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# Lectures on Contact 3-Manifolds, Holomorphic Curves and Intersection Theory

CHRIS WENDL
*Humboldt University of Berlin*

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# Preface

The main portion of this book is a lightly revised set of expository lecture notes written originally for a five-hour minicourse on the intersection theory of punctured holomorphic curves and its applications in 3-dimensional contact topology, which I gave as part of the LMS Short Course “Topology in Low Dimensions” at Durham University, August 26–30, 2013. These lectures were aimed primarily at students, and they required only a minimal background in holomorphic curve theory since the emphasis was on topological rather than analytical issues. The original appendices were relatively brief, their purpose being to provide a quick survey of analytical background material on holomorphic curves that I needed to refer to in the lectures without assuming that students already knew it. In revising the manuscript for publication, I have added a motivational introduction, and taken the opportunity to insert two further additions that I felt were lacking from the existing literature, as a result of which the appendices have become considerably more substantial. One (Appendix B) is a complete proof of local positivity of intersections, including just enough background material on elliptic regularity for a student familiar with distributions and Sobolev spaces to consider it “self-contained”; this notably includes a weak version of the Micallef–White theorem, which some readers may, I hope, find easier to comprehend than the deeper result in [MW95] that inspired it. The other (Appendix C) is a quick survey of Siefring’s intersection theory of punctured holomorphic curves, putting the essential facts and formulas in as compact a form as possible for the benefit of researchers who need a ready reference. Most of what is in Appendix C also appears in Lectures 3 and 4, but the latter are written in a more pedagogical style that develops the structure of the theory based on a few core ideas – which is presumably helpful if your goal is to understand why the main results are true, but less so if you just need to look up a specific formula, and Appendix C is there to help in that case.

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Intersection theory has played a prominent role in the study of closed symplectic 4-manifolds since Gromov's paper [Gro85] on pseudoholomorphic curves, leading to a myriad of beautiful rigidity results that are either not accessible or not true in higher dimensions. In the last 15 years, the highly nontrivial extension of this theory to the case of punctured holomorphic curves, due to Siefring [Sie08, Sie11], has led to similarly beautiful results about contact 3-manifolds and their symplectic fillings. These lectures begin with an overview of the closed case and an easy application (McDuff's characterization of symplectic ruled surfaces), and then explain the essentials of Siefring's intersection theory and how to use it in the real world. As a sample application, Lecture 5 concludes by discussing the classification of symplectic fillings of planar contact manifolds via Lefschetz fibrations [Wen10b].

**How to use these lecture notes:** I expect a variety of audiences to find these lecture notes useful for a variety of reasons. Since they were written with an audience of students in mind, I did not want to assume too much previous knowledge of symplectic/contact geometry or holomorphic curves, and most of the text reflects that. On the other hand, I also expect a certain number of readers to be experienced researchers who already know the essentials of holomorphic curve theory – including the adjunction formula in the closed case – but would specifically like to learn about the intersection theory for *punctured* curves. For readers in this category, I recommend starting with Appendix C for an overview of the basic facts, and then turning back to Lectures 3 and 4 for details whenever necessary. If, on the other hand, you are a student and still getting to know the field of symplectic and contact topology, you'll probably wish to start from the beginning.

Or if you really want to challenge yourself, feel free to read the whole thing backward.

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