TOPIC 1: Numbers and operations in context

In this topic, the focus is on estimations and percentages; ratios; rates; scientific notation and exponents; working with formulae, personal finances, investments and loans; entrepreneurship; and appreciation and depreciation.

Unit 1: Estimations and percentages

Estimations

- To estimate means to make a calculated, educated guess.
- An estimation is an approximation of an amount. A value is chosen using judgement rather than measurement or exact calculation.
- Use objects with known lengths or masses to help you when you have to make estimations. For example, you can use your mass when you have to estimate the mass of a large dog, and the length of a ruler or a sports field when you have to estimate the length of a cucumber and a path.

Percentages

- A percentage is a value out of a 100; example: $\frac{54}{100} = 54\%$
- You can calculate a percentage; example: $\frac{11,5}{35} \times 100 = 32,9\%$
- You can also use a calculator key when calculating a percentage.
- Always calculate rate as a percentage: example: $3\% = \frac{3}{100}$

Worked examples

1. When you have to estimate the number of people in a crowd, you cannot count each person. Count the people in a small area and use this number to estimate the number of people in the crowd.
   Estimate the number of learners in the photograph.

2. What is the percentage discount on the toothbrush?

PowerMate
Superior toothbrush

Incredible whiteness in seconds

● Removes plaque better than any other toothbrush
● Massages gums
● Rechargeable batteries

SPECIAL OFFER

Was R139,99
Now R109,99

3. Pearl would like to print digital photographs. She found the following advertisement in a magazine.

20 OR MORE DIGITAL PRINTS? SAVE 20%

YOU PAY **R2,99** PER PRINT

Valid from 15 November to 22 December

a) If Pearl wanted to print 18 photographs, how much would she have to pay?
b) If Pearl decided to have two copies printed of 7 of the 18 photographs and one copy of all the others, how much would she have to pay?
c) How much did Pearl save by making use of the special offer?

4. The government charges VAT (value-added tax) on most items that are sold – a few items are exempt from VAT. The government uses VAT to pay for services that are provided to the community. Presently VAT in South Africa is 15%. Calculate the VAT payable on each item.
a) a bag of maize meal that costs R32,50
**b)** a chocolate slab that costs R19,99

Solutions

1. Count the learners in one row and multiply this by the number of rows.
Estimated number of learners in the photograph: $14 \times 12 = 168$

The negative answer indicates a reduction in price.

2. $\frac{R109,99 - R139,99}{R139,99} \times 100\% = -R21,43\%$
The price was reduced by 21,43%.

Subtract 20% from the price.

3. **a)** Price for printing 18 photographs: $R2,99 \times 18 = R53,82$
b) $R2,99 \times (18 + 7) = R74,75$
$0,8 \times R74,75 = R59,80$
c) $R74,75 - R59,80 = R14,95$

4. **a)** VAT is not charged on maize meal.
b) 15% of $R19,99 = 0,15 \times 19,99 = R3,00$
Price of chocolate slab (including VAT): $R19,99 + R3,00 = R22,99$

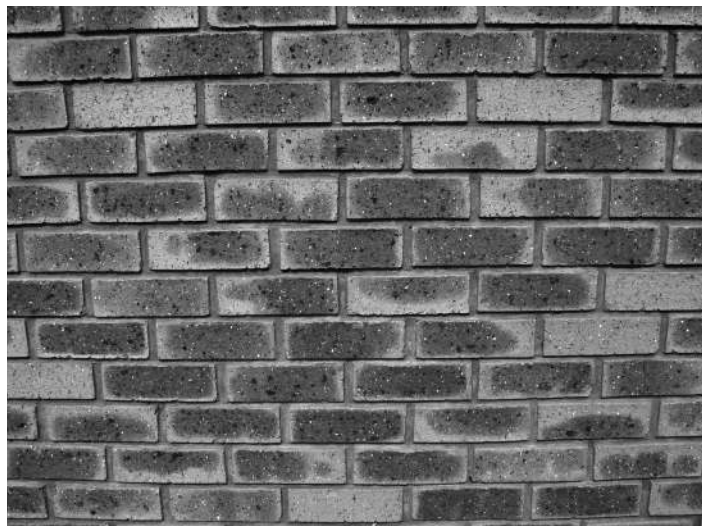## Practice questions

1. Estimate the number of paper clips in the drawing.
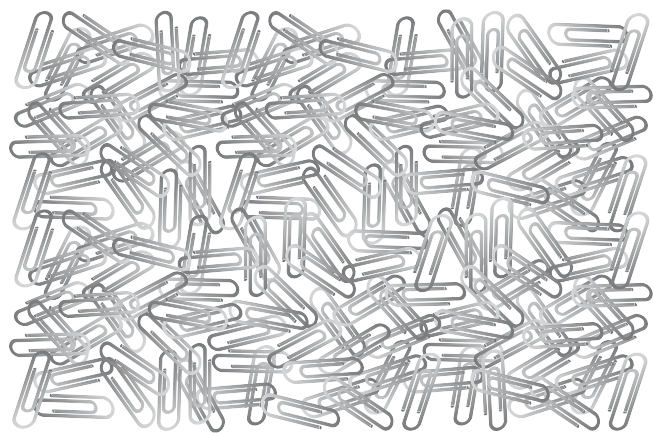
2. **a)** Estimate the number of bricks in the photograph.

   b) Did you use the same technique to estimate the number of bricks in the wall as to estimate the number of paper clips in question 1? If not, explain what you did differently.

3. **a)** Estimate how many flowers there are in the photograph.

b) If the photograph represents 0,06 m² of a flowerbed, and the size of the flowerbed is 2 m × 0,6 m, approximately how many flowers are there in the flowerbed?

4. a) Estimate how many leaflets there are in the leaf in the photograph. (There are many tiny leaflets in the leaf.)

b) If the leaf in the photograph were one of 12 500 leaves on a tree, approximately how many leaflets were there on the tree. (Write your answer in scientific notation.)

5. This question gives you the opportunity to see how well you can make estimations. Choose the best estimation from the box on the right for each value.

a) Length of a new unsharpened pencil: …
b) Body temperature of a healthy Grade 11 learner: …
c) Height of a standard door: …
d) Length of a stride of an adult female: approximately …
e) Length of a stride of an adult male: approximately …
f) Speed limit for vehicles in a residential area: …
g) Capacity of a standard teaspoon: approximately …
h) Capacity of a standard teacup: approximately …
i) Height of an average 18-year-old male: …

1.	250 ml
2.	49,9 °C
3.	40 cm to 50 cm
4.	2 m
5.	350 ml
6.	17 cm
7.	60 cm to 70 cm
8.	5 ml
9.	60 km/h
10.	36,9 °C
11.	10 ml
12.	1,8 m

Unit 2: Ratios

- A ratio is a comparison between the sizes of two or more quantities that have the same units.
- You can write ratios using a colon; first value : second value; example: 4 : 6
- When writing a ratio with a colon (:), do not include units; example: a ratio of 5 ml to 250 ml is 5 : 250.
- You can also write ratios as a common fraction: $\frac{\text{first value}}{\text{second value}}$; example: $\frac{4}{6}$
- Do not include decimal numbers in ratios; example: convert 2,2 cm to 1,3 cm as 22 mm to 13 mm and write the ratio as 22 : 13.

Example

Amahle receives a weekly allowance of R120. Her younger brother, Bandile, receives an allowance of R40 per week. Amahle's allowance is three times as much as Bandile's allowance. The ratio of Amahle's allowance to Bandile's allowance is 3 to 1 or 3 : 1.

Worked examples

1. Share R85,00 in a ratio of 4 : 6 between two brothers. How much will each brother receive?

2. Complete the ratios.
   a) 3 : 4 = 6 : …
   b) 12 : 10 = … : 5
   c) 400 : 240 = … : 6
   d) 1 : … = 4 : 10

3. The ratio of teachers to learners in South Africa is on average 1 : 37. How many teachers should there be at schools with the following numbers of learners?
   a) 1 554 learners
   b) 1 600 learners

4. Write each ratio in its simplest form. Remember that the units must be the same and that you cannot use decimal numbers in a ratio.
   a) 60 m : 40 m
   c) 1 m : 40 cm
   b) 0,6 cm : 0,005 cm
   d) 750 g : 2 kg

5. The ratio of unsaturated fatty acids to saturated fatty acids in a cell membrane is 1 : 8. If there are 86 billion fatty acid molecules, how many of these fatty acid molecules are unsaturated?

Solutions

1. The total number of shares in the ratio 4 : 6: 4 + 6 = 10
   One share: 85 ÷10 = 8,5
   First brother's share of 4: 4 × 8,5 = R34,00
   Second brother's share of 6: 6 × 8,5 = R51,00
   Check the answer: R34,00 + R51,00 = R85,00

2. a) 3 : 4 = 6 : 8
   c) 400 : 240 = 10 : 6
   b) 12 : 10 = 6 : 5
   d) 1 : 2,5 = 4 : 10

3. a) $\frac{1}{37} = \frac{x}{1\,554}$ (Cross-multiply.)

   $37x = 1\,554$

   $x = \frac{1\,554}{37}$

   $x = 42$

   42 teachers should be employed.

   b) $\frac{1}{37} = \frac{x}{1\,600}$

   $x = \frac{1\,600}{37}$

   $x = 43,2432…$

   You cannot get a fraction of a teacher and so round off your answer to 43 teachers. Nine classes will each have one more learner than the other classes.

Remember!
1 m = 100 cm
1 000 g = 1 kg

4. a) 3 : 2
   b) 120 : 1
   c) Convert both values to the same unit (cm), and simplify.
      1 m = 100 cm
      5 : 2

d) Convert both values to the same unit (grams), and simplify.
2 kg = 2 000 g
750 : 2 000 = 3 : 8

1 billion
= 1 000 million

5. $\dfrac{1}{8} = \dfrac{x}{86 \text{ billion}}$

$x = \dfrac{86 \text{ billion}}{8}$

$= 10{,}75$ billion

Practice questions

1. There are 42 tables and 168 chairs in a restaurant. What is the ratio of tables to chairs?

2. The price of denim jeans in brand A is R600 and in brand B it is R720. What is the ratio of the two prices?

3. The municipality planted roses next to the road in a ratio of 5 red roses to 2 white roses. If 6 400 red roses were planted, how many white roses were planted?

4. Simplify the ratio. Write each number as a whole number. (Remember, to convert a ratio to a whole number, start by changing the smallest number into a whole number.)
1,2 : 0,48 : 0,036

5. The RDA (recommended daily allowance) on a bottle of supplements gives guidelines about the quantity of each nutrient a person should take in one day.
   a) If there is 0,02 mg of vitamin C in one orange and this represents 0,5% of the RDA of vitamin C, what is the RDA of vitamin C in milligrams (mg)?
   b) How many oranges will give a person their RDA of vitamin C?
   c) Do you think it is possible to consume this number of oranges in one day? If not, how can someone get their RDA of vitamin C?
   d) If there is 0,137 g of protein in a chocolate bar and this represents 0,1% of the RDA for protein, what is the RDA for protein in grams (g)?
   e) If one litre of milk has a mass of 0,95 kg and there is 2% fat in one litre, how many grams of fat are there in one litre of milk?
   f) Use the information in question 5e to work out the percentage of fat in milk if there are 12 g of fat in one litre of milk (to two decimal places).
   g) If a breakfast cereal contains 5 g of fat and you pour $\frac{1}{4}$ litre of 2% fat milk over the cereal, how much fat will you consume when you eat the cereal?

6. The lengths of the sides of a triangle are in the ratio 2 to 3 to 5. If the perimeter of the triangle is 30 cm, what is the length of the shortest side?

7. An adult male with a mass of 75 kg has about 5,5 ℓ of blood in his body. An adult female with a mass of 72 kg has about 4,5 ℓ of blood in her body.
   a) How many millilitres of blood does an adult male have per kilogram of body mass?
   b) How many millilitres of blood does an adult woman have per kilogram of body mass?
   c) The total volume of plasma in the circulating blood of a grown man is approximately 2,9 ℓ. What percentage of blood is plasma?

d) Blood plasma consists of about 91% water. How many millilitres of water is this?

e) Normal blood has 4 000 000 000 to 11 000 000 000 leucocytes (white blood cells) per litre of blood. Write these numbers in scientific notation.

f) Why do you think someone with a mass of less than 50 kg should not donate blood?

8. A popular website carried out an opinion poll about the possibility of a female becoming president after the next national elections. The results of the opinion poll are shown in the table.

| Male voters		Female voters	
In favour	Against	In favour	Against
136	136	544	306

a) How many people took part in the opinion poll?
b) Give the ratio of males to females who took part in the opinion poll.
c) Give the ratio of males to females who voted in favour of a female president.
d) Give the ratio of females to males who voted against a female president.
e) Can you make a general conclusion after studying the outcome of the opinion poll?

9. Suzette wants to plant rosemary, grass onions and dill seedlings in a ratio of 4 to 3 to 2. How many of each type of herb should she plant if she wants to plant a total of 135 seedlings?

10. Ben, Bongi and Busi decided to play the Lotto together to improve their chances of winning. Ben spent R20, Bongo spent R40 and Busi spent R50. One ticket they bought had four correct numbers. They won R265 000. What was each person's share of the winnings?

11. The Department of Basic Education tries to maintain a policy of one teacher for every 35 learners in secondary schools. The table below shows the number of learners and teachers in five different secondary schools.

School	Teachers	Learners
A	21	819
B	45	1 575
C	60	1 600
D	13	520
E	11	385

a) Which school(s) adheres exactly to the Department's policy of maintaining a ratio of one teacher to 35 learners? Motivate your answer.
b) In which school(s) are there too many learners in a class according to the Department's prescribed ratio? Motivate your answer.
c) In which school(s) are there fewer than 35 learners in one class for every teacher? Motivate your answer.
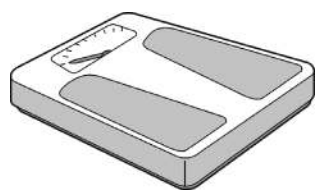
12. Body mass index (BMI) is a scientific way of assessing a person's mass to determine if it is within the range that is considered normal. To calculate BMI, you need the mass (in kilograms) and height (in metres).

$$\text{BMI} = \frac{\text{mass (kg)}}{(\text{height (m)})^2}$$

Classification of mass according to BMI:
- BMI lower than 18,5: person is underweight
- BMI between 18,5 and 25: person is of normal weight
- BMI over 25: person is overweight (obese)

a) Determine your BMI.
b) Ntuli and Tumi both have a mass of 50 kg. Ntuli is 1,2 m tall and Tumi is 2,4 m tall. Determine their BMIs.
c) Do Tumi and Ntuli have weight problems? Motivate your answer.
d) Although Tumi and Ntuli have exactly the same mass, their BMI is not the same. Explain why.

Unit 3: Rate

A rate compares two quantities with different units. Examples include metres per second (m/s), kilometres per hour (km/h) and rands per hour (R/h).

Example:

If petrol costs R12,49 per litre, how much will you pay for 25 ℓ of petrol?

12,49 × 25 = R312,25

How many litres of petrol could you buy for R200,00?

200 ÷ 12,49 = 16,01 ℓ

Worked examples

1. Using a camera with film, Joseph had to take and develop one photograph of each Grade 11 learner at his school. There were a total of 360 Grade 11 learners. Two film sizes were available – 24 photographs and 36 photographs. The table shows the cost of film and developing film.

Film size	Cost of film	Developing costs (per print)
24 photographs	R69,90	R2,99
36 photographs	R97,20	R2,70

a) Which film size works out cheaper?
b) Joseph only needed to take one photograph of each learner. How many films and which size film did he need to buy? Remember that he wanted to spend as little money as possible.
c) How much would the whole project cost?
d) Why do you think the cost of film is so high?

2. A popular cellphone provider offered the special SMS bundles shown in the table on the next page. They claimed that their SMS rates were lower than those of other companies.

SMS bundle size	Cost per SMS bundle	Cost per SMS
50	R25,00	
100	R33,00	
200	R45,00	
500	R112,50	
1 000	R225,00	
1 500	R337,50	
2 000	R450,00	

a) Complete a copy of the table above to show the cost per SMS when using the SMS bundles.

The table below gives the out-of-bundle SMS rates.

Bundle sizes	Peak	Off-peak
50–100	80c	35c
more than 100	80c	35c

b) Which option in the table would be the best for each person?
   i) A stockbroker sends daily SMSs to all her clients to notify them of drastic changes in the stock market.
   ii) A Grade 11 learner in a hostel is allowed to use her cellphone only for one hour at night.
   iii) A boy is so in love that he sends an SMS every two hours.
c) Would it be accurate to assume that the SMS in-bundle options will always be cheaper than out-of-bundle options? Motivate your answer.

3. A farmer purchased a square field. The fence along each side of the new field is 500 m long. If the farmer walks on average at a rate of 5 km per hour, how long will it take to inspect the whole fence? Give your answer in minutes.

Solutions

1. a) Price per photograph for the 24-photograph film:
$\frac{R69,99}{24} = R2,92$
Price per photograph for the 36-photograph film:
$\frac{R97,20}{36} = R2,70$
The 36-photograph film is cheaper.
b) $\frac{360 \text{ learners}}{36} = 10$
He needs 10 spools of film.
He needs to buy 10 spools of the 36-photograph films.
c) Cost for film: $10 \times R97,20 = R972,00$
Cost to develop film: $360 \times R2,70 = R972,00$
Total: $R972,00 + R972,00 = R1\ 944,00$
d) Very few people use film nowadays as most cameras are digital.

2. a)

Bundle size	Cost per bundle (R)	Cost per SMS (c) (in-bundle)
50	25,00	50,0
100	33,00	33,0
200	45,00	22,5
500	112,50	22,5
1 000	225,00	22,5
1 500	337,50	22,5
2 000	450,00	22,5

b) i) In-bundle SMS package
  ii) In-bundle SMS package
  iii) In-bundle SMS package

c) It would be wrong to assume that the in-bundle package is always cheaper. If you buy a bundle of 50 SMSs, the cost per SMS is 50 cents – this is more than the 35 cents that you pay for out-of-bundle SMSs that are sent in off-peak time.

2. Length of the fence:
4×500 m $= 4 \times 0,5$ km $= 2$ km
Time taken to inspect his fence:
time $= \frac{\text{distance}}{\text{speed}} = \frac{2 \text{ km}}{5 \text{ km/h}} = 0,4$ h
$0,4$ h $\times 60 = 24$ min.

Practice questions

1. Kaone wanted to earn pocket money. She looked for classified advertisements for part-time jobs on weekends. She found the following advertisements.

> **Advertisement A:** Wonder Videos where Kaone would earn a flat rate of R75 per hour. She would have to work on Saturdays and Sundays from 12:00 to 20:00.

> **Advertisement B:** Waiter at the Pizza Palace where she would have to work from 07:00 to 13:00 on Saturdays and from 08:00 to 15:00 on Sundays. She would earn 4% commission on everything she sold, and the tips she received would be additional income.
> • It is estimated that she would sell pizzas to the value of R7 500 on an average weekend.
> • A 10%-tip would be included in the bill.

> **Advertisement C:** Kaone could work at a call centre and earn R5 300 per month. She would have to work on Fridays from 15:30 to 19:00, on Saturdays from 08:00 to 16:00, and every second Sunday from 09:00 to 14:00.

a) Use a table to compare the three jobs. One column should show how many hours Kaone would work per month (assume four weekends) and another column should indicate how much she would earn.
b) Which job would be best for Kaone in each case?
  i) She desperately needs the money and time is not an issue.