

# Content

<i>List of Figures</i>	xv
<i>List of Tables</i>	xix
<i>Preface</i>	xxi
<i>Acknowledgments</i>	xxv
<b>1 Model and Analysis</b>	<b>1</b>
1.1 Computing Fibonacci Numbers	1
1.2 Fast Multiplication	3
1.3 Model of Computation	4
1.4 Randomized Algorithms: A Short Introduction	6
1.4.1 A different flavor of randomized algorithms	8
1.5 Other Computational Models	10
1.5.1 External memory model	10
1.5.2 Parallel model	11
Further Reading	12
Exercise Problems	13
<b>2 Basics of Probability and Tail Inequalities</b>	<b>16</b>
2.1 Basics of Probability Theory	16
2.2 Tail Inequalities	21
2.3 Generating Random Numbers	26
2.3.1 Generating a random variate for an arbitrary distribution	26

2.3.2 Generating random variables from a sequential file	27
2.3.3 Generating a random permutation	29
Further Reading	31
<i>Exercise Problems</i>	31
<b>3 Warm-up Problems</b>	<b>34</b>
3.1 Euclid's Algorithm for the Greatest Common Divisor (GCD)	34
3.1.1 Extended Euclid's algorithm	35
3.1.2 Application to cryptography	36
3.2 Finding the $k$ th Smallest Element	37
3.2.1 Choosing a random splitter	38
3.2.2 Median of medians	39
3.3 Sorting Words	41
3.4 Mergeable Heaps	43
3.4.1 Merging binomial heaps	44
3.5 A Simple Semi-dynamic Dictionary	45
3.5.1 Potential method and amortized analysis	46
3.6 Lower Bounds	47
Further Reading	50
<i>Exercise Problems</i>	50
<b>4 Optimization I: Brute Force and Greedy Strategy</b>	<b>54</b>
4.1 Heuristic Search Approaches	55
4.1.1 Game trees*	57
4.2 A Framework for Greedy Algorithms	60
4.2.1 Maximum spanning tree	64
4.2.2 Finding minimum weight subset	64
4.2.3 A scheduling problem	65
4.3 Efficient Data Structures for Minimum Spanning Tree Algorithms	66
4.3.1 A simple data structure for Union–Find	68
4.3.2 A faster scheme	69
4.3.3 The slowest growing function?	71
4.3.4 Putting things together	72
4.3.5 Path compression only*	73
4.4 Greedy in Different Ways	74
4.5 Compromising with Greedy	76

## Contents

ix

<b>4.6 Gradient Descent*</b>	<b>77</b>
4.6.1 Applications	83
Further Reading	87
<i>Exercise Problems</i>	88
<b>5 Optimization II: Dynamic Programming</b>	<b>92</b>
5.1 Knapsack Problem	94
5.2 Context Free Parsing	95
5.3 Longest Monotonic Subsequence	97
5.4 Function Approximation	99
5.5 Viterbi's Algorithm for Maximum Likelihood Estimation	100
5.6 Maximum Weighted Independent Set in a Tree	102
Further Reading	102
<i>Exercise Problems</i>	103
<b>6 Searching</b>	<b>109</b>
6.1 Skip-Lists – A Simple Dictionary	110
6.1.1 Construction of skip-lists	110
6.1.2 Analysis	111
6.1.3 Stronger tail estimates	113
6.2 Treaps: Randomized Search Trees	114
6.3 Universal Hashing	117
6.3.1 Existence of universal hash functions	120
6.4 Perfect Hash Function	121
6.4.1 Converting expected bound to worst case bound	122
6.5 A log log N Priority Queue*	122
Further Reading	124
<i>Exercise Problems</i>	125
<b>7 Multidimensional Searching and Geometric Algorithms</b>	<b>128</b>
7.1 Interval Trees and Range Trees	129
7.1.1 Two-dimensional range queries	131
7.2 $k-d$ Trees	132
7.3 Priority Search Trees	135
7.4 Planar Convex Hull	137
7.4.1 Jarvis march	139
7.4.2 Graham's scan	140
7.4.3 Sorting and convex hulls	141

7.5 Quickhull Algorithm	142
7.5.1 Analysis	143
7.5.2 Expected running time*	145
7.6 Point Location Using Persistent Data Structure	146
7.7 Incremental Construction	149
Further Reading	152
<i>Exercise Problems</i>	153
<b>8 String Matching and Finger Printing</b>	<b>157</b>
8.1 Rabin–Karp Fingerprinting	157
8.2 KMP Algorithm	161
8.2.1 Analysis of the KMP algorithm	165
8.2.2 Pattern analysis	165
8.3 Tries and Applications	165
Further Reading	168
<i>Exercise Problems</i>	169
<b>9 Fast Fourier Transform and Applications</b>	<b>171</b>
9.1 Polynomial Evaluation and Interpolation	171
9.1.1 Multiplying polynomials	172
9.2 Cooley–Tukey Algorithm	173
9.3 The Butterfly Network	175
9.4 Schonage and Strassen’s Fast Multiplication*	176
9.5 Generalized String Matching	179
9.5.1 Convolution based approach	180
Further Reading	182
<i>Exercise Problems</i>	182
<b>10 Graph Algorithms</b>	<b>184</b>
10.1 Depth First Search	184
10.2 Applications of DFS	188
10.2.1 Strongly connected components (SCC)	188
10.2.2 Biconnected components	191
10.3 Path Problems	193
10.3.1 Bellman–Ford SSSP algorithm	194
10.3.2 Dijkstra’s SSSP algorithm	195
10.3.3 All pair shortest paths algorithm	197

## Contents

xi

10.4 Computing Spanners for Weighted Graphs	198
10.5 Global Min-cut	201
10.5.1 The contraction algorithm	202
10.5.2 Probability of min-cut	203
Further Reading	204
<i>Exercise Problems</i>	205
<b>11 Maximum Flow and Applications</b>	<b>208</b>
11.0.1 Max-Flow Min-Cut	212
11.0.2 Ford and Fulkerson algorithm	213
11.0.3 Edmond–Karp augmentation strategy	214
11.0.4 Monotonicity lemma and bounding the number of iterations	215
11.1 Applications of Max-Flow	216
11.1.1 Disjoint paths	216
11.1.2 Bipartite matching	217
11.1.3 Circulation problems	222
11.1.4 Project planning	224
Further Reading	226
<i>Exercise Problems</i>	227
<b>12 NP Completeness and Approximation Algorithms</b>	<b>230</b>
12.1 Classes and Reducibility	233
12.2 Cook–Levin Theorem	235
12.3 Common NP-Complete Problems	237
12.4 Proving NP Completeness	240
12.4.1 Vertex cover and related problems	241
12.4.2 Three coloring problem	242
12.4.3 Knapsack and related problems	244
12.5 Other Important Complexity Classes	247
12.6 Combating Hardness with Approximation	249
12.6.1 Maximum knapsack problem	251
12.6.2 Minimum set cover	252
12.6.3 The metric TSP problem	253
12.6.4 Three coloring	253
12.6.5 Max-cut problem	254
Further Reading	254
<i>Exercise Problems</i>	255

<b>13 Dimensionality Reduction*</b>	<b>258</b>
13.1 Random Projections and the Johnson–Lindenstrauss Lemma	259
13.2 Gaussian Elimination	262
13.3 Singular Value Decomposition and Applications	264
13.3.1 Some matrix algebra and the SVD theorem	265
13.3.2 Low-rank approximations using SVD	267
13.3.3 Applications of low-rank approximations	269
13.3.4 Clustering problems	271
13.3.5 Proof of the SVD theorem	273
Further Reading	275
<i>Exercise Problems</i>	275
<b>14 Parallel Algorithms</b>	<b>277</b>
14.1 Models of Parallel Computation	277
14.2 Sorting and Comparison Problems	278
14.2.1 Finding the maximum	278
14.2.2 Sorting	282
14.3 Parallel Prefix	287
14.4 Basic Graph Algorithms	291
14.4.1 List ranking	292
14.4.2 Connected components	294
14.5 Basic Geometric Algorithms	298
14.6 Relation between Parallel Models	300
14.6.1 Routing on a mesh	301
Further Reading	303
<i>Exercise Problems</i>	304
<b>15 Memory Hierarchy and Caching</b>	<b>308</b>
15.1 Models of Memory Hierarchy	308
15.2 Transposing a Matrix	310
15.2.1 Matrix multiplication	311
15.3 Sorting in External Memory	313
15.3.1 Can we improve the algorithm?*	314
15.4 Cache Oblivious Design	316
15.4.1 Oblivious matrix transpose	317
Further Reading	320
<i>Exercise Problems</i>	321

## Contents

xiii

<b>16 Streaming Data Model</b>	<b>323</b>
16.1 Introduction	323
16.2 Finding Frequent Elements in a Stream	324
16.3 Distinct Elements in a Stream	327
16.4 Frequency Moment Problem and Applications	331
16.4.1 The median of means trick	334
16.4.2 The special case of second frequency moment	335
16.5 Proving Lower Bounds for Streaming Model	337
Further Reading	339
<i>Exercise Problems</i>	340
<b>Appendix A Recurrences and Generating Functions</b>	<b>343</b>
A.1 An Iterative Method – Summation	344
A.2 Linear Recurrence Equations	345
A.2.1 Homogeneous equations	345
A.2.2 Inhomogeneous equations	346
A.3 Generating Functions	346
A.3.1 Binomial theorem	348
A.4 Exponential Generating Functions	348
A.5 Recurrences with Two Variables	349
<i>Bibliography</i>	351
<i>Index</i>	363