

SYSTEMS OF FREQUENCY DISTRIBUTIONS FOR WATER AND ENVIRONMENTAL ENGINEERING

A multitude of processes in hydrology and environmental engineering are either random or entail random components that are characterized by random variables. These variables are described by frequency distributions. This book provides an overview of different systems of frequency distributions, their properties, and their applications to the fields of water resources and environmental engineering. A variety of systems are covered, including the Pearson system, the Burr system, and systems commonly applied in economics, such as the D'Addario, Dagum, Stoppa, and Esteban systems. The latter chapters focus on the Singh system and the frequency distributions deduced from Bessel functions, maximum entropy theory, and the transformations of random variables. The final chapter introduces the genetic theory of frequency distributions. Using real-world data, this book provides a valuable reference for researchers, graduate students, and professionals interested in frequency analysis.

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Vijay P. Singh , Lan Zhang
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VPS: wife Anita, son Vinay, daughter Arti, daughter-in-law Sonali, son-in-law
Vamsi, and grandchildren Ronin, Kayden, and Davin

LZ: husband Bret Rath and son Caelan

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Contents

<i>Preface</i>	<i>page</i> xiii
<i>Acknowledgments</i>	xvi
1 Introduction	1
1.1 Random Variables in Environmental and Water Engineering	1
1.1.1 Rainfall	1
1.1.2 Temperature	2
1.1.3 Frost, Fog, and Sunshine Hours	3
1.1.4 Wind	4
1.1.5 Snowfall	4
1.1.6 Runoff	4
1.1.7 Flood	4
1.1.8 Drought	5
1.1.9 Hydrogeology	6
1.1.10 Water Quality	6
1.2 Systems of Frequency Distributions	7
1.2.1 Stoppa System	8
1.2.2 Dagum System	8
1.2.3 Johnson System	8
1.2.4 General Classification	8
1.3 Need for Systems of Frequency Distributions	9
1.4 Organization of the Book	9
References	9
2 Pearson System of Frequency Distributions	11
2.1 Introduction	11
2.2 Differential Equation of Pearson System	11
2.3 Generalization of Pearson System	14
	vii

2.4	Pearson Distributions	16
2.4.1	Nonnegative Discriminant	16
2.4.2	Negative Discriminant	16
2.4.3	Pearson Type 0 Distribution	17
2.4.4	Pearson Type I Distribution	17
2.4.5	Pearson Type II Distribution	18
2.4.6	Pearson Type III Distribution	19
2.4.7	Pearson Type IV Distribution	20
2.4.8	Pearson Type V Distribution	21
2.4.9	Pearson Type VI Distribution	22
2.4.10	Pearson Type VII Distribution	22
2.4.11	Pearson Type VIII Distribution	23
2.4.12	Pearson Type IX Distribution	24
2.4.13	Pearson Type X Distribution	25
2.4.14	Pearson Type XI Distribution	25
2.4.15	Pearson Type XII Distribution	25
2.5	Graphical Representation of Shapes Based on the Relation of α_3^2 versus d and α_3^2 versus α_4	26
2.5.1	Graphical Representation of Pearson Distributions	26
2.5.2	Type I(U): $d = -0.8; \alpha_3^2 = 3$	26
2.5.3	Type III(B): $d = 0; \alpha_3^2 = 1.5$	32
2.5.4	Type IV(B): $d = 0.2, \alpha_3^2 = 1$	32
2.5.5	Type VI: $d = 0.2, \alpha_3^2 = 1.96$	33
2.6	Application	34
2.7	Conclusion	38
	References	38
3	Burr System of Frequency Distributions	40
3.1	Introduction	40
3.2	Characteristics of Probability Distribution Functions	40
3.3	Burr Hypothesis	41
3.4	Burr System of Frequency Distributions	43
3.4.1	Burr I Distribution	44
3.4.2	Burr II Distribution	44
3.4.3	Burr III Distribution	46
3.4.4	Burr IV Distribution	47
3.4.5	Burr V Distribution	49
3.4.6	Burr VI Distribution	50
3.4.7	Burr VII Distribution	52
3.4.8	Burr VIII Distribution	54

	<i>Contents</i>	ix
	3.4.9 Burr IX Distribution	55
	3.4.10 Burr X Distribution	57
	3.4.11 Burr XI Distribution	59
	3.4.12 Burr XII Distribution	61
	3.5 Parameter Estimation by Cumulative Moment Theory	62
	3.6 Application	69
	3.6.1 Peak Flow	73
	3.6.2 Annual Rainfall Amount	73
	3.6.3 Monthly Sediment Yield	77
	3.6.4 Maximum Daily Precipitation	77
	3.7 Conclusion	77
	References	81
4	D’Addario System of Frequency Distributions	82
	4.1 Introduction	82
	4.2 D’Addario System	82
	4.2.1 Pareto Type I Distribution	83
	4.2.2 Pareto Type II Distribution	85
	4.2.3 Lognormal (2-Parameter) Distribution	86
	4.2.4 Lognormal (3-Parameter) Distribution	88
	4.2.5 Davis Distribution	89
	4.2.6 Amoroso Distribution	92
	4.3 Application	105
	4.3.1 Peak Flow	108
	4.3.2 Monthly Discharge	108
	4.3.3 Deseasonalized TPN	109
	4.3.4 Daily Maximum Precipitation	111
	4.4 Conclusion	114
	References	114
5	Dagum System of Frequency Distributions	115
	5.1 Introduction	115
	5.2 Dagum System of Distributions	115
	5.3 Derivation of Frequency Distributions	117
	5.3.1 Pareto Type I Distribution	117
	5.3.2 Pareto Type II Distribution	118
	5.3.3 Pareto Type III Distribution	119
	5.3.4 Benini Distribution	121
	5.3.5 Weibull Distribution	122
	5.3.6 Log-Gompertz Distribution	123

5.3.7	Fisk Distribution	124
5.3.8	Singh-Maddala Distribution	125
5.3.9	Dagum I Distribution	126
5.3.10	Dagum II Distribution	127
5.3.11	Dagum III Distribution	129
5.4	Application	130
5.4.1	Monthly Sediment Yield	134
5.4.2	Peak Flow	136
5.4.3	Maximum Daily Precipitation	138
5.4.4	Drought (Total Flow Deficit)	139
5.5	Conclusion	140
	References	142
6	Stoppa System of Frequency Distributions	143
6.1	Introduction	143
6.2	Stoppa System of Distributions	143
6.3	Derivation of Frequency Distributions	145
6.3.1	Generalized Power Distribution (Stoppa Type I Distribution)	145
6.3.2	Generalized Pareto Type II Distribution	145
6.3.3	Generalized Exponential Distribution (Type III Distribution)	147
6.3.4	Stoppa Type IV Distribution	148
6.3.5	Stoppa Type V Distribution	149
6.3.6	Four-Parameter Generalized Pareto Distributions	150
6.4	Relation between Dagum and Stoppa Systems	151
6.5	Relations among Burr Distributions and Dagum and Stoppa Systems	153
6.6	Application	154
6.6.1	Monthly Suspended Sediment	155
6.6.2	Annual Rainfall Amount	156
6.6.3	Peak Flow	157
6.6.4	Maximum Daily Precipitation	158
6.6.5	Drought (Total Flow Deficit)	158
6.7	Conclusion	159
	References	160
7	Esteban System of Frequency Distributions	161
7.1	Introduction	161
7.2	Esteban System of Distributions	161

Contents

xi

7.2.1	Three-Parameter Gamma Distribution	162
7.2.2	Special Cases of Generalized Gamma Distribution	163
7.2.3	Generalized Beta Distribution of First Kind	169
7.2.4	Special Cases of GB1 Distribution	172
7.2.5	Generalized Beta Distribution of Second Kind	175
7.2.6	Special Cases of GB2 Distribution	176
7.3	Application	188
7.3.1	TPN	189
7.3.2	Peak Flow	190
7.3.3	Drought (Total Flow Deficit)	191
7.3.4	Annual Rainfall	191
7.4	Conclusion	193
	References	193
8	Singh System of Frequency Distributions	194
8.1	Introduction	194
8.2	Singh System of Distributions	194
8.3	Conclusion	206
	References	206
9	Systems of Frequency Distributions Using Bessel Functions and Cumulants	207
9.1	Introduction	207
9.2	Bessel Function Distributions	207
9.2.1	Moments of Bessel Function Distributions	208
9.2.2	Bessel Function Line	209
9.2.3	Inverse Gaussian Distribution	210
9.2.4	Other Distributions	211
9.3	Frequency Distributions by Series Approximation	214
9.3.1	Chebyshev (Probabilists')-Hermite Polynomials	214
9.3.2	Cumulants	215
9.3.3	Basic Concept of Approximating Frequency Distribution with Series Approximation	217
9.3.4	Gram-Charlier Type A Series	218
9.3.5	Edgeworth Series with Baseline Gaussian Distribution	219
9.3.6	Gram-Charlier/Edgeworth Series with Non-Gaussian Distribution	222
9.4	Applications	227
9.5	Conclusion	228
	References	230

xii	<i>Contents</i>	
10	Frequency Distributions by Entropy Maximization	231
10.1	Introduction	231
10.2	Entropy Maximization	231
10.3	Application	244
10.3.1	Peak Flow	244
10.3.2	Monthly Sediment Yield	245
10.4	Conclusion	246
	References	247
11	Transformations for Frequency Distributions	248
11.1	Introduction	248
11.2	Transformation to Normal Distribution	248
11.3	Transformation of Normal Distribution: The Johnson Family	250
11.4	Transformation Based on the First Law of Laplace	253
11.5	Transformation of Logistic Distribution	256
11.6	Transformation of Beta Distribution	257
11.7	Transformation of Gamma Distribution	263
11.8	Transformation of Student- <i>t</i> Distribution	265
11.9	Application	267
11.9.1	Peak Flow and Maximum Daily Precipitation	268
11.9.2	Monthly Sediment and Annual Rainfall	269
11.10	Conclusions	269
	References	272
12	Genetic Theory of Frequency	274
12.1	Basic Concept of Elementary Errors	274
12.2	General Discussion of Charlier Type A and B Curves	274
12.3	Charlier Type A Curve	275
12.4	Charlier Type B Curve	276
12.5	Extensions by Wicksell	276
	References	281
	<i>Appendix Datasets for Applications</i>	282
	<i>Index</i>	291

Preface

A multitude of processes in hydrometeorology, hydrology, geohydrology, hydraulics, and environmental and water resources engineering are either random or entail random components that are characterized by random variables. These variables are described by frequency distributions that encompass a broad range. In textbooks on statistical methods in hydrology and hydraulics, frequency distributions occupy a prominent place but there is seldom a discussion on where the distributions come from or how these distributions are derived. Statistical literature shows that there are different systems or families of distributions and the distributions used in hydrology and water and environmental engineering originate from one or the other of these systems. Understanding the origination of the distributions helps uncover their underlying hypotheses and may help estimate their parameters and make informed inferences. Currently, there does not appear to be a book covering these systems. This is what constituted the motivation for this book.

The subject matter of the book is divided into 12 chapters. Introducing the theme of the book in Chapter 1, the Pearson system is discussed in Chapter 2. This is the first system that was introduced almost a century and a quarter ago and can be considered as a foundational system, for the differential equation proposed for the system laid the seeds for some other systems. The Pearson system comprises 12 distributions some of which are frequently employed in environmental and water engineering. Each of these distributions is derived and estimation of their parameters is discussed.

Chapter 3 discusses the Burr system, which consists of a set of 12 distributions that exhibit different characteristics and some of these distributions are commonly used in environmental and water engineering. This system employs a hypothesis that relates the probability density function to the cumulative distribution function and its complement. Each distribution of the system is derived in the chapter and a method of parameter estimation is presented. An analogy is drawn between this system and the Pearson system.

Chapter 4 presents the D'Addario system, which is comprised of six distributions that result from the integration of a probability-generating function and a transformation function. Examples of the distributions include Pareto type I, Pareto type II, lognormal type I, lognormal type II, Amoroso, and Davis distributions. The Amoroso distribution leads to 11 special cases. These distributions are derived in the chapter.

The subject matter of Chapter 5 is the Dagum system, which consists of a set of 11 frequency distributions, some of which are commonly employed in water engineering. The system is based on a hypothesis for the elasticity of the cumulative distribution function. A set of logical-empirical postulates, including parsimony, interpretation of parameters, efficiency of parameter estimation, model flexibility, goodness of fit, ease of computation, and algebraic manipulation that should be used for deriving distributions are also discussed.

The Stoppa system is discussed in Chapter 6. This system employs the elasticity of the cumulative distribution function and a differential equation. Its special cases consist of generalized power distribution, generalized exponential distribution, generalized Pareto distribution, and different Stoppa distributions. The generalized distributions consist of several distributions as special cases. The Stoppa system is a generalized system of distributions and is closely related to the Dagum system. Also, several Burr distributions can be derived from the Stoppa or the Dagum system.

Chapter 7 deals with the Esteban system, which uses a slightly different definition of distribution elasticity. The system comprises generalized gamma distribution, generalized beta distribution of first kind, and generalized beta distribution of second kind. These distributions include as special or limiting cases a wide spectrum of frequency distributions used in hydrologic, hydraulic, environmental, and water resources engineering.

The subject matter of Chapter 8 is the Singh system, which may be considered as the generalized Burr and Stoppa system. The system employs certain hypotheses on the relation between probability density function (PDF) and cumulative distribution function (CDF), based on empirical data. A large number of distributions can be derived from this system. The chapter discusses the derivation of CDFs of these distributions.

Chapter 9 deals with frequency distributions that are derived using Bessel functions and cumulants. Beginning with a discussion of Bessel function distributions, including moments of distributions, Bessel function line, inverse Gaussian distribution, and other distributions, the chapter goes on to discuss frequency distributions by series approximations, including Chebyshev-Hermite polynomials, cumulants, series approximation of a frequency distribution with Gram-Charlier

Preface

xv

type A series, Edgeworth series with baseline Gaussian distribution, and Gram-Charlier/Edgeworth series with non-Gaussian distribution.

Chapter 10 employs the principle of maximum entropy. Entropy maximization provides a general framework for deriving any probability distribution subject to appropriate constraints. This chapter discusses this framework and derives a number of distributions that satisfy different constraints.

A wide spectrum of frequency distributions, used in hydrologic, hydraulic, environmental, and water resources engineering, are derived using transformations of some basic frequency distributions. The basic distributions that have been used are normal, logistic, beta, Laplace, and other distributions, and the transformations used are logarithmic, power, and exponential. Chapter 11 derives the distributions obtained by transformation and transformations applied to basic distributions.

The concluding Chapter 12 deals with the genetic theory of frequency distributions. Starting with the basic concept of elementary errors, the chapter discusses Charlier type A curve and Charlier type B curve. Then, it delves into the extensions that lead to different frequency distributions.

This book is meant for graduate students and faculty members who are in the fields of hydrology, hydraulics, geohydrology, water quality engineering, hydro-meteorology, environmental engineering, and water resources engineering. It can also be used as a reference book for courses on statistical methods in environmental and water engineering. It is hoped that the book will help people understand frequency distributions and their application.

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The subject matter of this book is based on the literature on mathematical statistics, econometrics, and environmental and water engineering. There are scores of researchers who have made seminal contributions in the area of frequency distributions without which the book would not have been possible. It is a pleasure to acknowledge their contributions as specifically as possible in the body of the text and any omission on our part has been entirely inadvertent and we offer our apologies in advance. We would be greatly benefitted and be obliged if the readers transmitted to us any errors, omissions, or criticisms.

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