefly, but masterfully, surveyed by Jean-Pierre Labesse. His chapter in Part IV is an important contribution to this book. In it, Labesse discusses the spectral decomposition for which Eisenstein series are central in accounting for both the continuous and the residual part of the spectrum.

It was Langlands' attempts to formulate a nonabelian reciprocity law that was among the most appealing yet challenging parts of the program to number theorists. They were formulated as conjectures, together with the principle of functoriality, again in his manuscript "Problems in the theory of automorphic forms." These arithmetic aspects are surveyed in this volume by Matthew Emerton's contribution in Part IV. His chapter touches upon Artin's L-functions, on which Langlands had produced a long manuscript with more or less complete local proofs of the existence of root numbers for these L-functions. A complete proof of the existence was later given by Pierre Deligne using local–global methods.

The most basic of all the automorphic L-functions are those studied, in particular, by a method generalizing Tate's thesis. They are usually addressed as "Principal L-functions" for the general linear group $GL(n)$. This basic but important case of automorphic L-functions is surveyed in a chapter in Part IV, contributed jointly by Dorian Goldfeld and Hervé Jacquet.

Langlands has acknowledged in several writings and interviews the influence that Harish-Chandra has had on his work. In particular, Harish-Chandra's study of orbital integrals for real reductive groups played an important role in the harmonic analysis that entered Langlands functoriality. This has influenced Diana Shelstad's early career, and her chapter in Part IV contains her presentation on the early history of functoriality.

Langlands' Ph.D. thesis at Yale, which has only been minimally published, is surveyed by Derek Robinson, with a delightful introduction, in his chapter in Part IV. As he explains, Langlands' thesis included a number of brilliant ideas addressing and providing full proof for density of analytic vectors and the Garding space for general continuous representations of any Lie group. Let us mention that Langlands' Ph.D. thesis is prior to his work on the "Langlands Program." Here are a few words from Langlands himself:

> I will tell you a story about my thesis. I wrote it on my own in the summer between my first and second years at Yale. Charlotte typed it and I submitted, but it was independently written and no-one understood it. The stair in the mathematics building was circular and that allowed me to overhear a conversation between Felix Browder and Kakutani. Kakutani wanted to decline it because no-one understood it. Browder, who was more familiar with me than Kakutani, wanted to accept it. He fortunately prevailed. I had, about the same time, a conversation with Ed Nelson, as I was taking a tourist visit to the Institute with friends from Yale. This conversation led, with no application whatsoever, to a position as instructor at Princeton. The thesis was never published but sometime later it was incorporated (and redone) in a book by Robinson. So, for me, there is considerable history in Robinson's article. It belongs to my first years and there is no connection to Hecke and automorphic forms. That began at Princeton, but my two years at Yale had made an analyst of me. The thesis is by the way impossible for me to read now.
>
> *Robert Langlands, 2020*

Some of Langlands' work in physics is presented in the two survey articles, with an excellent overview, in Part V by Thomas Spencer and Yvan Saint-Aubin.

Finally, we wish to express our sincere gratitude to all the contributors whose participation has given life to this book.

Julia Mueller
Freydoon Shahidi