TOPIC 1: Basic skills

In this topic, you will work with ratio, rate, percentages, scientific notation, estimations and rounding off. This material was covered in Grade 10 and revised in Grade 11. You will often need the material that is covered in this section when working on problems in other sections and you should revise these concepts before you start work on the other topics.

Ratio and rate

- We use ratios to compare the sizes of two or more quantities in the same units.
- Ratios can also be written as common fractions. Example: you can write 4 : 6 as $\frac{4}{6}$. When you write a ratio with a colon (:), you do not include units of measure and the units for the quantities must be the same. So, we write a ratio of 5 ml to 250 ml as 5 : 250.
- When using rates, we compare two quantities of different units. The word *per* is often used when describing rate. It means for every or for each. We often use a slash in the place of the word *per*. Examples: rand per hour (R/h), metres per second (m/s) and kilometres per hour (km/h).
- Rate is often calculated as a percentage. Example: 12% = $\frac{12}{100}$

Percentages

- A percentage is a way of expressing a value out of 100.
  Example: $\frac{68}{100}$ or 0,68 = 68%.
  To calculate the percentage 11,5 out of 35 represents:
  $\frac{11,5}{35} \times 100 = 32,9\%$
- You will find examples of percentage increase or decrease regularly in everyday life. For example, items for sale are usually marked down by a certain percentage. You can use the percentage key on a calculator to calculate percentage increase or percentage decrease. Or, if you do not have a calculator handy, use the following formulae:
  - Discount of a regular priced item = $\frac{\text{discount amount}}{\text{regular price}} \times 100$
  - Discounted price = retail price × (1 – discount percentage)
  - Retail price = cost price × (1 + mark-up percentage)

Rounding off

- Always keep common sense and logic in mind when rounding off values.
- To round off to:
  - one decimal place: Look at the digit in the second decimal place. If this digit is greater than 5, increase the first decimal value by 1. If the second decimal digit is less than 5, the first digit remains the same.
  - two decimal places: Look at the third decimal place and apply the principle that you apply for rounding off to one decimal place.
    Examples:
    3,348 ≈ 3,3 (rounded off to one decimal place)
    3,348 ≈ 3,35 (rounded off to two decimal places)
    3,358 ≈ 3,4 (rounded off to one decimal place)
    3,358 ≈ 3,36 (rounded off to two decimal places)

Scientific notation

- A number in scientific notation has only *one digit* (not zero) *before* the decimal comma.
- You can use your calculator for calculations in scientific notation.
- Use the EXP key or, on the latest calculators, the $\times 10x$ key for calculations in scientific notation.
- To convert numbers in scientific notation to standard notation (examples):
  $1{,}874 \times 10^6$: key in: 1,874 EXP 6 {=} 1 874 000
  $8{,}492 \times 10^{-3}$: key in: 8,492 EXP –3 {=} 0,008492
- To multiply, divide, subtract or add numbers in scientific notation (examples):
  $1{,}02 \times 10^4 + 2{,}6 \times 10^3$: key in: 1,02 EXP 4 {+} 2,6 EXP 3 {=} $1{,}28 \times 10^4$
  $1{,}5 \times 10^{-2} \div 2{,}0 \times 10^3$: key in: 1,5 EXP –2 {÷} 2,0 EXP 3 {=} $7{,}5 \times 10^{-6}$

Exponents (indices)

- An exponent (or index) is a positive number that indicates how many times the first number must be multiplied by itself.
  Example: $4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1\,024$
- Exponential growth indicates that the rate of change is always increasing.
- An exponential graph shows the relationship between variables that involves an expression in which one of the variables is written as an exponent (index).
  Exponents can be used to calculate growth in a population:
  $P_{new} = P_i \times \left(\frac{100 + \text{population growth } r}{100}\right)^n$
  where,
  P_{new} = the new population
  P_i = the start population
  population growth r = the population growth rate as a percentage increases in a period
  n = the number of years
- To calculate increase in growth as a percentage:
  percentage increase = $\frac{\text{difference between two values}}{\text{original value}} \times 100$.

Changing the subject of a formula

It is often necessary to change the subject of a formula.

- To calculate the area of a triangle use the formula:
  Area of triangle = $\frac{1}{2}$ base × perpendicular (⊥) height

But if the area of the triangle is given and the perpendicular height is unknown, change the subject of the formula:

Area of triangle = $\frac{1}{2}$ base × ⊥ height

$$A = \frac{\text{base} \times \perp \text{ height}}{2} \quad \text{(Multiply both sides by 2.)}$$
$$2A = \text{base} \times \perp \text{ height} \quad \text{(Divide both sides by the base.)}$$
$$\frac{2A}{\text{base}} = \perp \text{ height} \quad \text{(The height is the subject of the formula.)}$$

- To calculate the volume of a cube, use the formula $V = \text{length}^3$, but if the volume of a cube is given and you need to calculate the length of the side, you need to make length the subject of the formula:
  $V_{cube} = l^3$ (Find the cube root of both sides.)
  $\sqrt[3]{V} = \sqrt[3]{l^3}$
  $\sqrt[3]{V} = \text{length of side}$

Remember!

- Use brackets when you substitute values into any formula.
- Use the correct order of procedures when you use your calculator.

- To calculate the area of a square, use the formula A = length^2, but if the area of a square is given and you need to calculate the length of the side, you must make length the subject of the formula.

$A_{square} = l^2$ (Find the square root of both sides.)

$\sqrt{A} = \sqrt{l^2}$

$\sqrt{A}$ = length of side

Worked examples

1. Carlos finds the information below on a 125-g packet of potato chips.

Nutrition information		
Nutrients	**Per 100 g**	**Per 30 g**
Energy	2 178 kJ	653 kJ
Proteins	7,4 g	2,2 g
Carbohydrates	45,0 g	13,5 g
Sugars	0,7 g	0,2 g
Fats	35,0 g	10,5 g
Dietary fibre	4,0 g	1,2 g
Sodium	0,7 g	0,2 g

a) What ratio of fats to carbohydrates will Carlos consume when he eats the potato chips? Give your answer in its simplest form.

b) Calculate his nutrient intake if Carlos eats the whole 125-g packet of potato chips.

c) The recommended daily nutrient intake for an 18-year-old girl with height 161 cm and mass of 54 kg is given as 2 100 calories. If an 18-year-old girl eats the whole 125-g packet of potato chips, what percentage of her recommended daily energy allowance has she consumed?

Note

1 calorie = 4,186 kJ

2. The price of an umbrella that sells for R198,79 is reduced by 14% at a sale. Calculate the sale price of the umbrella.

3. Altogether 45 000 candidates are writing the final Grade 12 examinations. If each learner uses a minimum of seven pieces of paper for each of his or her seven subjects, how many pieces of paper will all the candidates use in total for the final Grade 12 examinations? Give your answer in scientific notation, rounded off to two decimal places.

4. During a gymnastics competition, the judges use the formula S = 0,6DT to calculate the score of a particular event, where S = score, D = degree of difficulty and T = total score of judges.

a) Calculate the score of a competitor who executed an event with a degree of difficulty of 2,5 and for which the total score of the judges was 36,9.

b) What was the degree of difficulty if the score was 85,5 and the total score of the judges was 38,5? Give your answer correct to one decimal place.

c) What was the total score of the judges if the degree of difficulty was 2,5 and the score obtained was 29,5? Give your answer correct to one decimal place.

5. A remote rural settlement receives a drum containing 600 litres of water to share amongst 36 people living in the settlement. If the water is shared equally, how many litres of water will each person receive?

Answers

1. a) Fat : carbohydrate
= 35 : 45
= 7 : 9

b) The nutrients on the packet are given per 100 g of potato chips. So you need to calculate the nutrients for 25 g potato chips and then add the two quantities.
$100 \div 4 = 25$
Calculate $\frac{1}{4}$ of each nutrient.

Nutrients	Per 100 g	Per 25 g	Per 125 g
Energy	2 178 kJ	544,5 kJ	2 722,5 kJ
Proteins	7,4 g	1,85 g	9,25 g
Carbohydrates	45,0 g	11,25 g	56,25 g
Sugars	0,7 g	0,175 g	0,875 g
Fats	35,0 g	8,75 g	43,75 g
Dietary fibre	4,0 g	1,0 g	5,0 g
Sodium	0,7 g	0,175 g	0,875 g

c) First convert 2 100 calories to kilojoules.
2 100 calories = 4,186 × 2 100 = 8 790,6 kJ
$\frac{x}{100}$ of 8 790,6 = 2 722,5

$$x = \frac{2\,772{,}5 \times 100}{8\,790{,}6}$$

$x = 30{,}97$

By eating the 125-g packet of potato chips, she has consumed 31% of her recommended daily energy intake.

2. Two methods you can use for the calculation are shown below.

Method 1
Calculate 14% of the price:
$198{,}79 \times \frac{14}{100} = 27{,}83$
Subtract R27,83 from the original price:
R198,79 – R27,83 = R170,96

Method 2
Let the original price represent 100%.
100% – 14% = 86% of the original price:
$198{,}79 \times \frac{86}{100} = \text{R}170{,}96$

3. 7 pages × 7 subjects = 49
45 000 × 49 = 2 205 000
$= 2{,}21 \times 10^6$ pages

4. a) S = 0,6DT
= 0,6(2,5)(36,9)
= 55,35

b) S = 0,6DT
85,5 = 0,6(D)(38,5)
85,5 = 23,1D
$\frac{85{,}5}{23{,}1} = D$
3,7 = D

c) S = 0,6DT
29,5 = 0,6(2,5)T
29,5 = 1,5T
$\frac{29{,}5}{1{,}5} = T$
19,7 = T

5. $\frac{600}{36} = 16{,}666666\ldots$

The logical way to round off 16,666666… is to 16 litres per person. There will probably be some spillage when the water is distributed.

$16 \times 36 = 576\ \ell$

An amount of approximately 24 litres has been allowed for spillage.

Practice questions 1.1

1. A farmer buys food for his farm animals in 50-kg bags. This information is printed on each bag.

Nutrition facts		WINTER LICK (50 kg)			
	g/kg				mg/kg
Protein	400	Cobalt	2,5	Manganese	400
Fibre	50	Copper	50	Zinc	350
Moisture	120	Iodine	2,6	Magnesium	250
Calcium	32	Iron	220	Selenium	0,6
Phosphorus	16	**Recommended intake:**		Sheep/goats	78 g/day
Sulphur	7,5			Cattle	375 g/day

a) Calculate the amount of nutrients (in grams (g) and milligrams (mg)) one cow will consume if she receives the recommended amount of food per day.

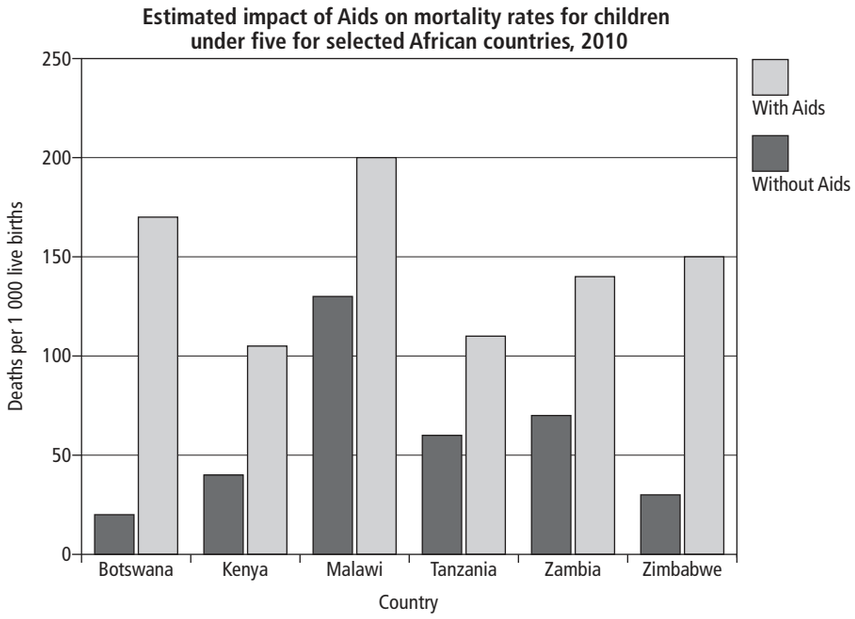
b) If a cow receives the recommended amount of food per day, in what ratio will she receive the following? Give each answer in its simplest form.

i) protein to manganese **ii)** cobalt to iodine

iii) manganese to magnesium

2. The ratio of money a girl spends on skin care to the money a boy spends on skin care is 5 : 2. If a girl spends R195,50 on skin care, how much does a boy spend on skin care?

3. The graphical representation below shows the estimated impact of Aids on the mortality rate of children below five years of age for 2010.



(*Source:* US Census Bureau, 27 June 2010)

a) Give the estimated rate of children under the age of five who die of Aids in the six African countries listed on the graph. Give your answer as a simplified fraction and a percentage.

b) What is the ratio in Malawi and Zimbabwe of children under five dying of Aids to children under five dying without Aids?

4. The table shows call charges between two licensed operators.

Number range		**Retail rates (R/min.)**	
From	**To**	**Standard time** Weekdays from 07:00 to 19:00	**Talkmore time** Weekdays from 19:00 to 07:00 and from Friday 19:00 to Monday 07:00
087 350 0000	087 359 9999	0,82	0,82
087 780 0000	087 781 9999	1,07	1,07
087 805 0000	087 805 0999	1,07	1,07
087 750 0000	087 754 9999	0,59	0,59
087 810 0000	087 810 4999	1,00	0,50
087 700 0000	087 701 9999	1,00	0,50
087 845 0000	087 845 1999	0,90	0,90
087 940 0000	087 940 4999	0,91	0,46
087 870 0000	087 870 1999	0,91	0,46
087 830 0000	087 830 0999	0,73	0,42
087 855 0000	087 855 1999	0,84	0,84
087 960 0000	087 960 6999	0,79	0,42
087 985 0000	087 985 2999	1,07	1,07
087 820 0000	087 820 2999	0,82	0,41

* Calls are charged per second, with a minimum charge of R0,72 for all calls.

a) Use the table to calculate the cost for each phone call from telephone number 087 700 2491 to telephone number 087 701 0909.

i) a call of $3\frac{1}{2}$ minutes at 16:00 on a Tuesday

ii) a call of two seconds at 14:00 on a Sunday

b) Use the table to calculate the cost for each phone call from telephone number 087 964 0410 to telephone number 087 961 3139.

i) a call of 5 minutes 20 seconds at 07:30 on a Monday

ii) a call of 5 minutes 20 seconds at 06:20 on a Monday

c) What is the difference per minute between the cost of a call during Standard time and a call during Talkmore time from number 087 820 2569 to number 087 820 2984? Give your answer as a ratio.

5. Shafiek's medical aid contribution amounts to R2 184,00 per month for himself as principal member of his medical aid scheme. His wife and two children are also on this medical aid scheme. Members who belong to a medical scheme may deduct a fixed amount per month from their income for tax purposes:
Single member: R310,00
First registered member: R310,00
All other registered dependants: R209,00 (each).

a) The ratio of the monthly payment for employer and employee is 2 : 1. How much do Shafiek's employer and Shafiek each pay into the medical aid per month?

b) How much may Shafiek deduct from tax per month?

6. Post Office domestic parcel rates

Product	Rate
Ordinary parcel (counter to counter)	R52,50 for the first kilogram (kg) R7,05 for each additional kg or part of a kg
Cash-on-delivery (COD) charges (in addition to the parcel rate)	R28,20 per parcel
Insurance is available at a rate of 3% of the declared value to a maximum of R5 000 with a minimum of R4,20 per parcel.	

(Source: *The South African Post Office*, November 2018)

What will it cost to send each of these parcels?

a) a counter-to-counter parcel with a mass of 32,4 kg

b) a counter-to-counter parcel with a mass of 12,5 kg that is insured to the value of R150

c) a COD parcel with a mass of 19,2 kg that is insured to the value of R1 500,00

7. According to health researchers, the unofficial rule for a handbag or school bag's mass is that it should not exceed 10% of your body mass. If you carry more than this mass on a regular basis, you could experience back pain.

a) What should the maximum mass of a school bag be for a learner with a mass of 51 kg?

b) The average mass of one school textbook is 750 g. If a learner needs to take seven textbooks to school, will the mass of the bag adhere to the unofficial rule for a learner with a mass of 41 kg?

8. The Greeks of long ago believed that the golden ratio for the sides of a rectangle are in a proportion of 1 : 1,618. They believed that rectangles that adhere to this ratio are the most pleasing to the eye. The width of a rectangle is 4,5 cm. What should its length be for it to adhere to the golden ratio?

9. The ratio of a concrete mix is determined by the purpose for which the concrete will be used:

- Concrete more than 75 mm thick: cement, river sand and washed aggregate in a ratio of 1 : 2,5 : 4
- Concrete less than 75 mm thick: cement, river sand and washed aggregate in a ratio of 1 : 2 : 3
- Rough bedding concrete: cement, river sand and washed aggregate in a ratio of 1 : 3 : 6.

If 50 buckets of cement are used, determine, for each type of concrete, how much river sand and washed aggregate are needed.

10. In June 2006, the inflation rate in Zimbabwe was 1 043%, and in August 2007, it was about 5 000%.

a) Explain in your own words what the term inflation means.

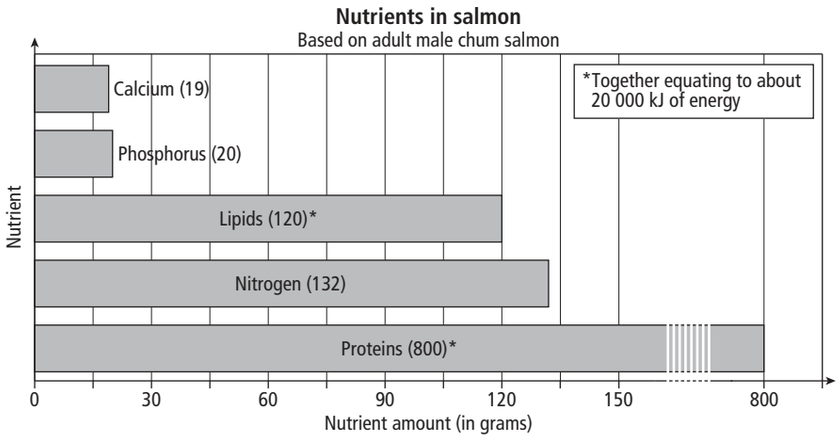
b) Determine the rate of increase in inflation in Zimbabwe between June 2006 and August 2007. Round off your answer to one decimal place.

c) Is an inflation rate of 1 043% normal? Explain.

d) What does the inflation rate say about the financial situation of a country?

11. Altogether 930 000 of 2,4 million tickets that were available for Rugby World Cup tournaments were sold a few months before the first match. What percentage of tickets was sold a few months before the first match? Round off your answer to one decimal place.

12. The graph shows the nutrients that are present in an adult male chum salmon.



(*Source: Scientific American,* August 2006)

Note

1 calorie = 4,186 kJ

a) What is the ratio of protein to nitrogen to fats (lipids) in an adult male salmon?

b) If another salmon contained 690 g of protein, how many grams of fats (lipids) and nitrogen would it contain?

c) Convert 20 000 kJ of energy to calories.

13. Liza is a university student. She arrived on campus at 08:00 for her first class. Her last class finished at 17:00. She spent her time at university in class, having lunch, socialising and exercising in the gym in the ratio 4 : 2 : 5 : 1. Calculate the time (in hours and minutes) she spent on each activity.

14. According to the Engineering Council of South Africa's records, between 1998 and 2004, altogether 50 570 students enrolled at South African universities for engineering courses, and 8 900 graduated.

a) Would it be correct to say that one out of every six first-year engineering students will complete their degrees? Give a reason for your answer.

b) Calculate the graduation rate of engineering students between 1998 and 2004. Give your answer correct to one decimal place.

15. The population of Niagara Falls in New York in 1997 was 60 000, and 25 years earlier it was 100 000.

a) Was there an increase or a decrease in the population of Niagara Falls between 1972 and 1997? What was the percentage increase or decrease?

b) By how much did this percentage increase or decrease on average per year? Give your answer as a percentage, correct to one decimal place.

16. The table shows part of a municipal account for two different properties before and after increases in rates and taxes, and other municipal services.

Land/site value	**Property A: R10 000**		**Land/site value**	**Property B: R190 000**	
	Before increase	**After increase**		**Before increase**	**After increase**
Rates/taxes on land value (R)	–*	–*	Rates/taxes on land value	1 387,32	1 484,06
Electricity R/kWh	20,49	21,72	Electricity	942,31	1 020,61
Water (R/kl)	33,72	51,00	Water	611,14	796,54
Sanitation (R/kl)	21,60	33,52	Sanitation	73,28	90,88
Waste services (R)	18,40	19,70	Waste services	52,16	55,82
Total	**94,21**	**125,94**	**Total**	**3 066,21**	**3 447,91**

* Property owners whose land has a value of R10 000 or less receive a rebate of 100%.

a) Explain what a rebate of 100% means.
b) How much more did each household have to pay after the increases?
c) What was the total percentage increase in municipal costs for each household? Give your answers to one decimal place.

17. Caren bought R100 fuel for her lawn mower and received 6,09 ℓ of fuel. What was the price of fuel per litre?

18. Transfer duty is an amount of money you pay to the government when you buy a house or flat. Calculate the transfer duty payable on each property below.

New rates of transfer duty	
R0 – R900 000	0%
R900 001 – R1 250 000	3% of the value above R900 000
R1 250 001 – R1 750 000	R10 500 + 6% of the value above R1 250 000
R1 750 001 – R2 250 000	R40 500 + 8% of the value above R1 750 000
R2 250 001 – R10 000 000	R80 500 + 11% of the value above R2 250 000
R10 000 001 and above	R933 000 + 13% of the value above R10 000 000

(Source: *sars.gov.za*, November 2018)

a) a town house sold for R850 000
b) a house sold for R1,3 million
c) a house sold for R2 200 000

19. The Lethe Power Station at Eskom burns approximately 50 000 tonnes of coal per day, producing 18 000 tonnes of ash.

Note

1 tonne (t) = 1 000 kg

a) If one tonne of coal is burnt per day, how many tonnes of ash will be produced?
b) Convert your answer to kilograms.

20. The table shows the municipal accounts for two households.

Items	Household A	Household B
Land value (R)	10 000,00	190 000,00
Property rates and taxes (R)	–	1 387,32
Electricity (R/kWh)	100,00	945,00
Water (R/kl)	35,00	611,10
Sanitation (R/kl)	21,60	73,28
Waste services (R)	18,40	52,16
Total	**175,00**	**3 068,86**

a) Use the information on household tariff increases to calculate how much each account will be after the increases have been introduced.

Municipal tariff increases/changes
» 100% rebate on site with a value of R10 000 and less
» 6% increase in electricity
» 7% increase in waste services, property rates, water and sanitation

b) How much more will each household account be after the increases?
c) What was the total percentage increase for each household?
d) Explain in your own words what a rebate is.
e) In the same month, the price of petrol increased from R16,91 per litre to R17,10 per litre. What was the percentage increase in the price of petrol that month?

21. A shop has a sale and marked down the prices of various items as shown in the table below. Give the missing information (a to i). Round the percentages in your answers off to the nearest percentage.

Remember!

Percentage discount $= \frac{\text{original value} - \text{decreased value}}{\text{original value}} \times 100$

Item	Original price (R)	New price (R)	Percentage discount
Toaster	126,55	a)	5%
Kettle	245,89	191,79	b)
Six mugs	c)	40,94	10%
Alarm clock	39,95	17,98	d)
Photo frame	16,00	e)	25%
Bedside lamp	f)	179,82	60%
Duvet cover	288,64	184,73	g)
Curtains (two drops)	h)	756,92	15%
Blanket	59,99	i)	12%

22. You need to buy flour and sugar for your mother, who is baking a cake. She instructs you to buy 2,5 kg of each. These are the prices of the different masses in which the products are sold.

Flour		Sugar	
Mass (kg)	**Price (R)**	**Mass (kg)**	**Price (R)**
0,5	9,49	0,5	10,99
1,0	14,99	1,0	17,99
2,5	39,99	2,5	33,99
5,0	60,90	5,0	77,99

a) From the options given above, what is the cheapest bag of flour you can buy? Show all your calculations.

b) From the options given above, what is the most expensive pack of sugar you can buy? Show all your calculations.

23. The director of a business shares the company's annual profits in a ratio of 5 : 2 as bonuses (5 parts go to him and 2 parts go to his employees). If the company's profit in a particular year is R430 899 and the director has six employees, calculate the respective bonuses for the director and one employee.

24. A netball team plays 31 matches. They win 18 of these matches. What is the ratio of their number of wins to their number of losses?

25. The ratio of boys to girls in a class is 9 : 7.

a) If there are 14 girls in the class, how many boys are there?

b) How many learners are in the class?

26. In a particular season the Blue Bulls won 8 games, the Lions won 14 games and the Sharks won 11 games. What is the ratio of wins of the Bulls and the Lions to the wins of the Sharks? Give your answer in its simplest form.

27. On average there are 40 sweets in a 300-g packet of sweets. The ratio of green sweets to yellow sweets is 5 : 3.

a) How many of each colour sweet should there be in a packet?

b) How many grams of sweets would be yellow?