The Metaphysics and Mathematics of Arbitrary Objects

Building on the seminal work of Kit Fine in the 1980s, Leon Horsten here develops a new theory of arbitrary entities. He connects this theory to issues and debates in metaphysics, logic, and contemporary philosophy of mathematics, investigating the relation between specific and arbitrary objects and between specific and arbitrary systems of objects. His book shows how this innovative theory is highly applicable to problems in the philosophy of arithmetic, and explores in particular how arbitrary objects can engage with the nineteenth-century concept of variable mathematical quantities, how they are relevant for debates around mathematical structuralism, and how they can help our understanding of the concept of random variables in statistics. This worked-through theory will open up new avenues within philosophy of mathematics, bringing in the work of other philosophers, such as Saul Kripke, and providing new insights into the development of the foundations of mathematics from the eighteenth century to the present day.

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For CAA

Contents

Figures

Preface

For a long time I had a plan to write a book on mathematical structuralism. In this book I would put my cards on the table and try to make a contribution to the ongoing philosophical debate. But I just did not seem to be able to make solid progress with this project.

When I was honest with myself about it – and this was not very often the case – I had to admit that I did not share some of the essential presuppositions in the current debate on mathematical structuralism. At the same time, the task of re-thinking the framework of the debate was too big for me: I lacked the courage. So the book project did not really get anywhere for years.

At some point – I do not clearly remember when – some of the work of Kit Fine came to have a deep impact on me in two different ways.

First, I discovered Fine's work on the theory of arbitrary objects. I am one of the proud handful of owners of a copy of *Reasoning with Arbitrary Objects* (first edition, 1985). I was impressed by it, and instinctively felt that it had unrealised philosophical potential. Many metaphysical questions about arbitrary objects immediately came to mind, and Fine himself listed many possible applications. At the same time, I was astonished to see that Fine's theory had received so little attention in the philosophical literature.

I came to suspect that Fine's theory of arbitrary objects can be fruitfully connected to the discussion on mathematical structuralism and subsequently found out that Fine had, in an all too brief final section of his article on *Cantorian Abstraction*, already noticed this. When I read this succinct section carefully, I saw that Fine's way of relating mathematical structuralism to arbitrary object theory was not quite the way in which I would do it. In any case, as Fine himself said in his paper, the connection needed to be worked out in detail. All this gave me courage to spell out my ideas on this subject. In the course of doing this, important differences of view between Fine and me about the nature of arbitrary objects came more clearly into focus. What started as an idea for a paper grew into something that is too extensive to be crammed into a few articles.

Secondly, and just as importantly, by reading Fine's work on arbitrary object theory, I felt – there is no other way to put it – released from presuppositions in current analytical metaphysics that had bothered me for a long time. I sensed that Fine used a very different methodology, one that immediately and viscerally appealed to me. Then I found out that Fine had started to reflect on his metaphysical methodology in some of his writings from the early 2000s onward. These reflections reached (I think) a mature expression in his 2017 article on naive metaphysics. But here, too, I found that I might have something to add to what Fine wrote. I felt that Fine's departure from the received methodology in metaphysics was not radical enough: too much of the received metaphysical methodology remained in Fine's new proposal. This convinced me that I should start what had by now developed into the idea for a monograph with a chapter on the methodology of the metaphysics of mathematics. From then onwards, I felt able to make real progress in writing this book.

Fine took a shot in the dark when he wrote his book on arbitrary object theory. Today, almost 35 years later, there still is almost no philosophical literature on the subject. So I am worried that it may still be too early for a research monograph on these matters. All I can say in response to this concern is that this book is intended as an *essay* in the original sense of the word. It may well be miles off the mark: time will tell. If this book encourages more philosophers to work on the theory of arbitrary objects, then it has achieved its aim. Anyway, already now I sense an increased interest in arbitrary object theory. I take it as an encouraging sign that by the time this monograph is published, a new edition of Fine's *Reasoning about Arbitrary Objects* will (probably) finally be out.

Now that I am at it, let me relate an early influence on the writing of this book. As a visiting postdoctoral researcher, I attended a graduate seminar on modal logic by Saul Kripke in Princeton in 1999. Even though I did not know it at the time, this was a defining moment in my philosophical life. It was a strange experience. I understood very little. I was much too shy to ask questions, and I was intimidated by the environment. Foolishly, I thought that I should not spend too much time trying to understand what Kripke was trying to do but should instead concentrate on making progress with the projects that I was working on. Today I thoroughly regret not dropping all these projects completely and spending all my time on at least trying to understand what the problems were that Kripke was trying to make progress on. But Kripke's lectures influenced me more deeply than I could imagine possible at the time. As far as the present monograph is concerned, what is important is that Kripke drew attention to the expressive powers and possible applications of the Carnapian framework of quantified modal logic. (I recall that Kripke was trying somehow to prove the Fundamental

Theorem of Calculus in Carnapian quantified modal logic.) This stayed with me. Thinking about arbitrary objects from within the framework of Carnapian quantified modal logic has influenced my views on the nature and properties of arbitrary objects, as you will see.

Moving on to more recent influences, a few conversations with my former colleague Øystein Linnebo stand out in my mind. Especially his exhortation to think the philosophy of arbitrary objects through carefully, rather than to jump immediately to technical-looking applications, was important. At a conference on mathematical structuralism in Prague in 2016 he told me: *do not try to fly before you can walk.*[1] I have tried to keep that in mind. In particular, I have tried to keep my eye on the metaphysics at all times. Indeed, even though only a few applications of arbitrary object theory have been worked out in any degree of detail, what has been holding the subject back is the fact that the metaphysics of arbitrary objects is at present still ill understood and underdeveloped. We have jumped too quickly to applications.

There is not much background literature to fall back on, so most of the material in this book is new. Nonetheless, there are a few places in this book where I use material from articles that I have published or are in press. In Chapter 8, I draw in places on my article Horsten (2019). Chapter 9 is partly based on my article Horsten and Speranski (2018). In Chapter 10 I rely on a theory of non-Archimedean probability that I have developed together with Vieri Benci and Sylvia Wenmackers (Benci et al. 2013) and on an unpublished article in which this framework is applied to the set theoretic universe (Brickhill and Horsten 2019).

[1] My only complaint to Øystein is that he used the word ‘walk’ instead of ‘crawl’, or ‘roll over’.

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I have presented material in this book at various conferences and workshops, and I am grateful for all the comments that I have received. In particular, I benefited from encouraging comments and suggestions by members of the audience at the conference on *The Emergence of Structuralism and Formalism* (Prague, 2016), where I first presented some of my ideas on the connection between arbitrary object theory and mathematical structures. Special thanks to Oliver Tatton-Brown and Volker Halbach for carefully going through a version of the entire manuscript and generously commenting on it.

This book was written mostly during the academic year 2017–2018, when I was on research leave from the University of Bristol. I spent the first term (Michaelmass, 2017) as a Visiting Fellow in Oxford. Attending Tim Williamson's postgraduate metaphysics seminar during that period played an important role in shaping my views on metaphysical method. I will never forget how I ended up, completely by accident, with Tim Williamson, Ofra Magidor, David Wiggins, and Volker Halbach at a small coffee table in the Senior Common Room in New College. As Volker remarked: where else in the world could such a thing happen? I spent the rest of that academic year as a Visiting Professor in Kyoto University. In this wonderful environment most of this book was written. I am immensely grateful to Professor Tetsuji Iseda for the welcoming academic environment that he provided, and for his kind invitation to give a research seminar lecture (CAPE lecture) on metaphysical methodology. I also enjoyed all sessions of the logic and metaphysics reading group (organised by Ryo Ito and Takuro Onishi) in Kyoto. And I am very grateful to the logicians at Kobe University (especially to Professor Sakaé Fuchino) for their interest in my work during my time in Japan.

In July and August of 2017, Stanislav Speranski was in Bristol on a Benjamin Meaker Visiting Professorship. I had an excellent time collaborating with him on the formalisation of reasoning about the generic ω-sequence in quantified Carnapian modal logic.

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Hilary Gaskin of Cambridge University Press has been very patient with me during the long time when I was not able to make real progress with the manuscript of this book. I also want to express my gratitude to Sophie Taylor of Cambridge University Press. Together with Hilary Gaskin, she provided invaluable assistance and encouragement in the last stages of the writing of this book.

Addressing you, Hazel, is the most difficult: I do not know where to begin. Putting up with my philosophical questions to you for a couple of years is the least of it. You got me out of philosophical confusions on so many occasions, and showed me where and how conceptional distinctions needed to be made. There are just so many things that we have thought through together by talking about it to each other. (I don't know what you think of it all.)

Symbols and Abbreviations