# SYSTEMS OF FREQUENCY DISTRIBUTIONS FOR WATER AND ENVIRONMENTAL ENGINEERING

A multitude of processes in hydrology and environmental engineering are either random or entail random components that are characterized by random variables. These variables are described by frequency distributions. This book provides an overview of different systems of frequency distributions, their properties, and their applications to the fields of water resources and environmental engineering. A variety of systems are covered, including the Pearson system, the Burr system, and systems commonly applied in economics, such as the D'Addario, Dagum, Stoppa, and Esteban systems. The latter chapters focus on the Singh system and the frequency distributions deduced from Bessel functions, maximum entropy theory, and the transformations of random variables. The final chapter introduces the genetic theory of frequency distributions. Using real-world data, this book provides a valuable reference for researchers, graduate students, and professionals interested in frequency analysis.

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VPS: wife Anita, son Vinay, daughter Arti, daughter-in-law Sonali, son-in-law Vamsi, and grandchildren Ronin, Kayden, and Davin

LZ: husband Bret Rath and son Caelan

# Contents

# Preface

A multitude of processes in hydrometeorology, hydrology, geohydrology, hydraulics, and environmental and water resources engineering are either random or entail random components that are characterized by random variables. These variables are described by frequency distributions that encompass a broad range. In textbooks on statistical methods in hydrology and hydraulics, frequency distributions occupy a prominent place but there is seldom a discussion on where the distributions come from or how these distributions are derived. Statistical literature shows that there are different systems or families of distributions and the distributions used in hydrology and water and environmental engineering originate from one or the other of these systems. Understanding the origination of the distributions helps uncover their underlying hypotheses and may help estimate their parameters and make informed inferences. Currently, there does not appear to be a book covering these systems. This is what constituted the motivation for this book.

The subject matter of the book is divided into 12 chapters. Introducing the theme of the book in Chapter 1, the Pearson system is discussed in Chapter 2. This is the first system that was introduced almost a century and a quarter ago and can be considered as a foundational system, for the differential equation proposed for the system laid the seeds for some other systems. The Pearson system comprises 12 distributions some of which are frequently employed in environmental and water engineering. Each of these distributions is derived and estimation of their parameters is discussed.

Chapter 3 discusses the Burr system, which consists of a set of 12 distributions that exhibit different characteristics and some of these distributions are commonly used in environmental and water engineering. This system employs a hypothesis that relates the probability density function to the cumulative distribution function and its complement. Each distribution of the system is derived in the chapter and a method of parameter estimation is presented. An analogy is drawn between this system and the Pearson system.

Chapter 4 presents the D'Addario system, which is comprised of six distributions that result from the integration of a probability-generating function and a transformation function. Examples of the distributions include Pareto type I, Pareto type II, lognormal type I, lognormal type II, Amoroso, and Davis distributions. The Amoroso distribution leads to 11 special cases. These distributions are derived in the chapter.

The subject matter of Chapter 5 is the Dagum system, which consists of a set of 11 frequency distributions, some of which are commonly employed in water engineering. The system is based on a hypothesis for the elasticity of the cumulative distribution function. A set of logical-empirical postulates, including parsimony, interpretation of parameters, efficiency of parameter estimation, model flexibility, goodness of fit, ease of computation, and algebraic manipulation that should be used for deriving distributions are also discussed.

The Stoppa system is discussed in Chapter 6. This system employs the elasticity of the cumulative distribution function and a differential equation. Its special cases consist of generalized power distribution, generalized exponential distribution, generalized Pareto distribution, and different Stoppa distributions. The generalized distributions consist of several distributions as special cases. The Stoppa system is a generalized system of distributions and is closely related to the Dagum system. Also, several Burr distributions can be derived from the Stoppa or the Dagum system.

Chapter 7 deals with the Esteban system, which uses a slightly different definition of distribution elasticity. The system comprises generalized gamma distribution, generalized beta distribution of first kind, and generalized beta distribution of second kind. These distributions include as special or limiting cases a wide spectrum of frequency distributions used in hydrologic, hydraulic, environmental, and water resources engineering.

The subject matter of Chapter 8 is the Singh system, which may be considered as the generalized Burr and Stoppa system. The system employs certain hypotheses on the relation between probability density function (PDF) and cumulative distribution function (CDF), based on empirical data. A large number of distributions can be derived from this system. The chapter discusses the derivation of CDFs of these distributions.

Chapter 9 deals with frequency distributions that are derived using Bessel functions and cumulants. Beginning with a discussion of Bessel function distributions, including moments of distributions, Bessel function line, inverse Gaussian distribution, and other distributions, the chapter goes on to discuss frequency distributions by series approximations, including Chebyshev-Hermite polynomials, cumulants, series approximation of a frequency distribution with Gram-Charlier type A series, Edgeworth series with baseline Gaussian distribution, and Gram-Charlier/Edgeworth series with non-Gaussian distribution.

Chapter 10 employs the principle of maximum entropy. Entropy maximization provides a general framework for deriving any probability distribution subject to appropriate constraints. This chapter discusses this framework and derives a number of distributions that satisfy different constraints.

A wide spectrum of frequency distributions, used in hydrologic, hydraulic, environmental, and water resources engineering, are derived using transformations of some basic frequency distributions. The basic distributions that have been used are normal, logistic, beta, Laplace, and other distributions, and the transformations used are logarithmic, power, and exponential. Chapter 11 derives the distributions obtained by transformation and transformations applied to basic distributions.

The concluding Chapter 12 deals with the genetic theory of frequency distributions. Starting with the basic concept of elementary errors, the chapter discusses Charlier type A curve and Charlier type B curve. Then, it delves into the extensions that lead to different frequency distributions.

This book is meant for graduate students and faculty members who are in the fields of hydrology, hydraulics, geohydrology, water quality engineering, hydro-meteorology, environmental engineering, and water resources engineering. It can also be used as a reference book for courses on statistical methods in environmental and water engineering. It is hoped that the book will help people understand frequency distributions and their application.

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The subject matter of this book is based on the literature on mathematical statistics, econometrics, and environmental and water engineering. There are scores of researchers who have made seminal contributions in the area of frequency distributions without which the book would not have been possible. It is a pleasure to acknowledge their contributions as specifically as possible in the body of the text and any omission on our part has been entirely inadvertent and we offer our apologies in advance. We would be greatly benefitted and be obliged if the readers transmitted to us any errors, omissions, or criticisms.

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