Part I

Atomic and Few-Body Physics

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A Single Atom

Learning Objectives

- Discuss the Coulomb interaction, the spin-orbit coupling, and the hyperfine coupling as the three effects that determine atomic structure.
- Highlight the importance of the separation of energy scales of these three effects.
- Introduce different atomic structures of alkali-metal, alkaline-earth-metal, and magnetic atoms.
- Introduce the long-lived excited states in alkaline-earth atoms, and their applications, such as to atomic optical clocks.
- Discuss the Zeeman structure of atoms in a magnetic field.
- Discuss the idea of magnetic trapping, which can naturally lead to the emergence of a synthetic gauge field.
- Introduce the scalar light shift and its applications, such as laser trapping, optical lattices, and laser cooling.
- Introduce the vector light shift and its applications, such as the light-induced Zeeman field and synthetic spin-orbit coupling.
- Discuss the synthetic spin-orbit coupling and various kinds of gauge fields generated by the vector light shift.
- Introduce the basic idea of the stimulated Raman adiabatic passage.

1.1 Electronic Structure

Let us first consider a general Hamiltonian of Z electrons moving around a nucleus that contains the Coulomb interaction, the spin-orbit coupling, and the hyperfine coupling. These are the three effects that determine the electronic structure of an atom. Here we should emphasize the important role of the separation of energy scales; that is to say, the typical energy scales of these three terms are quite different. Thanks to the separation of energy scales, we can analyze them one by one, which enables us to obtain a clear picture of the electron structure.

Coulomb Interaction between Electron and Nucleus. Each electron moves around the nucleus with an attractive Coulomb interaction between the electron and the nucleus, which is described by

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$$\hat{H}_{0} = \sum_{i=1}^{Z} \left(-\frac{\hbar^{2} \nabla_{i}^{2}}{2m^{*}} + V_{\text{ei}}(\mathbf{r}_{i}) \right), \qquad (1.1)$$

where i = 1, ..., Z labels the electrons; \mathbf{r}_i labels the coordinate of electron centering at the nucleus; $m^* = mM/(m + M)$ is the reduced mass, where *m* is the electron mass and *M* is the nucleus mass; $V_{ei}(\mathbf{r}) = -Z\kappa/r$ is the Coulomb potentials between the electron and the nucleus, where $\kappa = e^2/(4\pi\epsilon_0)$; *e* is the electron charge; and ϵ_0 is the vacuum permittivity. The eigenstates are characterized by three quantum numbers (n, l, m). Usually for the spherical symmetric potential, because of the *SO*(3) rotational symmetry, the energy spectrum only depends on *n* and *l* and does not depend on *m*. However, for the 1/r Coulomb potential, such as with a hydrogen atom, the eigenspectrum is

$$E = -\frac{m^* Z^2 \kappa^2}{2\hbar^2 n^2},\tag{1.2}$$

which only depends on the principal quantum number n and is independent of angular momentum quantum number l; l can take integer values from 0 to n - 1. This extra degeneracy is a consequence of 1/r potential, which leads to an SO(4) symmetry larger than the three-dimensional rotational symmetry [102]. The separation of these energy levels is of the order of electron volts ($\sim 10^{14}$ Hz) because it originates from the Coulomb interaction. The energy levels, usually named as the term-diagrams, are schematized in Figure 1.1, where 1, 2, 3, ... label the principal quantum number n and s, p, d, ... represent the angular momentum quantum number l. The term-diagram for a hydrogen atom is shown in Figure 1.1(a).

Coulomb Interaction between Electrons. The repulsive Coulomb interaction between electrons is given by

$$\hat{V}_{c} = \sum_{i < j} V_{ee}(\mathbf{r}_{i} - \mathbf{r}_{j}), \qquad (1.3)$$

Figure 1.1

Schematic of the term-diagram: (a) the hydrogen atom without screening effect and (b) an alkali-metal atom with screening effect. A color version of this figure can be found in the resources tab for this book at cambridge.org/zhai.

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where $V_{ee}(\mathbf{r}) = \kappa/r$. Here we discuss a couple of physical consequences of this term. First, the inner electrons of the fully filled levels screen the positive Ze charge of the nucleus, and thus, the valence electron experiences a reduced Coulomb potential. When the electronic orbit of the outermost electron is far from the nucleus, approximately, it experiences a fully screened field of all the rest of the Z – 1 electrons. That is to say, for large enough r, the effective attraction between the electron and the nucleus becomes a Coulomb potential with effective charge unity, that is, $-\kappa/r$. The closer this electron approaches toward the nucleus, the more it experiences the unscreened nuclear potential with charge Ze. The attraction between the electron and the nucleus recovers $-Z\kappa/r$ for sufficiently small r. Therefore the effective potential seen by the valance electron is no longer proportional to 1/r, and hence, the enlarged SO(4) symmetry no longer exists. Consequently, the eigenstates with the same n but different l are no longer degenerate, and the energy level becomes

$$E = -\frac{m^* Z^2 \kappa^2}{2\hbar^2 (n - \delta(n, l))^2},$$
(1.4)

where $\delta(n, l)$ is a function depending on *n* and *l* and is also called the "quantum defect" [157]. The term-diagram with the screening effect is schematized in Figure 1.1(b). Normally, the energy level with larger *l* becomes higher. This energy splitting is also of the order of electron volts, because it also originates from the Coulomb interaction.

Second, let us consider two electrons in two orbits, say, $\psi_1(\mathbf{r})$ and $\psi_2(\mathbf{r})$. Because the total wave function of two electrons has to be antisymmetric, and if these two electrons form a spin singlet, the wave function in the spin space is antisymmetric, and their spatial wave function has to be symmetric, that is, $\psi_1(\mathbf{r}_1)\psi_2(\mathbf{r}_2) + \psi_1(\mathbf{r}_2)\psi_2(\mathbf{r}_1)$. If these two electrons form a spin triplet, the wave function in the spin space is symmetric, and their spatial wave function should be antisymmetric, that is, $\psi_1(\mathbf{r}_1)\psi_2(\mathbf{r}_2) - \psi_1(\mathbf{r}_2)\psi_2(\mathbf{r}_1)$. In the latter case, the wave function vanishes when two electrons come close enough, which reduces the repulsive interaction energy. Thus, the energies of the triplet states are lower than the energy of the singlet state. In other words, the Coulomb repulsion favors the total spin *S* of electrons and to cases with more than two quantum states, which gives the early day explanation of the first Hund's rule.¹ Also, for a given *S*, the short-range repulsion is minimized when the total angular momentum *L* is maximized, which gives the second Hund's rule. The characteristic energy scale of the Hund's rules is also of the order of electron volts.

The Spin-Orbit and Hyperfine Couplings. The Hamiltonian for the spin-orbit coupling is given by

$$\hat{H}_{\rm so} = \sum_{i} \alpha_{\rm f}^{i} \hat{\mathbf{S}}_{i} \cdot \hat{\mathbf{L}}_{i}, \qquad (1.5)$$

¹ There are more advanced discussions of the origin of the first Hund's rule in later quantum chemistry calculations that we will not discuss in detail here.

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and it describes the coupling between the electronic spin $\hat{\mathbf{S}}_i$ and its orbital angular momentum $\hat{\mathbf{L}}_i$ with strength α_f^{i} , giving rise to the fine structure. The origin of the spin-orbit coupling can be intuitively understood as follows. Sitting in the rest frame of an electron, the nucleus moves around the electron. Because the nucleus is charged, the circulating motion of the nucleus gives rise to an electric current, and the strength of the current is proportional to the angular momentum of the relative motion between the electron and the nucleus. The circulating current further induces a magnetic field, which acts on the spin of electrons. This leads to the spin-orbit coupling given by Eq. 1.5.

As one can see from this picture, because this process involves the magnetic effect induced by the electric current, it is naturally weaker than the Coulomb interaction, because the latter is purely electronic. In fact, the characteristic energy scale of the spin-orbit coupling is typically of the order of 10^{-3} eV (~ 10^{11} Hz), and in many cases it is much weaker than the Hund's rule coupling originating from the Coulomb interaction. Originally, this spin-orbit coupling is between the spin and orbital angular momentum of each individual electron; however, because the Hund's rule coupling locks the electron spins of all valance electrons to an eigenstate of the total electron spin \hat{S} , and locks the angular momentum of all valance electrons to an eigenstate of the total angular momentum $\hat{\mathbf{L}}$, it is more convenient to express the leading order effect of the spin-orbit coupling in terms of \hat{S} and \hat{L} as $\alpha_f \hat{\mathbf{S}} \cdot \hat{\mathbf{L}} + \cdots$. Here the first term represents the coupling between $\hat{\mathbf{S}}$ and $\hat{\mathbf{L}}$ with strength $\alpha_{\rm f}$, which is called the LS coupling. The residual terms represented by \cdots denote the difference between the actual coupling (Eq. 1.5) and the LS coupling term. Because S and L are not really good quantum numbers for Eq. 1.5, these residual terms compete with the Hund's rule and can change the quantum number S and L. Nevertheless, $\hat{\mathbf{J}} = \hat{\mathbf{S}} + \hat{\mathbf{L}} = \sum_{i} \hat{\mathbf{J}}_{i}$ still commutes with this coupling.

The hyperfine interaction couples the electronic degrees of freedom $\hat{\mathbf{S}}$ and $\hat{\mathbf{L}}$ to the nucleus spin $\hat{\mathbf{l}}$. In general, $\hat{\mathbf{S}}$ and $\hat{\mathbf{L}}$ couple to $\hat{\mathbf{l}}$ differently. Nevertheless, the characteristic energy scale for the hyperfine coupling is of the order of 10^{-6} eV (~ 10^8-10^9 Hz), which is much smaller compared with the spin-orbit coupling. This is because the nuclear magneton is much smaller than the Bohr magneton. Since the LS coupling already locks $\hat{\mathbf{S}}$ and $\hat{\mathbf{L}}$ to an eigenstate of $\hat{\mathbf{J}}$, we express the leading order effect of the hyperfine coupling in terms of $\hat{\mathbf{J}}$ and $\hat{\mathbf{l}}$ as $\alpha_{\rm hf} \hat{\mathbf{J}} \cdot \hat{\mathbf{l}} + \cdots$, where $\alpha_{\rm hf}$ is the strength of this coupling. This gives rise to the hyperfine structure. Only with the first term, *J* is still a good quantum number, but the residual term represented by \cdots can change the quantum number *J*, which is due to $\hat{\mathbf{S}}$ and $\hat{\mathbf{L}}$ coupled to $\hat{\mathbf{l}}$ differently.

Zoo of Ultracold Atoms. So far, three classes of atoms have been cooled to quantum degeneracy in cold atom experiments. They are (1) alkali-metal atoms, including hydrogen (H), lithium (Li), sodium (Na), potassium (K), rubidium (Rb), and cesium (Cs); (2) alkaline-earth-metal (-like) atoms, including strontium (Sr), calcium (Ca), and ytterbium (Yb). In the periodic table, ytterbium does not belong to the alkaline-earth-metals, but its outer electronic structure is the same as alkaline-earth-metal atoms; and (3) atoms with large electronic magnetic moments, which are called "magnetic atoms" here. These include chromium (Cr), dysprosium (Dy), and erbium (Er). We also anticipate that more atomic

² In general, $\alpha_{\rm f}^i$ should also depend on spatial position. Here we ignore this dependence for simplicity.

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Table 1.1 The electronic structure and the nuclear spin of the alkali-metal atoms used in current experiments					
Atom	Valance electron	Label $2S+1L_J$	Nuclear spin I		
Li	$2s^{1}$	${}^{2}S_{\frac{1}{2}}$	⁷ Li $(I = 3/2, B)$; ⁶ Li $(I = 1, F)$		
Na	$3s^{1}$	${}^{2}S_{\frac{1}{2}}^{2}$	²³ Na ($I = 3/2, B$)		
К	$4s^{1}$	${}^{2}S_{\frac{1}{2}}^{2}$	40 K ($I = 4, F$);		
		2	³⁹ K ($I = 3/2$, B); ⁴¹ K ($I = 3/2$, B)		
Rb	$5s^{1}$	${}^{2}S_{\frac{1}{2}}$	⁸⁵ Rb ($I = 5/2$, B); ⁸⁷ Rb ($I = 3/2$, B)		
Cs	$6s^1$	$2S_{\frac{1}{2}}^{2}$	133 Cs ($I = 7/2$, B)		

Note: F denotes fermion, and B denotes boson. Here the symbol ${}^{2S+1}L_J$ labels the electronic structure of each atom.

species can be cooled to quantum degeneracy in the future. Here we will discuss the electronic structure and the spin structure at zero magnetic field of these three classes based on the aforementioned terms.

Alkali-Metal Atoms. So far, all atomic species in Table 1.1 have been cooled to quantum degeneracy, among which ⁸⁷Rb and ²³Na are the most-studied ultracold bosonic isotopes and ⁴⁰K and ⁶Li are the most-studied fermionic isotopes. Following are a few key points about this class of atoms:

- In the ground state, since there is only one electron in the *s*-orbital, S = 1/2, L = 0, and J = 1/2. The ground state is always labeled by ${}^{2}S_{1/2}$. There is no spin-orbit coupling in the ground state, and the atomic spin structure is determined by the hyperfine coupling. The hyperfine spin is defined as $\hat{\mathbf{F}} = \hat{\mathbf{I}} + \hat{\mathbf{J}}$, and *F* is a good quantum number for an alkali-metal atom at the zero magnetic field. For instance, for 87 Rb, I = 3/2, so the total *F* can be either 1 or 2. At zero magnetic field, the energy splitting between F = 1 states and F = 2 states is a few times 10^{9} Hz.
- For the first excited state, the valance electron is the *p*-orbital, and thus L = 1. Due to the LS coupling, the total J can be either 1/2 or 3/2. Thus the excited states are split into ²P₃/₂ and ²P₁/₁, as shown in Figure 1.2(a). Historically, this splitting was discovered in the absorption spectra of lights due to sodium atoms, and they are named as D₁ and D₂ lines. Using sodium as an example, this fine splitting is 2.1 × 10⁻³ eV (≃ 5 × 10¹¹Hz), and the splitting between the ground state ²S₁/₂ and these two states is about 2.1eV (≃ 5 × 10¹⁴Hz). Because the fine-structure splitting is much smaller compared with the excitation energy, normally both ²P₃/₂ and ²P₁/₂ should participate in the optical transition, which are key processes for trapping and manipulating alkali-metal atoms, as we shall discuss in Section 1.3 in detail. In addition, ²P₃/₂ and ²P₁/₂ are further split by the hyperfine coupling, and the hyperfine splitting is even smaller compared with the fine-structure splitting.

Alkaline-Earth-Metal (-Like) Atoms. Table 1.2 contains the alkaline-earth-metal atoms (Ca and Sr) and alkaline-earth-metal-like atom (Yb) that have been cooled to quantum

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8	A Single Atom Table 1.2 The electronic structure and the nuclear spin of alkaline-earth-metal (-like) atoms used in current experiments				
	Atom	Valance electron	Label ^{2S+1} L _J	Nuclear spin I	
	Yb	$4f^{14}6s^2$	${}^{1}S_{0}$	174 Yb ($I = 0, B$);	
	G	. 2	1 c	¹⁷¹ Yb $(I = 1/2, F)$; ¹⁷³ Yb $(I = 5/2, F)$	
	Ca Sr	$4s^2$ $5s^2$	$\frac{1}{S_0}$	¹³ Ca $(I = 0, B)$ ⁸⁴ Sr $(I = 0, B)$; ⁸⁷ Sr $(I = 9/2, F)$	

Note: F denotes fermion, and B denotes boson.



Figure 1.2

Schematic of the electronic structure: (a) an alkali-metal atom and (b) an alkaline-earth-metal atom. The dashed line in (a) denotes the excitation energy, and the dashed lines in (b) denote that these states are coupled either by the spin-orbit coupling (SO) or by the hyperfine coupling (HF) process. A color version of this figure can be found in the resources tab for this book at cambridge.org/zhai.

degeneracy. Alkaline-earth-metal atoms have several unique properties compared with the alkali-metal atoms:

- For the ground state, two electrons occupy the *s*-orbital, and therefore, the total electron spin S = 0 and the angular momentum L = 0. All bosonic isotopes of alkaline-earthmetal atoms have zero nuclear spin, and the fermionic isotopes have nonzero nuclear spin *I*, which can be very large. However, because of J = 0, and consequently, the absence of the hyperfine coupling, the nuclear spin is decoupled from the electronic spin degree of freedom. Therefore, the nuclear spin nearly does not participate in two-body interactions, and the interaction possesses an SU(N) symmetry with large-N [193]. We will discuss this in Section 2.3.
- For the first excited states, one electron still occupies the *s*-orbital, but the other electron is excited to the *p*-orbital. Thus, these states have L = 1. The electronic structure of these excited states is shown in Figure 1.2(b). First of all, because of the Hund's rule, the Coulomb energies for the S = 1 states $({}^{3}P_{J})$ are lower than that for the S = 0 state

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 $({}^{1}P_{1})$. Second, due to the LS coupling, all states within the S = 1 manifold split into J = 0, 1, and 2, denoted by ${}^{3}P_{0}, {}^{3}P_{1}, {}^{3}P_{2}$, respectively.

- As we will see in Section 1.3, because the optical transition is dominated by the dipole transition, and the dipole transition does not change the electronic spin quantum number *S*, the direct coupling between these excited states with S = 1 (${}^{3}P_{J}$) and the ground state with S = 0 is forbidden because of different quantum number *S*. That is to say, at the leading order, the dipole transition can only couple the ground state to the S = 0 excited state (${}^{1}P_{1}$) and cannot couple the ground state to the S = 1 manifold (${}^{3}P_{J}$).
- As we have mentioned above, the spin-orbit coupling term does not conserve the quantum number S and L, and it can mix two states as long as they have the same J. Thus, among the three states with S = 1, ${}^{3}P_{1}$ states can be coupled to ${}^{1}P_{1}$ states by the spin-orbit coupling. Through this coupling, there exists a small but finite dipole transition matrix element between the ${}^{3}P_{1}$ states and the ground state. This gives rise to a lifetime for ${}^{3}P_{1}$ states of about a few hundred nanoseconds. And for ${}^{3}P_{0}$ and ${}^{3}P_{2}$, because their quantum numbers J are different from that of ${}^{1}P_{1}$, they cannot be coupled to ${}^{1}P_{1}$ by the spin-orbit coupling term.
- For fermionic isotopes, the coupling between ³P₀ or ³P₂ and ¹P₁ can be induced by the hyperfine coupling. As we have discussed above, after including the hyperfine coupling, *J* is also not a good quantum number. However, the coupling mediated by the hyperfine coupling is much weaker, and hence, the lifetimes of ³P₀ and ³P₂ states are much longer than for ³P₁ states, and the lifetime can be many seconds. These long-lived electronic excited states can be used as an important tool for precision measurement. On one hand, the spontaneous emission rates of these states are so small, and on the other hand, the coupling is not completely forbidden because of these residual couplings. Taking advantage of these properties, the transition between ³P₀ and ¹S₀ induced by laser coupling can be used for the purpose of realizing the atomic optical clock. Therefore, these states are also called the "clock state." The atomic optical clock has reached an accuracy of 10⁻¹⁹s nowadays, and it is the most accurate clock we have now [25]. If one were to start to run such a clock from the beginning of the universe until now, this clock would be expected neither to gain nor to lose even one second. Such a clock can now be used to test fundamental physics [92].
- For bosonic isotopes, due to the absence of the nuclear spin, there is absolutely no onephoton dipole transition for ${}^{3}P_{0}$ and ${}^{3}P_{2}$. In this case, the coupling to ground state has to be induced by higher-order processes. The lifetime of these two states can be many years long, and for all practical purposes, these states can be viewed as not decayed.

Magnetic Atoms. Table 1.3 contains three atoms whose total angular momentum of electron J is very large. For chromium, five d-orbitals and one s-orbital are all half-filled, and thus all six electrons are spin polarized because of the first Hund's rule, which gives S = 3 and L = 0. Dysprosium and erbium are open-shell lanthanide atoms. For dysprosium, 7 f-orbitals are filled by 10 electrons, and thus there are 4 unpaired electrons. Because of the first Hund's rule, these four unpaired electrons are spin polarized, which gives rise to a total electronic spin S = 2. And because of the second Hund's rule, these four unpaired electrons give maximized angular momentum L = 6. Similarly, for erbium, 7 f-orbitals

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Table 1.3 The atomic structure of high-spin magnetic atoms like Cr and lanthanide Dy and Er						
Atom	Valance electron	Label $^{2S+1}L_J$	Nuclear spin I			
Cr	$3d^{5}4s^{1}$	⁷ S ₃	52 Cr ($I = 0, B$); 53 Cr ($I = 3/2, F$)			
Dy	$4f^{10}6s^2$	${}^{5}I_{8}$	¹⁶² Dy $(I = 0, B)$; ¹⁶³ Dy $(I = 5/2, F)$			
Er	$4f^{12}6s^2$	${}^{3}H_{6}$	168 Er ($I = 0,B$)			

Note: F denotes fermion, and B denotes boson.

are filled by 12 electrons, and thus there are 2 unpaired electrons, which gives S = 1 and a maximized angular momentum L = 5. Furthermore, it turns out that for both Dy and Er, the spin-orbit coupling favors a maximum J, that is, J = 8 for dysprosium and J = 6 for erbium. The atomic structures of these atoms also have strong effects on the interaction between these atoms:

- In the presence of a finite magnetic field, **J** can be easily polarized, which results in a magnetic moment $d = 6\mu_{\rm B}$ for chromium, $d = 10\mu_{\rm B}$ for dysprosium, and $d = 7\mu_{\rm B}$ for erbium. Therefore, the magnetic moment is about one order of magnitude larger than that of the alkali-metal atoms, and hence the magnetic dipole interaction between two atoms is two orders of magnitude larger.
- In the presence of a finite magnetic field, because the angular momentum *L* is nonzero for dysprosium and erbium, the electron cloud is anisotropic, so that the short-range Van der Waals potential is also anisotropic. This effect does not exist in chromium, whose angular momentum is zero.
- In the limit of a vanishing magnetic field, **J** becomes depolarized, and the spin rotational symmetry is restored. These atoms exhibit the aspects of high-spin particles, and their interactions depend on spin, as we will discuss in Section 4.3.

1.2 Magnetic Structure

Now we consider the effect of a static magnetic field on the atomic structure. Because electrons are charged, in principle, the electron motion inside an atom can also be affected by the presence of magnetic field. However, this effect is too small compared with the Coulomb interaction, such that we can safely ignore the change of electron orbital due to the magnetic field. We only focus on the Zeeman effect acting on the electron spin **S**, orbital angular momentum **L**, and nuclear spin **I**. The energy scale of the Zeeman splitting is comparable with the hyperfine splitting for a typical magnetic field of hundreds of Gauss in the laboratory.

Hence, here we consider an atom as a point neural particle carrying **S**, **L**, and **I**. Now let us focus on the ground state of alkali-metal atoms. For example, for ⁸⁷Rb atoms with S = 1/2, L = 0, and I = 3/2, the ground state spin structure is determined by

$$\hat{H}_{\rm s} = B(\mu_{\rm B}g_{\rm S}\hat{S}_z + \mu_{\rm N}g_{\rm I}\hat{I}_z) + \alpha_{\rm hf}\,\hat{\mathbf{J}}\cdot\hat{\mathbf{I}},\tag{1.6}$$





Schematic of the Zeeman energy structure. Here we consider the electronic ground state of a ⁸⁷Rb atom with J = 1/2 and I = 3/2. F_z labels the good quantum number of each state. A color version of this figure can be found in the resources tab for this book at cambridge.org/zhai.

where μ_B and μ_N are the Bohr magneton and the nuclear magneton, respectively, and $\mu_N \ll \mu_B$. g_S and g_I are the Landé *g*-factors. Here we first consider the situation that the magnetic field is spatially uniform and its direction is chosen as the \hat{z} direction. For this Hamiltonian, F_z is a good quantum number, and its spectrum can be solved exactly. Here we analyze the behavior in the small *B* and large *B* regimes, respectively. By smoothly connecting the small *B* and large *B* regimes, one naturally obtains the qualitative feature for the energy diagram, as shown in Figure 1.3

- In the small *B*-field regime, when $B\mu_Bg_s \ll \alpha_{hf}$, the hyperfine coupling dominates. The hyperfine coupling splits the energy between states with F = 1 and the states with F = 2. Within the three F = 1 states, or the five F = 2 states, the Zeeman field simply creates a linear Zeeman energy and a quadratic Zeeman energy proportional to F_z and F_z^2 , respectively. The reason that there exists a quadratic Zeeman effect is precisely because the hyperfine spin contains both electronic spin and nuclear spin components, and they couple to the external magnetic moment differently.
- In the large *B*-field regime, when $B\mu_Bg_S \gg \alpha_{hf}$, the Zeeman energy of electron spin dominates. The energies of four states with $S_z \approx -1/2$ decrease as *B* increases, and the energies of the other four states with $S_z \approx 1/2$ increase as *B* increases.

Magnetic Trap. In the presence of a magnetic field, the energies of some spin states increase with an increasing magnetic field strength. That is to say, if an atom is prepared in such a state, it can be trapped in the regime where the magnetic field strength has a local minimum. Atoms in these states are called the "low-field seeking" atoms. The energies of some other states decrease with an increasing magnetic field strength. These states can be trapped in the regime where the magnetic field strength. These states can be trapped in the regime where the magnetic field strength has a local maximum. Atoms in these states are called the "high-field seeking" atoms. This is the basic idea of the magnetic