

Ultracold Atomic Physics

The field of ultracold atomic physics has developed rapidly during the last two decades. It currently encompasses a broad range of topics in physics, with a variety of important applications in topics ranging from quantum computing and simulation to quantum metrology, and can be used to probe fundamental many-body effects, such as superconductivity and superfluidity. Beginning with the underlying and including the most cutting-edge experimental developments, this textbook covers essential topics, such as Bose–Einstein condensation of alkali atoms, studies of BEC-BCS crossover in degenerate Fermi gas, synthetic gauge fields and Hubbard models, and many-body localization and dynamical gauge fields. Key physical concepts, such as symmetry and universality, highlight the connections between different systems, and theory is developed with plain derivations supported by experimental results. This self-contained and modern text will be invaluable for researchers, graduate students, and advanced undergraduates studying cold atom physics, from both a theoretical and experimental perspective.

Hui Zhai is Changjiang Chair Professor of Physics at the Institute for Advanced Study, Tsinghua University, and his research is focused on ultracold atomic physics, condensed matter physics, and machine learning. He was awarded a National Natural Science Foundation of China (NSFC) award for distinguished young scholars and the Rao Yutai Prize by the Chinese Physical Society.

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HUI ZHAI

Tsinghua University



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Preface

Since 1995, the field of ultracold atomic physics has been developing very quickly, and it is still expanding. The physics of ultracold atomic systems has covered a broad range of topics and has had an impact on several other fields, such as condensed matter physics, quantum information and computation, nuclear physics, and high-energy physics. This textbook tries to cover most of the major achievements of ultracold atomic physics in the past 25 years, although it is not possible to cover all of them. These achievements range from the early-stage developments, such as Bose–Einstein condensation of alkali atoms, to the studies of BEC–BCS crossover in degenerate Fermi gas, synthetic gauge fields, and the Hubbard models, and recent progress, such as many-body localization and dynamical gauge fields. To cover these topics, the book consists of four parts. Full-color versions of certain figures can be found in the resources tab for this book at cambridge.org.

- Part I introduces basic atomic physics relevant to ultracold atomic systems, in order to be self-contained. The part consists of two chapters, one on single-atom physics, such as atomic structures and atom–light interaction, and the other on two-body collision physics. This part provides basics for readers to understand, for instance, how to trap and manipulate ultracold atoms with light and how to tune the interaction by magnetic field. It is precisely these control tools that make the ultracold atomic physics possible. When discussing atomic structure and two-body collision, we not only cover the widely used alkali-metal atoms but also introduce the alkaline-earth-metal atoms, which have been used by more laboratories in recent years. When discussing atom–light interaction, we not only the scalar light shift, which is the basics mechanism for optical trapping and optical lattices, but also vector light shift and STIRAP, which are essential for generating a synthetic gauge field and creating ultracold molecules. We also extend the discussion of the two-body problem to the three-body problem, where the famous Efimov effect has been extensively studied in ultracold atomic systems.
- Part II is about interacting Bose gas. This part consists of two chapters, one focusing on the interaction effect and the other focusing on topology and spin effects. The interaction effect is mainly about Bose condensate and superfluidity. One exception is the one-dimensional system, where we highlight that the interaction effect is so strong that it destroys condensate. The topology effect mainly concentrates on topological defects in a Bose–Einstein condensate, both spinless and spinful. Finally, we also discuss the spin-orbit coupling effect, arising from the synthetic gauge field, in a Bose–Einstein condensate. The spin-orbit coupling effect has been studied extensively in electronic systems in condensed matter physics, but in ultracold atomic systems, it is the first time

that the spin-orbit coupling effects are studied in the Bose system, which has been a major topic in ultracold atomic physics in the past decade.

- Part III is about Fermi gas. This part also consists of two chapters, one on Fermi liquids and the other on Fermi superfluids. For Fermi liquids, we use polaron as an example to discuss a number of basic qualities and their universal relations in a Fermi liquid, and we use quantum point contact as an example to discuss the transport property of Fermi liquids. Both polaron and quantum point contacts have been focused experimental topics of ultracold atomic physics over the past 10 years or so. For Fermi superfluid, we first introduce the basics of the BCS theory, also for the purpose of being self-contained. Then, we generalize the BCS theory to discuss the BEC-BCS crossover across a Feshbach resonance. We both introduce the theoretical concepts and describe the crossover, and we also review the representative experimental results for the crossover.
- Part IV is about lattice physics. Part II and Part III consider uniform systems, and this part considers lattice effects by applying optical lattices to ultracold atoms. This part also consists of two chapters, one on the noninteracting band effect and the other on the interacting effect. The noninteracting band effect mainly focuses on various kinds of topological bands, including how to realize such topological bands and how to reveal the unique physical effects of topological bands in ultracold atomic systems. The interaction effect mainly focuses on Bose and Fermi Hubbard models, and we also discuss the interplay between interaction and disorder potential, which has led to the new developments in many-body localization seen in the past 10 years. Being an isolated system, an ultracold atomic gas is an ideal platform for experimental studies of many-body localization, and so far, most experiments about many-body localization have been carried out in ultracold atomic systems.

When I selected and organized the topics for this book, I paid special attention to the following considerations. I hope that, with these considerations, this book is accessible for most readers, especially for experimentalists; for junior researchers, including senior undergraduate students; and for readers outside the field of ultracold atomic physics.

- A few key physics concepts are emphasized throughout the book for example, symmetry and universality. Many studies in ultracold atomic physics have illustrated the importance and power of these concepts. I hope that by introducing these examples, the book can also benefit readers outside the field of ultracold atomic physics.

Symmetry plays a crucial role in many physics discussions, and it is one of the key concepts that we continually highlight in this book. For example, first of all, the concepts of the symmetry of Hamiltonian and the symmetry of the state are discussed in Sections 1.1, 3.5, and 4.5, which lead to the relation between symmetry and degeneracy, as well as the concept of symmetry breaking. In Section 4.5, we have also emphasized how these concepts can help us understand the orders of phase transition. In Section 8.1, these concepts are revisited by introducing the concept of emergent symmetry. Second, the relation between symmetry and topology, especially the symmetry-protected topological phenomenon, is introduced in Sections 7.2 and 7.3. Third, in Section 8.2, we discuss another use of symmetry, that is, two different systems are related by a symmetry, and how this can help us understand one system with the knowledge of the other system.

Here the example is the Fermi Hubbard model, where the repulsive Fermi Hubbard is related to the attractive Fermi Hubbard model by the particle-hole symmetry. Finally, a special symmetry, known as the scaling symmetry, is encountered several times in the discussion of the Efimov effect in Section 2.6, of the Tonks–Girardeau gas in Section 3.4, and of the unitary Fermi gas in Section 6.2.

Universality is another important concept in physics, which states that many microscopically different systems can share the same low-energy physics described by very few parameters. We have discussed several such examples in this book. In Section 2.2, the low-energy scattering of different interatomic potentials can be universally described by the s -wave scattering length. In Section 5.1, different Fermi liquids can be described by a few parameters known as the Fermi liquid parameters. In Section 8.1, the quantum critical regime at different microscopic models can be described by a very few critical exponents.

- Connections between different physics contents are highlighted. Many seemingly different physics can have connections, sometimes because of a common mathematical structure behind them. For example, the synthetic gauge field has become a major topic of study since about 2010 in ultracold atomic physics, created by the atom–light interaction; however, this effect actually already existed in the magnetic trapping of atoms, even prior to the birth of the field. This connection is explicitly discussed when I introduce magnetic trapping in Section 2.1. Other examples include discussion of topology and mean-field theory. For topology, in Sections 4.2 and 4.4, we discuss various kinds of topological defects in a Bose–Einstein condensate, and in Sections 7.2 and 7.3, we discuss various kinds of topological band structures for noninteracting fermions. These two are different physics, but they share the same mathematical descriptions. For mean-field theory, we discuss the BCS mean-field theory for fermions in Section 6.1 and the mean-field theory for the Bose–Hubbard model in Section 8.1. The physics of these two systems are also very different, and these two mean-field theories also look quite different. However, there are common physical insights behind these two, which are highlighted in Section 8.1. In this book, we use many boxes to discuss the concepts and connections across different chapters. By building up connections between different physics phenomena, we hope this can help readers to understand the physics more deeply.
- Discussions of theories are always supported by experimental results in ultracold atomic systems. If one looks at the literature on ultracold atoms, there are a lot more theory papers than experimental works. When I choose topics for this book, aside from discussing some open issues, I only selected those theories that have been confirmed by experiments. All the discussions of theories are supported by experimental results. Though I do not go into the experimental details, I hope this can provide readers with direct physical pictures and intuitions. In addition, for the discussion of theories, I try to use the back-of-envelope calculations, though sometimes being less rigorous. I try to avoid using advanced theoretical tools, such as Green’s functions and field theory approaches. Students with quantum mechanics and statistical mechanics backgrounds should be able to understand most parts of the results. I hope this can make this book

friendly to students, including senior undergraduate students and those mostly focusing on experiments.

I have taught a course on Ultracold Atomic Physics at Tsinghua University, Beijing, since 2012. For the past eight years, every year, about 80–100 students attend this course. More than half of them are not from Tsinghua University but rather from other universities and institutions in the Beijing area, or even from other cities. Although they do not get grades from the course, they attend all the lectures because of their interest in the physics. They ask many excellent questions in and after class. These questions help me improve my lecture notes, which have led to this book as it stands now. Here I give special thanks to all of them. This book would not have been possible without their enthusiasm in the course.

In the past 10 years, I have enjoyed fruitful collaborations with my previous students and postdocs, and many things of which I have written in the book I learned from them during these collaborations. The book would also not have been possible without their contributions. I thank them for many valuable suggestions and for help in finalizing the manuscript: Chao Gao, Zheyu Shi, Pengfei Zhang, Yanting Cheng, Ran Qi, Zeng-Qiang Yu, Wei Zheng, Yu Chen, Ren Zhang, Boyang Liu, Mingyuan Sun, Zhigang Wu, and Juan Yao. Most of them are currently already faculty in different universities and institutions in China. I wish them great success in their careers.

I sincerely thank my thesis advisor, Professor Chen-Ning Yang, who brought me into the field of ultracold atomic physics nearly 20 years ago. As Professor Yang always said, it is good luck for someone to grow up together with a young field. His taste in physics and style of doing research, his guidance and encouragement, have had an important impact on my scientific career. This book is a special gift to Professor Yang’s coming one-hundredth birthday. I thank Professor Tin-Lun Ho and many other senior scientists for their support over all these years. The Institute for Advanced Study of Tsinghua University (IASTU) provides a very special academic environment, and I am very glad that I can carry out my research there.

Last, but not least, I am grateful to all my family members for their support all these years. Together with my wife, it was great fun to watch two kids grow up during the eight years over which I wrote this book.