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A Glimpse at the Genesis of the Langlands Program

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1.1 Introduction

This chapter[1] serves as an introduction to the volume as a whole, and it is aimed at a general mathematical audience. Our presentation of the early life and work of Robert Langlands, creator and founder of the Langlands program, is narrated, to a large extent, by Langlands himself. We focus on two of Langlands' major discoveries: automorphic L-functions and the functoriality conjecture. Langlands' desire to communicate his excitement about his newly discovered objects resulted in his famous letter to André Weil in January 1967, and the Langlands program was launched soon afterwards.

Section 1.2 of this chapter focuses on Langlands' early years, from 1936 to 1960, and the material is taken from an interview given by Langlands to a student, Farzin Barekat, at the University of British Columbia (UBC), Langlands' alma mater, in the early 2000s. A copy of this interview is available on Langlands' website at http://publications.ias.edu/rpl/.

Section 1.3 is an overview of Langlands' early work and professional life from 1960 to 1967. It contains a brief descriptive account of the essential events that led him to his discoveries of automorphic L-functions and the functoriality conjecture. Our source material in this part is taken largely from our correspondence and interviews with Langlands himself over the past few years. Those correspondence can also be found on Langlands' website.

1.1.1 A Summary of Langlands' Early Life (1936–1960)

Let us list a few important events in Langlands' early life which, in our view, shaped his professional life later on.

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(1) Toward the end of his twelfth grade (see Section 1.2.4), an English teacher, Crawford Vogler, took up an hour of class time to explain to him, in the presence of all the other students, that it would be a betrayal of God-given talents for him not to attend university. Langlands had no intention of attending college because none of his classmates did. At the time, very, very few students, at the best one or two, did so in any year. School meant little to him. However, Langlands was flattered by his teacher's comments and his ambition was aroused.

(2) Taking the aptitude tests at UBC in the fall of 1953 (see Section 1.2.5) was a decisive event in his early life. From the results of his tests, the university counselor suggested at first that he might want to become an accountant, but Langlands rejected that idea right away. The counselor then suggested mathematics or physics, but cautioned him that this would require a master's degree or even a PhD. Langlands did not know what a PhD was, but he decided on the spot that he would become a mathematician or a physicist.

(3) Another important event was the discovery of his natural talent for languages (see Section 1.2.6). Sometime during the year of 1954, Langlands spoke to Professor S. Jennings about his intention to choose honors in mathematics. Professor Jennings declared that to be a mathematician, one had to learn French, German, and Russian. Taking Professor Jennings' advice, Langlands became fluent not only in French, German, and Russian, but also in several other languages, including Italian and Turkish.

(4) Let us take a brief look at Langlands' performance as a student at UBC. What stood out was his fierce independence in conducting his research. However, the result was not completely successful. For example, the topic of his master's thesis was chosen by himself, but unfortunately, the theorem he was trying to prove was false (see Section 1.2.7). There was some question whether the thesis could be accepted, but the committee decided to let him go on to Yale.

(5) Langlands' unique and profound talent in mathematics was first successfully revealed during his first year as a graduate student at Yale University (see Section 1.2.8). He not only completed all the required courses and examinations for the degree, but also submitted his PhD thesis at the end of his first year. Consequently, he was completely free to pursue his own research during his second year as a graduate student at Yale.

While all things seemed to point toward an illustrious career in mathematics, it is important to note that Langlands, as a young man, harbored ambitions of a much more general nature. He did not envision a career as a mathematician. Here he is, in his own words:

> I began my mathematical, or better my university, studies more than sixty years ago just before my seventeenth birthday, after a childhood and adolescence that were in no sense a preparation for an academic career, or a career in any sense. Various

influences came together that suggested that I attend university not so much to prepare myself for a profession but to become what one might think of as a savant. A major influence was perhaps Ernest Trattner's *The Story of the World's Great Thinkers'*, a book that was very popular in the late thirties, so that there are still many copies available on the used-book market at very modest prices. What I envisioned, after enrolling in the University of British Columbia in 1953, was myself, in a jacket with leather elbow-patches and perhaps a pipe in my mouth, gazing over the lawns and the trees while reflecting on a still undetermined, but certainly abstruse, topic. That topic became mathematics, not out of a strong preference for that subject, but because it pretty much required no preparation, only native ability.

1.1.2 A Summary of Langlands' Early Works (1960–1966)

During the first six years in the 1960s, Langlands' research output was prodigious. The important ideas in the discoveries of his automorphic *L*-functions and the functoriality conjecture were rooted mainly in the works of Selberg, Harish-Chandra, and Gelfand; as well as the formulation of adèlic structure of groups by Godement, Tamagawa, and Satake (see Section 1.3.12). Of course, Artin's reciprocity law (see Section 1.3.11.5) also played an important role. Let us highlight some of his important contributions.

(1) Langlands' spectral theory (see Section 1.3.8) was certainly one of his first major contributions; particularly, the most general Eisenstein series he constructed in this piece of work which played an important role in the discovery of his automorphic *L*-functions later on; for example, it became the archimedean component $\pi\infty$ of the "automorphic representation π." Let us note that his success in generalizing Selberg's result on spectral theory (see Section 1.3.7) from $SL_2(R)$ to arbitrary reductive groups was very much helped by his knowledge in harmonic analysis and representation theory that he acquired while working on his PhD thesis.

(2) Langlands' calculation of the constant term of his generalized adèlic Eisenstein series in the fall of 1966 (see Section 1.3.14) was a decisive factor in his discovery of automorphic *L*-functions. It not only led him to create the *L-group*, but also to introduce the *Langlands isomorphism.* Please note that *Langlands isomorphism extends* the Satake isomorphism and connects the Hecke algebra with the representation ring of the *L*-group while the Satake isomorphism only relates the Hecke algebra with a certain polynomial ring (see Sections 1.3.15.2 and 1.3.15.3). In Langlands' own words: "The expression 'Satake parameter' is completely inappropriate and could very well be replaced by 'Frobenius–Hecke parameter'."

(3) An Artin *L*-function is defined as an Euler product which does not have analytic properties, but when the number field extension *F* over **Q** is abelian, Artin's reciprocity law allows his *L*-function to acquire "nice" analytic

properties from Hecke's degree-1 L-series (see Sections 1.3.11.3 to 1.3.11.5). However, in a nonabelian number field extension, Artin was unable to provide such analytic properties to his L-functions.

Langlands automorphic L-functions $L(s, \pi, \rho)$, as Langlands discovered, depend on both the *automorphic representation* π and the *finite-dimensional representation* ρ, where π is an isomorphism from a "generalization" of Hecke's degree-2 L-series to the L-group, and ρ is linked, via the L-group, with the Frobenius classes of Artin's L-functions. Hence, Langlands L-functions simultaneously generalize Hecke's L-functions as well as those of Artin.

(4) We learn (see Section 1.3.16) that Langlands was contemplating the problem of establishing "nice" analytic properties for those automorphic L-functions when he created the functoriality conjecture. Apparently, one of the major moments in his mathematical career was the first hint of the genesis of functoriality. That happened during the Christmas vacation of 1966, while looking through the leaded windows of his office distractedly, he was suddenly struck by the epiphany: "The Artin reciprocity law has to be replaced by what I now refer to as functoriality." (See Section 1.3.16.2.)

There are 17 subsections in Section 1.3. A large part of the last section, Section 1.3.17, is a presentation of an interesting and illuminating example of Langlands, which gives a rough sketch on how the functoriality conjecture was discovered.

We hope that by prefacing the story of the evolution of Langlands' mathematical landscape with a summary of crucial life events, we provide the reader with greater insight into both the man and his work.

1.2 Langlands' Early Years (1936–1960)

My time as a student at UBC (University of British Columbia) was the major transitional phase of my life, and it is not possible to appreciate its significance for me without understanding what I brought to it.

1.2.1 Preschool Years

I was born in October 1936, in New Westminster, British Columbia, Canada. My first years were not, however, spent there. They were spent on the coast in a hamlet south of Powell River and slightly north of Lang Bay. It was a very small group of houses, to the best of my recollection, three summer cottages. My father had found work at the nearby Stillwater dam.

One of the other two cottages, perhaps better described as cabins, was occupied by an elderly woman and a child, who I recall was her granddaughter. That they had a goat has also fixed itself in my mind. I also remember a field surrounding their

house, but it could have been small. The other house and its inhabitants I do not recall. It may have been occupied only in the summer. Like ours, it was bounded on one side by the beach and the Straits of Georgia, and on the other by a forest or swamp. The only vegetation that fixed itself in my mind was skunk-cabbage.

The five or six years there must have been broken occasionally, by trips to New Westminster to my parents' families. But I have no recollection of such excursions, simply of my mother, my father, a younger sister, and towards the end a second baby sister.

1.2.2 Families in New Westminster

When time came for me to attend school, and perhaps for other reasons, we returned to New Westminster, then a small, in my memory, delightful city with chestnut-lined streets, the trees planted in what was called a boulevard, between the sidewalk and the curb.

New Westminster, during the war years was agreeable for a child. There were very few automobiles. The available area for me was from the north at 8th Ave. to the south at Columbia St., and from Queen's Park in the east to 12th St. in the west. So I had about one-half a square mile in which to freely roam, although I was not very nomadic. I had grandparents and many cousins close to 12th St., or on the other side of 12th, but it never occurred to me to visit them on my own.

Although we were only a short time in New Westminster, it was the period when I made the acquaintance of my many aunts and uncles. My mother's family large, my father's smaller but substantial, so that all in all there were thirteen aunts and uncles together with their husbands or wives and children.

In England my grandparents may have been Methodists but in Canada they were members of the United Church. My father's parents were not so much stern as pious. In my father there were only un-reflected remnants of the piety. I have seen a photo of my father and two sisters, all infants because he was a twin, with their mother before a tent in the forest, probably on Vancouver Island, but he himself never once mentioned those early years. As a child I had found my father's parents and his sisters and brother, in comparison with my mother's family, a little colorless and distant.

We were used to parents, especially fathers, with little schooling. My own father had eight years. His arithmetic was excellent but his reading was on the whole confined to the sport pages. My mother's family was, at least for a child, much warmer. Her parents, had drifted west from Halifax, trying their hand at land-grant farming in Saskatoon before reaching New Westminster.

My grandmother was already a widow when I was born and even before my grandfather's death had to assume a good deal of responsibility in difficult financial circumstances for her ten surviving children. She had many grandchildren, a few older than me, several of about my age, and a good number younger. Most of the latter I never met. My grandmother had affection and energy enough for all of us, although she cannot have been well. Her heart failed after a bus-ride to White Rock and a walk up a steep, dusty hill on a very warm day as she was coming to visit mother when I was ten.

Since I left British Columbia I have seen my aunts, uncles, and cousins on both sides only infrequently, but it has always been a pleasure. I still correspond occasionally with my mother's only surviving sister.

I was enrolled by my mother, a Catholic, in the parochial school, St. Ann's Academy. It was a school, taught partially by nuns, especially in the first two or three years. They were young, pretty I suppose, and encouraging, so that I enjoyed these first years, taking three years in two, or four in three, but then I was moved for the later years of the elementary school to the companion academy, St. Peter's I believe, which was less friendly, with a morose beadle, prompt to resort to his baton, and I grew restive. By the time I reached 6th grade, just before my 10th birthday, we had moved again, to White Rock, where I learned less, indeed pretty much nothing at all, but which also gave me a great deal.

1.2.3 White Rock – the Town of my Youth

I have not visited White Rock for a very long time, several decades. I would suppose that the town of my youth was quite different than the town of today. The logs washed up along the shore were there and for me, for all of us, part of the natural environment, to be enjoyed like the sea itself.

My first two or three years in the town had been carefree. I was a little older than in New Westminster, a little more independent than I had been there. The sea and the shore were immediately accessible; the pier was, at least after I learned to swim and dive, an attraction. I found my way there when free of responsibility.

On Sunday, the main street, which ran along the shore, was devoted entirely to cruising. In my early adolescence, I admired these people, their clothes, above all their freedom, their independence, but I was too young to imitate them. By the time I might have been in a position to do that, I had other goals. Each of us is a child only once, so that it is difficult to distinguish what is particular from the atmosphere in which we grew up from what was particular in us.

My parents did succeed for some years in establishing and maintaining a business, millwork and builders supplies, so that we were better off than many. After the age of twelve or thirteen I worked afternoons, weekends and summers there, and continued working there in the summer even after I began university. It was typical then, and may still be now, for students at UBC to work in the summer to earn enough for room, board and tuition during the university's relatively short fall and winter terms.

There were other sources of income, a newspaper route for the Vancouver's *The Province* to which I was faithful for a year before I tired of the inevitable loss of freedom. For six days of every week, one and a half hours of the afternoon had to be given over to collecting and delivering the newspaper. I also remember changing the marquee at the movie theater, next to the dance hall, but visited only by local residents, so that the featured movie ran just for two days, changing three times each week. This means three visits a week at 10:30 in the evening to the theater in exchange for free access. At first I was delighted with that, but as I was then old enough that the obligatory visits interfered with my social life, I soon abandoned the marquee even though the theatre was only a few hundred yards from our home and business.

School, now a public school, was mixed. I enjoyed that. Except that it was a place frequented by girls and my friends, meant little to me. For a short period I was large for my age, I was also younger than almost all my classmates and maladroit. I had little success with the limited athletic possibilities.

I was probably the despair of the teachers, who, perhaps from the results of IQ tests, were aware that I had considerable untapped academic potential, from which I refused to profit.

1.2.4 Grade 12 – a Very Special and Important Year

In my last year, in grade 12, we had an excellent teacher, Crawford Vogler, with a newly designed textbook, and a newly designed course on English literature. He is one of the people to whom I owe most and for a very specific reason. Toward the end of the year, he took up an hour of class time to explain to me, in the presence of all the other students, that it would be a betrayal of God-given talents for me not to attend university.

I had had no intention of doing that. None of my classmates did: at the time very, very few students, at the best one or two, did so in any year. I was flattered by his comments, my ambition was aroused, and I decided then and there to write the entrance examinations. I worked hard and was successful, even winning a small fellowship from the University.

There was another factor in my changing stance. I have acknowledged elsewhere that I was tainted even at a fairly early age by ambition, but could never satisfy it.

I also was not incapable of intellectual or moral passion. I had decided at the age of seven or eight, not long after beginning to attend the parochial school, that I would become a priest, building myself an altar with improvised paraphernalia in my bedroom. It probably delighted my mother to see this early sign of a vocation, but it soon passed.

In the course itself, Vogler singled me out as one of the students who could usefully present a report on a novel and assigned me Meredith's novel: *The ordeal of Richard Feverel*. He had overestimated me. I read the novel but had no idea what might be said about it, or perhaps I hesitated to express my feelings. It is, after all, a novel of young love.

Unfortunately, by the time I undertook to thank him personally, it was too late. Although still alive after a successful teaching career in the lower mainland, he was, as I learned from his son, no longer in any condition to appreciate expressions of gratitude.

By the time I read Meredith, I had met the girl, Charlotte, whom I was to marry and to whom, although she is no longer a girl, I am still married many years later.

In retrospect, I am astounded that by good luck, certainly not with any foresight, I found in a little town at an early age and with no guidance, someone who could give me so much, in so many ways, in so many different circumstances, and for so long, without sacrificing her independence or totally neglecting her talents.

Her father, having lost his mother as an infant, grew up in a Gaelic-speaking foster family on Prince Edward Island, but went off to the logging camps at the

very early age of twelve. When I met him he had spent the best part of his life in
Ontario and the Lower Mainland, some of it during the Depression, unemployed
and drifting.

My wife's father had only one year of schooling. It was at the age of 30, during
the Depression, that he learned how to read, and in principle to write, at the classes
that the parties on the left organized for the unemployed. He also acquired a small
library, but he could not really read with ease. By his books, however, in particular
one with biographies of those savants who were heroes to the socialists, Marx,
Freud, Hutton, Darwin and several others, I was inspired with the ambition to be a
savant. It is odd, but this ambition has never faded. It is the ambition with which
I arrived at the University.

1.2.5 The Aptitude Tests at UBC (1953) – a Decisive Event

At the University, I took, as was common at the time and as was perfectly
appropriate for anyone with my lack of academic experience, aptitude tests.

The results were predictable. In those domains, mathematics and physics,
where, at least in the context of such tests, only native talent matters I did
extremely well. In the others I also did well, but not so well. So the university
counselor, whom one was encouraged to consult after the tests, suggested at
first that I might want to become an accountant, even a chartered accountant.
This lacked all glamour. So he then suggested mathematics or physics, cautioning
me that this would require a master's or even a PhD. The latter meant nothing
to me, but I did not acknowledge this. I decided on the spot that I would become
a mathematician or physicist. The PhD whatever it was, could take care
of itself.

As soon as I returned to White Rock from the University, I looked up my future
father-in-law, found him in bed, and learned from him what a PhD was. Having set
out to become a mathematician and a savant, I was as systematic as possible, even
buying a copy of Euclid in the Everyman's edition to remedy, as I thought, my
neglect of elementary geometry.

1.2.6 A Talent for Languages Revealed via Mathematics

At some point in the first or second year, because of my intention to choose honors
in mathematics, I spoke with Professor S. Jennings. For me, he became later a
somewhat comical figure, dapper but a little soiled, who reminded me of the
Penguin in Batman comics, although I cannot say for certain that he always carried
an umbrella with a crook-handle.

He gave me a piece of advice for which I am grateful to this day. He declared
that to be a mathematician one had to learn French, German and Russian. The
romantic desire to penetrate the present and the past of the enchanting, mysterious
or seductive tones of a tongue not sung at my cradle, was planted in me as a
student. This desire came to me simultaneously with mathematics.

Through the advice of S. Jennings, thus through mathematics, and through
opportunities given me as a mathematician, I was eventually led to the very words
and the very sounds not only of Gauss, Galois or Hilbert but of Thomas Mann,

Proust, Pasternak, or even Giuseppe Tomasi di Lampedusa and Almet Hamidi
Tanpinar, not to speak of Robert de Roquebrune or Michel Tremblay, or Mommsen
and Michelet. I am afraid such possibilities are no longer available. I myself have
not yet reached the classical languages or any truly exotic language, except in an
inadequate way. I like to think there is time left to remedy this.

Although it may appear that I quickly abandoned the desire to become a
physicist or even to acquire some understanding of physics, the desire persisted and
I took a large number of courses that I enjoyed, both as an undergraduate and
during my year as a graduate student at UBC.

My experience suggests nevertheless that it is easier to learn mathematics on
one's own than physics. It also suggests that my natural aptitude for mathematics
was greater than my natural aptitude for physics. I fear, however, that I was more
attracted to the mathematical explanations than to the physical phenomena
themselves.

It was not possible at the time I was an undergraduate to take books from the
library at UBC – there was only the main library – or to visit the stacks. It was
nevertheless my first encounter with a library after that on Carnarvon Street in New
Westminster and I profited from it as much as whatever knowledge I had would
allow. It was a pleasure to handle the books.

1.2.7 An Incomplete Master's Thesis

During my year as a candidate for a master's degree at UBC I had probably tried to
read both Weyl's book on algebraic number theory and Lefschetz's *Introduction to
Topology*. The first is, of course, a difficult book and God only knows what
I understood, but I did come away with a clear notion that the law of quadratic
reciprocity, which never appealed to me as an undergraduate, was in the context of
the theory of cyclotomic equations genuinely a thing of beauty. Weyl opened my
eyes. Lefschetz's book would have been my introduction to topology. I never did
take to topology. Whether Lefschetz is to blame or my intellectual limitations
I cannot say.

My master's thesis, which was influenced by an undergraduate seminar at UBC,
was not a successful undertaking because I discovered just as I was submitting it
that I was trying to prove a false theorem. I could not recover much useful from
what I had written. There was, I believe, some question about whether it could be
accepted. My guess is that the committee was generous, gave me credit for
independence and enterprise, and let me go on to Yale and the next step, for which
I am still grateful.

1.2.8 Yale University – an Excellent Choice

I was eager to finish the Master's degree as soon as possible and to continue with
what seemed to me genuine graduate work. I applied to three institutions: Harvard,
Wisconsin and Yale. I was accepted by all three.

Wisconsin was without aid and I would have had to teach. I had discovered in
my year as a candidate for a master's degree at UBC that teaching interfered with
learning mathematics. So I did not hesitate to decline Wisconsin.

Yale offered a fellowship that would, with almost no help from my family, support both me and my wife, who would not be allowed to work in the USA. Besides I had some familiarity with the mathematics of the faculty at Yale, above all of Hille and Dunford, thus functional analysis, but informed by classical analysis. I was accepted at Harvard but with no support. So the choice was evident.

In retrospect, it was extremely fortunate for me that Harvard did not offer a fellowship. I would have gone there and missed in one way and another a great deal. At Harvard, I would have had to deal with fields that were both popular and extremely difficult and with fellow students who were already initiated into them. That would have taken an incalculable toll.

1.2.9 A Stressful Oral Examination

At Yale I was on my own and allowed to follow my own inclinations. I finished at Yale in two years, indeed the thesis was written in one, so that I had a great deal of time after it was finished in which to think about various problems and to learn various techniques.

I did not prepare for the oral examinations to take place at the end of the first year. In some sense I took them cold and it began badly because I could not prove the simplest things about Noetherian rings. I had obviously read Northcott's book on ideal theory too quickly and too superficially.

I had certainly read with some care the first edition of Zygmund's book on Fourier series available at the time in a Dover edition. I had also read in Burnside's book on finite groups, with dreams of solving the famous conjecture that all simple groups were of even order. Fortunately for me, when Shizuo Kakutani, one of the examiners, discovered that I knew something about Fourier series he began to question me closely about interpolation theorems, which take up a certain amount of space in Zygmund's book and are probably still popular among Fourier analysts, but otherwise little known. Having recently spent considerable time reading the book, and with considerable pleasure, I could respond quickly and correctly to his examination. Thanks to this – so far as I know – I was saved. I am particularly grateful to him for discovering that, even if I did not know what I should have known, I did know something.

Formally, the director of my thesis was Cassius Ionescu Tulcea. The first half of my thesis was a solution of an open problem in the somewhat obscure domain of Lie semi-groups and their representations. It became my first paper, appearing in the Canadian Journal of Mathematics. The second half of my thesis, never properly published, but available, because Derek Robinson incorporated it into his book *Elliptic Operators and Lie Groups*. It had in another way an important effect, because it drew me to the attention of Edward Nelson, then an assistant professor at Princeton, and on his recommendation alone, with no application, with no information whatsoever about me, the department appointed me as an instructor.

1.2.10 A Fateful Future

What I really hoped to do when I completed my PhD was to stay at Yale. I had fallen in love with the atmosphere there: I had a freedom to study and think that