

1 Fluids and Fluid Mechanics



The wind blows into the sails, the ship sails through the huge waves, and the seabirds fly in the sky. The forces exerted by fluids are everywhere in our daily lives.

1.1 Fluids: Basic Concepts

Fluids are substances that deform more easily than solids, and includes liquids and gases. However, an object deforms only under forces, and whether it deforms easily depends on the intensity and type of forces acting on it. As a solid, nylon rope has high tensile strength but offers practically no resistance when pressing on it, and can also be easily cut by scissors. Therefore, we need a more rigorous definition to distinguish between fluids and solids.

The key difference between solids, liquids, and gases lies in the microstructure of the material. Figure 1.1 shows the microstructure in the three states of water, with oxygen and hydrogen atoms represented by small balls. The close-packed molecules of a solid strive to retain a fixed arrangement, while those of a liquid have no such

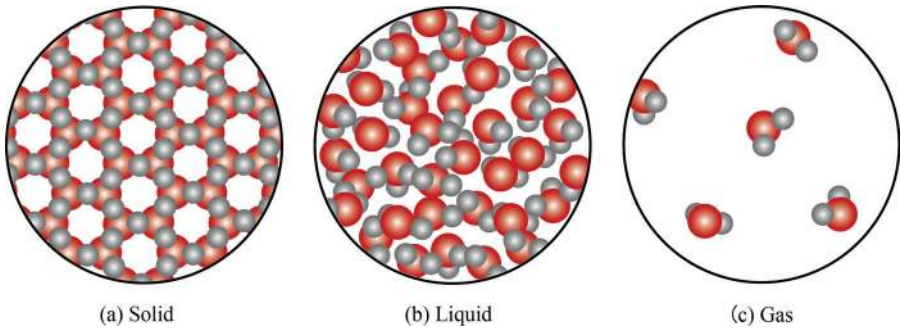


Figure 1.1 Microstructure in the three states of water.

intention. The wide-spaced molecules of a gas neither huddle together nor aim to retain a fixed arrangement.

Now we can try to understand the features of the three states of matter. A solid or liquid, being composed of closely packed molecules with strong cohesive forces, tends to retain its volume unless it is subjected to enormous pressure. On the other hand, under the action of external pressure the distance between the molecules of a gas will decrease, producing a change in overall volume.

Liquids share with gases the ability to flow because the atoms and molecules in both states have no tendency to be rigidly packed in a fixed pattern. Although tightly packed, the molecules in a liquid can be arranged randomly, causing the liquid to have a definite volume but no definite shape. Since the molecules in a gas move freely and independently of each other, the forces between them can be ignored, except when they collide with each other.

We understand that if the resultant force-torque acting on a rigid body equals zero, the object remains stationary or in uniform motion. However, this is not true for a real solid or fluid. Any solid material has a strength limit. Even if the resultant force-torque acting on it equals zero, the solid material inside may experience a tensile stress, a compressive stress, or a shear stress. A solid deforms under such stresses. Furthermore, stresses beyond the strength limit cause the solid material to yield and to fracture. At the microscopic level, the intermolecular (or interatomic) chemical bonds at the fracture point are broken, losing their interactive forces; this means the material cannot retain its original size and shape.

A fluid differs substantially from a solid in this respect. If a fluid is subjected to compressive stresses only, it will deform like a solid and remain stationary. When subjected to shear stresses, a fluid will deform continually and be unable to resist the external shear stresses in static equilibrium. This is somewhat similar to the case in which the forces acting on a solid far exceed its strength limit. But a fluid at rest has zero “shear strength” and will continually deform under the action of any small amount of shear force. Therefore, the essential difference between fluids and solids is that *a fluid cannot generate internal shear stresses by static deformation alone*.

To better understand the difference between fluids and solids, let us discuss the static friction between two solid surfaces. Place a solid block on a plane and gradually

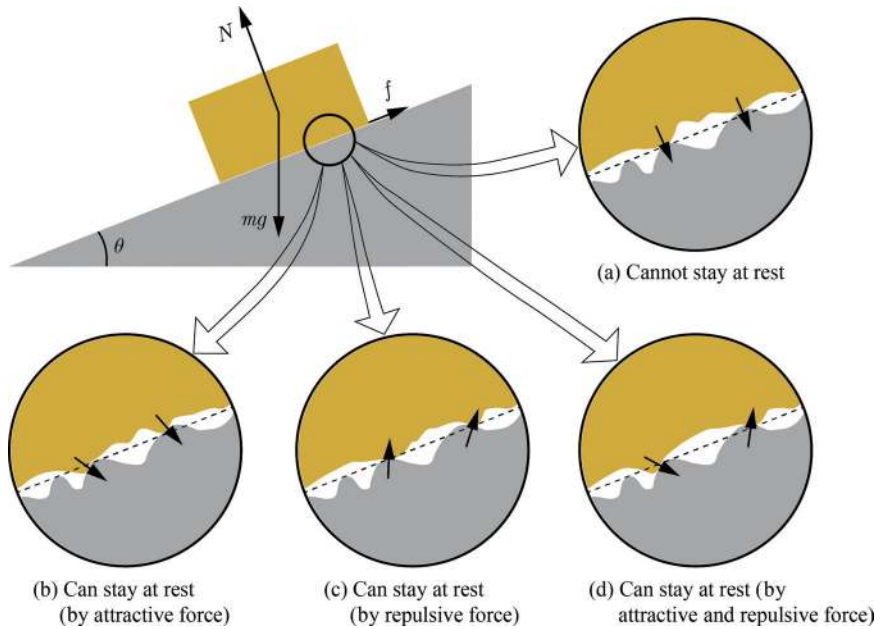


Figure 1.2 The forces exerted on a solid block on an incline and corresponding microscopic explanation of the static friction between two solid surfaces.

lift one end of the surface. Within a certain range of inclination angle the block will not move because the component of gravity acting along the inclined plane equals the static friction. Figure 1.2 shows the forces acting on the block (represented by arrows) and the corresponding microscopic explanation of the static friction between the two surfaces. As is well known, static friction results from the electromagnetic forces between molecules (or, similarly, atoms) that are in contact. In order to generate shear forces, the electromagnetic attractive and repulsive forces should not be perpendicular to the contact surface but have one component acting along it. This component may be caused by one of two factors, corresponding to rough and flat surfaces, respectively.

In general, as shown in Figure 1.2, neither the bottom of the block nor the inclined plane is a perfectly flat surface. There are some attractive and repulsive forces only between the molecules on the very small contact area between the two surfaces. When the plane starts to tilt from horizontal, the block will slide down an imperceptible distance. At the microscopic level, when the block slides down, as shown in Figure 1.2(b)–(d), the attractive and repulsive forces between the bulges of the block and the plane offer an upward component acting along the contact surface. This generates a shear force that keeps the block in static equilibrium.

If two perfectly flat planes at the molecular level are placed in contact with each other, is there any friction between them? Not only friction is present in this case, but it can be very high. This is because the contact area is quite large, and a certain form of molecular bond forms between the molecules of the two objects, both of which are similar to the situation inside a single object, resulting in a remarkable shear stress.



Figure 1.3 Water droplets at rest on a glass pane.

The contact between a liquid and solid interface is somewhat like that of two perfectly flat solid surfaces – that is to say, a full intermolecular contact. However, unlike the static frictional force occurring between two solid surfaces, there is no such force between a liquid and a solid interface. A typical example is a boat floating on water, which can be moved by an arbitrarily small amount of thrust. The liquid molecules adjacent to the solid surface will stick to it through the intermolecular attractive forces, forming a relatively fixed structure. However, these stuck molecules cannot apply a shear force to the adjacent molecules further inside the liquid. As a result, the solid has no way of exerting a shear force on the fluid through static displacement alone.

Even so, we notice water may flow at a uniform velocity along an inclined riverbed, which means there is a frictional force between the water and the inclined ground to balance the component of gravity along the riverbed. That is to say, *a frictional force does indeed occur in a moving fluid*. This property of a fluid is known as *viscosity*, and will be discussed in detail in the next section.

Here's an interesting counterexample: Water can remain stationary on inclines, even on vertical walls or glass. Consider water droplets clinging to window panes, for example, as shown in Figure 1.3. The water droplet is like a type of solid cantilever structure inside which internal shear stress occurs. Consequently, shouldn't there be shear stress in these water droplets?

Conclusion first: In this case there is still no shear stress. To further analyze this situation, we need to take into account the surface tension of a liquid, a concept that will be discussed later.

1.2 Some Properties of Fluids

The definitions of many properties of fluids, including density, pressure, and temperature, are consistent with those that apply to solids. However, liquids possess

other properties as well, such as viscosity and surface tension. Viscosity is an important characteristic of fluids from the viewpoint of fluid mechanics. The surface tension arises due to cohesive interactions between the molecules on the liquid surface.

1.2.1 Viscosity of Fluids

Any shear force externally applied to a fluid, no matter how small, will cause a continuous deformation of that fluid. Viscosity is a property of a fluid that describes the shear stress caused by such a continuous shear deformation. Common liquids and gases are viscous, and only superfluids (such as liquid helium near absolute zero) can be assumed to be inviscid.

Before discussing the viscosity of a liquid, we need to clarify the difference between viscous and adhesive forces: these are in fact completely different concepts. The viscous force is the shear force exerted in a moving fluid, while the adhesive force is the attractive force between molecules of two different materials. It is the adhesive force between the molecules of the glue and those on the solid surface that holds them together, rather than the viscous force discussed here. They cannot be held together by the viscous force since it does not exist in a static fluid. When a solid object moves through a fluid it is subjected to the frictional force on its sides. This force results from the combined action of the adhesive force between the solid surface and its adjacent fluid molecules, and the viscous force within the fluid.

We know that honey has a much higher viscosity than water. Let us discuss the intrinsic factors that determine the magnitude of viscous forces in a fluid by analogy with the frictional force between two solid surfaces. For a block sliding down an incline, its dynamic friction is equal to the product of the coefficient of friction and the normal force between the two solids in contact. The coefficient of friction is a numerical representation of the magnitude of the intermolecular force between neighboring molecules of the solid and the contact surface roughness. This is easy to understand. But why is the dynamic friction proportional to the normal force between the two solids in contact? After all, the normal force is perpendicular to the motion and should not affect the magnitude of the friction force.

The reason is that the greater the normal force, the larger the contact area between the two solid surfaces. Since there is essentially a linear relationship between contact area and normal force, the frictional force is proportional to the normal force. If two solids come into gapless contact with each other at the molecular level, the friction force should be independent of the normal force.

For solids, the coefficient of dynamic friction does not in general equal that of static friction due to different mechanisms behind frictional force. Static friction is a consequence of the balance of forces exerted on a stationary solid, while dynamic friction also involves the conversion of kinetic energy into internal energy. When two adjacent solid surfaces slide relative to each other, not only do forces similar to static friction occur on the contact area, as in Figure 1.2, but also the bonds between the molecules of both objects are constantly broken and reformed, with molecules and molecular groups falling from the objects. Accompanied by the change in molecular vibrational

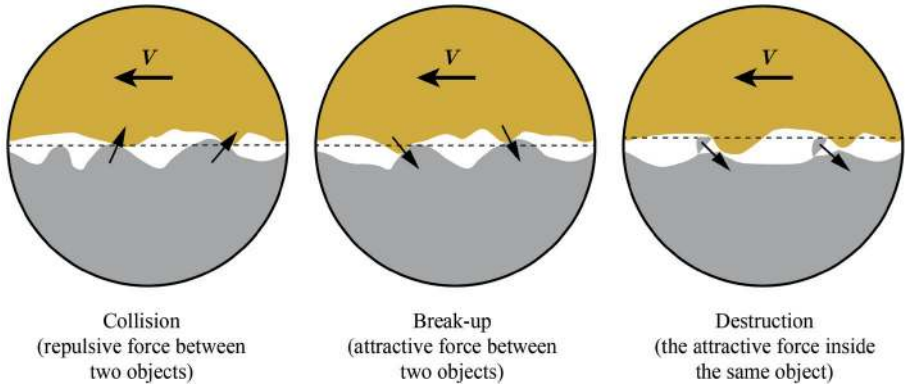


Figure 1.4 Microscopic explanation of the dynamic friction between two solid surfaces.

energy and the directional disorder of molecular motion, friction inevitably produces heat. Figure 1.4 shows three typical phenomena occurring at the contact area.

Whenever a fluid and a solid come into contact with each other, the molecules of the fluid right next to the solid surface will stick to and move with it. Thus, *the friction between a fluid and a solid surface is actually that between two fluid layers*. In other words, the friction between solid and fluid surfaces relates only to properties of fluids, independent of the properties of solid materials.

Since the liquid and the solid surface come into full contact at the molecular level, the frictional force should be independent of the normal force (i.e., the pressure). This explains why the magnitude of the viscous force in a liquid is basically independent of pressure. On the other hand, viscous force is closely related to the temperature of the liquid. *In general, for liquids, the higher the temperature, the lower the viscosity*. For example, a cold, thick syrup becomes less sticky when heated. Contrary to liquids, *the viscosity of gases increases with temperature*. The difference in viscosity between liquids and gases is directly related to the physical mechanism generating it.

If the velocities between two adjacent layers of liquid are different, the slower-moving layer is dragged along by the faster-moving one by intermolecular force. As the momentum of the molecules in the faster-moving layer decreases, that of the molecules in the slower-moving one increases. From a macroscopic point of view, the momentum transfer is caused by the frictional forces between the two fluid layers. Since friction is an internal force when we look at the two layers of liquid as a single system, the total momentum of the system is conserved. Figure 1.5 shows how the slower-moving liquid layer is dragged along by the adjacent faster-moving layer at the molecular scale.

The friction caused by the intermolecular forces in a liquid is somewhat similar to that acting between two solid surfaces in contact with one another. Unlike solid molecules, liquid molecules do not always flow in their respective layers, but instead move freely among different layers. In other words, the liquid molecules in each layer have random diffusion perpendicular to the direction of motion. Some molecules

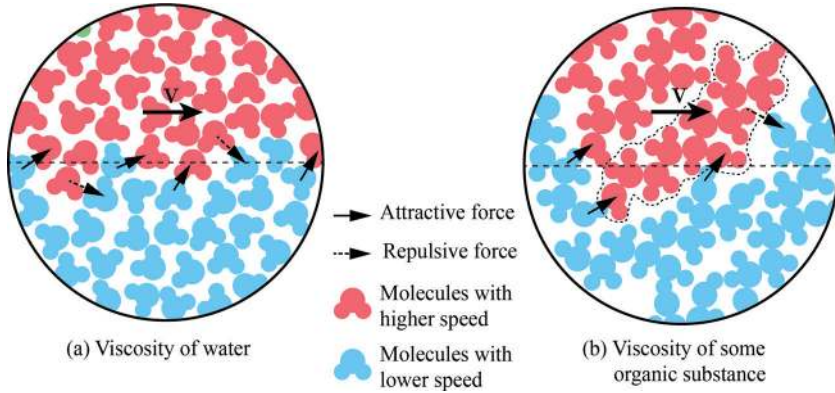


Figure 1.5 Microscopic explanation of the viscous forces in a liquid.

from the faster-moving layer may intrude into the slower-moving one, where after one or several collisions they are decelerated to move along with their neighbors. At the same time, some molecules from the slower-moving layer may intrude into the faster-moving one, where they are accelerated to move along with their neighbors after undergoing one or multiple collisions. The transfer of momentum taking place through the interaction between two adjacent layers is reflected as frictional forces along the direction of motion. For liquid molecules in laminar flow, the frictional forces generated by momentum transfer, being much smaller than their intermolecular attractive and repulsive forces, are usually ignored. Consequently, the viscosity of a liquid is considered to be caused by intermolecular forces, among which the attractive forces dominate.

Daily life experiences show that liquids of organic substances, such as paint, honey, and plasma, generally exhibit high viscosity. The large, irregular, and polar molecules of these substances are difficult to move but get easily tangled with each other. For example, glycerin molecules are not very large, but they are noncircular and polar. Glycerol molecules are more likely to get stuck when they pass over rough surfaces, and the molecules get tangled easily, so glycerol is highly viscous. On the other hand, mercury molecules are larger than glycerol molecules, but they are spherical and nonpolar. Therefore, mercury, though much more viscous than water, is less viscous than liquid organic substances of the same molecular weight. Figure 1.5(b) shows the shearing motion in a liquid organic substance, where the region enclosed by dashed lines represents a large organic molecule. As can be intuitively seen, the molecules of this organic substance, compared to the water molecules in Figure 1.5(a), are large and irregular in shape, inevitably experiencing substantial tangle and therefore having higher viscosity.

As the temperature of a liquid increases, the vibrations of individual molecules intensify and the tangle between them becomes more easily loosened, leading to a decrease in viscosity. This can be explained by analogy to the following example: When carefully positioned on a wet and muddy surface, a cellphone can be kept from

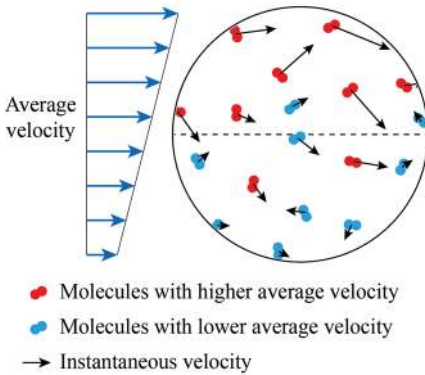


Figure 1.6 Microscopic explanation of the viscous force in a gas.

sinking for quite a long time; if someone calls, the cellphone will vibrate and quickly sink. Generally speaking, the more intense the tangle between molecules, the more sensitive viscosity is to temperature. Syrup, with elongated molecules, shows a rapid reduction in viscosity as temperature rises, but temperature has a much smaller impact on the viscosity of mercury, which has spherical molecules.

However, not all liquids show a reduction in viscosity with increasing temperature. For example, the viscosity of a single-component lubricant decreases rapidly with increases in temperature. This causes huge problems for the lubrication of mechanical parts, such as those in engines used in a wide operating temperature range. To overcome this shortcoming, a substance with large molecules whose shape changes from spherical to elongated as the temperature increases is added to modern lubricants in order to make their viscosity less sensitive to temperature.

Attractive and repulsive forces between gas molecules are negligible. The momentum transfer due to frequent collisions between molecules produces pressure force in a gas. The collisions between molecules can produce not only a normal stress such as pressure, but also a shear stress, such as viscous force. The latter may be considered as a frictional force acting between adjacent fluid layers traveling at different velocities. For instance, the molecules in the faster-moving layer constantly fly into the slower one due to random thermal molecular motion, dragging the slower neighboring molecules to accelerate; the molecules in the slower-moving layer also constantly fly into the faster one, pulling back the faster neighbors to decelerate. From a macroscopic point of view, the viscosity of a gas represents the drag forces caused by the momentum transfer between two adjacent layers of the gas. As remarked earlier, this kind of momentum transfer also exists between two adjacent layers of a liquid, but it is negligible compared to intermolecular forces. In a gas, however, molecules exert almost no forces on each other, but are themselves very active. It is the momentum transfer between the molecules that creates the viscosity of the gas. Figure 1.6 shows the mechanism responsible for gas viscosity. As slower molecules frequently collide with faster ones, the former accelerate while the latter decelerate, thereby resulting in viscosity.

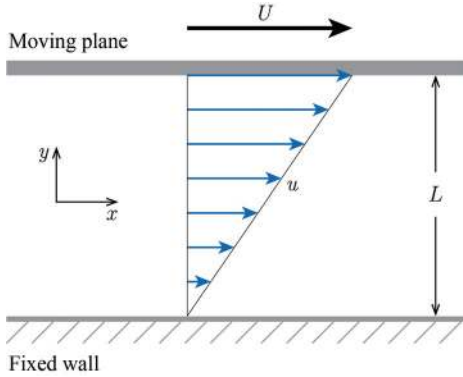


Figure 1.7 Newton's experiment for the viscous force in a liquid.

As the temperature of a gas increases, the thermal motion of the molecules intensifies, and the frequency of their collisions increases, generating more exchange of momentum between the individual gas layers. This is why the viscosity of gases increases with temperature. Just as in liquids, viscosity in gases is essentially pressure-independent. How can this phenomenon be explained?

For an ideal gas, pressure is determined by temperature and density. When the temperature is constant, a change in pressure results in a change in density and vice-versa. Therefore, we reach a conclusion that the viscosity of an ideal gas is independent of its density at constant temperature. It may seem odd that a denser gas does not generate a larger viscous force. When density increases, the collision frequency between molecules must also increase. Why is the viscosity of a gas independent of its density?

As shown in Figure 1.6. If the density of a gas is increased, more molecules in the faster-moving layer indeed fly into the slower layer in unit time. However, due to the decrease in mean free path, the two molecules in collision would have less velocity difference and therefore the momentum transfer caused by a single collision decreases. The percentage of decrease in the momentum transfer due to a single collision is equal to the percentage of increase in density, resulting in a constant momentum transfer. As a result, the change in density does not affect the magnitude of viscous force.

The above analysis, based on kinetic molecular theory, is qualitative. Quantitative derivation of viscosity can be found in some textbooks and will not be discussed here.

Classical mechanics is mainly concerned with the motion of macroscopic objects. Isaac Newton (1642–1727) was the first to attempt a quantitative study of the viscosity of liquids. In 1686, he measured the viscosity of liquids experimentally and established Newton's friction law to describe the frictional force in a fluid.

To make the flow model as simple as possible, Newton attempted an experiment in which the liquid in question was sandwiched between two large parallel horizontal plates, as shown in Figure 1.7. The bottom plate is fixed, and the top plate moves at a steady horizontal speed. When a fluid flows over a solid surface, the layer right next to the surface will attach to it. The fluid attached to the bottom plate remains stationary, while that attached to the top plate moves with a constant velocity of U . Newton's

law of viscosity states that the force required to pull the upper plate is proportional to the velocity with which the top plate moves and inversely proportional to the distance between the plates – that is,

$$\frac{F}{A} \propto \frac{U}{L},$$

where F is the force required to pull the upper plate; A is the area of the contact surface between the fluid and the plate; U is the top plate velocity; and L is the distance between the two plates.

Each successive fluid layer from the top down exerts a force to the right on the one below it. Since the pressure is constant in the fluid, the shear forces between any adjacent fluid layers are the same, and relate to the pulling force to the upper plate. Newton found that the fluid velocity varies linearly with y . He stated that the shear stress between adjacent fluid layers is proportional to the velocity gradient between two layers, written as:

$$\tau = \mu \frac{\partial u}{\partial y}, \quad (1.1)$$

where τ is the shear stress; u is the horizontal velocity in the fluid; μ is a measure of the viscosity of a fluid, known as the coefficient of viscosity, or coefficient of dynamic viscosity. The coefficient of viscosity varies greatly from one fluid to another, while that of the same fluid changes with temperature. There exist tables listing the coefficients of viscosity that have been experimentally determined for various liquids and gases.

The fluids used in Newton's experiments satisfy Equation (1.1). All gases and many common liquids with small viscosity obey Newton's law of viscosity and are accordingly called Newtonian fluids. *A Newtonian fluid is a fluid in which the viscous stresses are linearly correlated to the local strain rate.*

But there are many fluids in nature that do not satisfy this linear relationship; these are called non-Newtonian fluids. Generally speaking, Newtonian fluids are less viscous, while most more viscous fluids, such as paint, honey, and plasma, are non-Newtonian. Unlike Newtonian fluids, most non-Newtonian fluids are composed of molecules or particles that are much larger than water molecules. Since these molecules are tangled up in the flow field where the velocity gradients exist, non-Newtonian fluids exhibit a more complex relationship between shear stress and velocity gradient than simple linearity. Furthermore, some non-Newtonian fluids have a shear stress, or viscosity, that depends not only on velocity gradient but also on time or deformation history.

1.2.2 Surface Tension of Liquids

Surface tension is an amazing force, often used in science shows to draw a crowd, such as through coins floating on water and oversized soap bubbles. Figure 1.8 shows one of the most common surface tension phenomena. Figure 1.8(a) shows an ellipsoidal droplet of water on a plane, and Figure 1.8(b) shows the diagram of forces on water molecules near the droplet's surface. As can be seen, any water molecules inside the