

Intermediate Solid Mechanics

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“The Lubardas, a father-son duo, deliver a unique and well-balanced textbook on solid mechanics. The material is presented at the intermediate level, and is tested by many years of well received classroom instruction by both authors in their respective institutions. The authors take the reader from basic concepts of traction, stress, and strain, to boundary-value problems in elasticity, and finish with more advanced topics, such as contact, variational principles, and failure criteria. The book is well suited for advanced undergraduate students as a course textbook, as well as for first- and second-year graduate students as a reference for more advanced courses in solid mechanics. The book strikes an excellent balance between theory and application examples, and presents a perfect jumping-off point to study more advanced topics in solid mechanics, such as damage, plasticity, fracture, and advanced numerical approaches such as the finite element method.”

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Horacio Espinosa,
Northwestern University

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Preface

The mechanics of solids is a fundamental engineering discipline which deals with the analysis of strength and deformation of structural members made of metals, polymers, ceramics, concrete, wood, and other materials. It plays an essential role in the design of machines, automobiles, airplanes, ships, bridges and other structures, robots, biomedical devices, and modern materials. This book represents a concise yet comprehensive treatment of the fundamentals of the mechanics of solids. It is written in the form of a textbook for an upper-division undergraduate course in solid mechanics, which comes after an introductory strength of materials course in mechanical, aerospace, civil, structural, and materials engineering departments. It can also serve as a textbook or supplemental reading for an introductory graduate course in solid mechanics or theory of elasticity, being particularly well suited for the Master of Engineering Programs.

The book consists of two parts. Part I is devoted to the basic concepts and ingredients of the theory. It comprises five chapters. Chapter 1 covers the analysis of stress, the most fundamental concept in the mechanics of deformable bodies. The methods of determining the principal stresses and the maximum shear stress are presented, and the stress equations of equilibrium are derived. The analysis of strain, strain–displacement relations, and the Saint-Venant compatibility equations are presented in Chapter 2. The generalized Hooke’s law, relating three-dimensional states of stress and strain for small deformations of isotropic elastic materials, is introduced in Chapter 3. The effects of temperature are incorporated in the Duhamel–Neumann constitutive law of thermoelasticity. Beltrami–Michell compatibility equations expressed in terms of stresses, with and without temperature effects, are also introduced in Chapter 3. A summary of the governing equations in Cartesian coordinates and the formulation of the boundary-value problem of linear elasticity whose solution specifies the stress, strain, and displacement fields is presented in Chapter 4. Several example boundary-value problems are solved to illustrate the solving procedure, including the use of the so-called semi-inverse method which greatly facilitates the solution. The derivation of the strain–displacement relations, the equations of equilibrium, and the compatibility equations in cylindrical coordinates is presented in Chapter 5, which also includes the solution of the Lamé problem of a pressurized cylinder, and its application to shrink-fit problems. A brief referral to problems with spherical symmetry is also given.

Part II contains six chapters on the application of the general theory from Part I to solve a variety of boundary-value problems of solid mechanics, and two chapters

on energy methods and failure criteria. Chapter 6 addresses two-dimensional, plane stress and plane strain problems of elasticity expressed in Cartesian coordinates. The Airy stress function is introduced. Its governing biharmonic differential equation is derived and solved for several cases of in-plane bending of thin beams. Chapter 7 is devoted to two-dimensional problems expressed in polar coordinates. The solutions to the classic problems of a concentrated force at the boundary of a half-space (Flamant problem), diametral compression of a circular disk (Michell problem), stretching of a plate weakened by a circular hole (Kirsch problem), and a rotating disk problem are presented. The stress fields near a crack tip and around an edge dislocation are also derived and discussed. Chapter 8 covers some basic problems of antiplane shear, with a focus on the stress concentration around holes, the stress field near a crack tip, and the stress field around a screw dislocation in an infinite medium, or near a circular hole in an infinite medium. Torsion of prismatic rods is considered in Chapter 9. The Prandtl stress function is introduced and its governing Poisson's differential equation is derived. The stress and displacement fields, including warping of non-circular cross sections, are determined and discussed for twisted rods of elliptical, triangular, rectangular, thin-walled open and thin-walled closed cross sections. The stress and deformation analysis of a cantilever beam bent by a transverse force is presented in Chapter 10. The stress function is introduced and the governing Poisson-type partial differential equation and the accompanying boundary conditions are derived for simply and multiply connected cross sections of the beam. The stress and deformation fields are obtained for circular, semi-circular, hollow-circular, elliptical, and rectangular cross sections. Approximate formulas for shear stresses in thin-walled open and thin-walled closed cross sections, including multicell cross sections, are derived and applied to different profiles of interest in structural engineering. Chapter 11 is a brief coverage of contact problems. It begins with the analysis of three-dimensional axisymmetric problems of elasticity expressed in cylindrical coordinates, followed by the solutions to the fundamental problems of a concentrated force within an infinite medium (Kelvin problem) and at the boundary of a half-space (Boussinesq problem). The stress field in a half-space loaded by an elliptical or uniform pressure distribution over a circular portion of its boundary is then discussed. The chapter ends with the analysis of indentation and the elastic contact of two spherical bodies pressed against each other (Hertz problem). Chapter 12 is devoted to the three-dimensional energy analysis of elastically deformed solids and the corresponding energy methods. The expressions for the total, volumetric, and deviatoric strain energy are derived. Betti's and Castigliano's theorems are formulated and applied to various structural mechanics problems. An introduction to the approximate Rayleigh–Ritz method and the finite element method is also given. The final Chapter 13 is a survey of the failure criteria for brittle and ductile materials. This includes the maximum principal stress and strain criteria, Tresca, von Mises, Mohr, Coulomb–Mohr, and Drucker–Prager criterion. The failure criteria based on fracture mechanics, stress intensity factors, and the energy release rate associated with the crack growth are also formulated. To further facilitate the understanding of the theoretical foundation of the subject and its application, numerous exercise problems and solved examples are included throughout the book. There are also ten representative problems at the end of each of the thirteen chapters, which are

intended for homework exercise. The solutions manual is available to instructors, with the solutions to all 130 problems, at www.cambridge.org/lubarda.

Although being a concise coverage of solid mechanics, the book comprises material too extensive to be covered in a one-quarter, or even one-semester, course. However, if the focus of the course is on the fundamentals of solid mechanics and the solution of boundary-value problems, one may cover in one semester selected material from Chapters 1 through 10. If the focus of the course is on the fundamentals and design issues related to energy considerations and failure criteria, one may choose to cover in one semester most of the sections from Chapters 1 through 5, and Chapters 12 and 13. Chapter 11 on contact mechanics may be the most challenging to cover together with other material in a short 10- or 15-week-long course, but is included as a reference because of the great importance of indentation and contact problems in engineering design and materials testing. Much of the contents of the book can also be used to build an introductory graduate course of solid mechanics, particularly within contemporary Master of Engineering Programs in mechanical, aerospace, civil, and structural engineering departments.

In writing this book, we have used our lecture notes from the solid mechanics courses that we taught at several universities in Europe and the USA (University of Montenegro, University of Donja Gorica, Arizona State University, and University of California, San Diego). We have also consulted numerous textbooks and reference books on the subject written by other authors, as cited in the Further Reading section at the end of the book. We are grateful to many colleagues with whom we discussed the topics of the book and to our students for their valuable feedback from the mechanics and materials classes that we taught.