Condensed Matter Field Theory

The methods of quantum field theory underpin many conceptual advances in contemporary condensed matter physics and neighboring fields. This book provides a praxis-oriented and pedagogical introduction to quantum field theory in many-particle physics, emphasizing the application of theory to real physical systems. This third edition is organized into two parts: the first half of the text presents a streamlined introduction, elevating readers to a level where they can engage with contemporary research literature from the introduction of many-body techniques and functional integration to renormalization group methods, and the second half addresses a range of advanced topics including modern aspects of gauge theory, topological and relativistic quantum matter, and condensed matter physics out of thermal equilibrium. At all stages the text seeks a balance between methodological aspects of quantum field theory and practical applications. Extended problems with worked solutions provide a bridge between formal theory and a research-oriented approach.

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Third edition

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Preface

Most students who have entered the physics Master's curriculum will have some familiarity with condensed matter physics. But what of "condensed matter *field theory*," the subject of this text? *Fields* are effective continuum degrees of freedom describing macroscopically large numbers of "atomistic" objects. Mundane examples of fields include water waves formed from the interaction of molecules or currents inside a conductor describing the collective motion of electrons. The language of fields reduces the complexity of many-particle systems to a manageable level, the natural degrees of freedom of condensed matter systems.

In condensed matter physics, we neither can, nor want to, trace the dynamics of individual atomistic constituents. Instead, we aim to understand the observable collective properties of matter, their thermal excitations, transport properties, phase behavior and transitions, etc. The art of condensed matter theory is to identify the nature and dynamics of the low-energy degrees of freedom – articulated as fields, and formulated with the framework of effective theories encapsulating universal properties of matter. This program has a long history, and it unfolded in a succession of epochs: in the 1950s and 1960s, the development of high-order perturbation theory; in the 1970s, the advent of renormalization group method; in the 1980s, the development of powerful non-perturbative methods; and, up to the present day, advances in topological field theories. These developments often paralleled, and drew inspiration from, particle physics, where quantum field theory was just as important, if from a slightly different perspective. In the course of its development, field theory has become a lingua franca, providing a unifying framework for the exploration of core concepts of condensed matter physics, as follows.

- **Universality:** A comparatively small number of "effective theories" suffices to describe the physics of myriads of different forms of matter. For example, the quantum field theory of vortices in superfluid helium films is the same as that of a plasma of dipoles. Despite different microscopic realizations, these systems fall into the same universality class.
- **Emergence:** In condensed matter physics, the conspiracy of large numbers of fundamental degrees of freedom often leads to the emergence of a smaller number of effective ones. For example, in two-dimensional electron systems subject to magnetic fields the emergent degrees of freedom may be effectively pointlike objects carrying *fractional* charge. These quasi-particles are responsible for the observable physics of the system. If one did not know their emergent nature, one might consider them as fractionally charged fermions.

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- **Broken symmetries and collective fluctuations:** States of matter often show lower symmetry than that of the underlying microscopic theory. For example, a ferromagnetic substance may be magnetized along a specific direction while its Hamiltonian is invariant under global spin rotations. Under such conditions, large collective fluctuations, representing continuous changes between states of different local symmetry, are prevalent. In the vicinity of transition points between different phases, they can induce criticality.
- **Criticality**: Fluctuation-induced phenomena are characterized uniquely by just a few dimensionless parameters known as critical exponents. A relatively small number of different critical theories suffices to explain and describe the critical scaling properties of the majority of condensed matter systems close to phase transitions. Yet, the critical theories for some of the most well-known transitions (including, for example, the integer quantum Hall transition) remain unknown, presenting open challenges to future generations of field theorists.

In this third edition, we have separated the text into two major components. In the first part, we introduce core concepts of condensed matter quantum field theory. These chapters will furnish readers with fluency in the language and methodologies of modern condensed matter theory research. No prior knowledge is assumed beyond familiarity with quantum mechanics, statistical mechanics and solid state physics at bachelor's level. We aim to introduce the subject gently, in a language that changes gradually from being prosaic in the beginning to more scientific in later chapters. The subjects covered in the first part reflect developments in condensed matter theory that took place in the second half of the last century. However, in contrast with traditional approaches, the text does not recount these advances in chronological order. Instead, it emphasizes the comparatively modern methods of functional *field integration* – the generalization of the Feynman path integral of quantum point particles to continuum degrees of freedom. We introduce this concept early on and rely on it as an organizational principle throughout the text.

The second part of the text addresses more advanced developments, most of which have come to the fore over the past 30 years. During this period, developments in quantum field theory have proceeded in concert with revolutionary progress in experiment, both in solid state physics and in the neighboring fields of ultracold atom and optical physics. For example, while previous generations of experiments in condensed matter were conducted under close to thermal equilibrium conditions, the micro-fabrication of devices has reached levels such that *nonequilibrium* phenomena can be accessed and controlled. At the same time, we are seeing the advent and impact of *topological* forms of matter, whose physical properties are governed by the mathematical principles of topological order and long-range quantum entanglement. Combined with advances in the ability to manipulate and control quantum states, these developments are beginning to open a window on computational matter, i.e., realizations of condensed matter systems capable of storing and processing quantum information. Indeed, although separate, these new developments are surprisingly

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interrelated: quantum information may be protected by principles of topology, while nonequilibrium phases of quantum matter may be characterized by principles of topological gauge theory, etc. While it is too early to say where the field may evolve in the next 30 years, concepts from condensed matter field theory will play a key role in shaping new directions of research and in exposing their common themes. The contents and style of this more advanced part of the text reflect these structures.

While the first, introductory, component of the text is arranged in a structured manner, with each new chapter building upon previous chapters, the chapters of the second part of the text can be read independently. Moreover, the writing style of the more advanced chapters is often more succinct, drawing attention to primary, and often contemporary, literature. Perhaps most importantly, a key objective of the second part of the text is to draw readers into modern areas of condensed matter research. Alongside the core material we have also included several forms of supplementary material:

- **Info** sections place methodological developments into a given context, contain details on specific applications, or simply provide auxiliary "information" that may enrich the narrative. For example, in chapter 1, an info section is used to describe the concrete realization of "vacuum fluctuations" of fields in condensed matter systems, in the context of Casimir or van der Waals forces.
- **Example** sections are used to develop general concepts. For example, the two-sphere is used as an example to illustrate the general concept of differentiable manifolds.
- **Remarks** appear as preambles of some sections. They may indicate, for example, whether a section may require knowledge of previous material; this is particularly valuable in the second part of the text, where the chapters are non-sequentially ordered or interlinked. The text also includes sections that, while important, may be safely skipped at a first reading. In such cases, the remarks section provide advice and guidance.
- In-text **exercises** (some answered,[1] and some not) provide opportunities for the reader to test their methodological understanding. Alongside these small exercises, each chapter closes with a problem set.
- **Problem** set: These problems differ from the in-text exercises both in depth and level, and are chosen to mirror as much as possible the solution of "realistic" condensed matter problems by field-theoretical methods. Their solution requires not just methodological but also conceptual thinking. Many of them reflect the narrative of research papers, some of which are of historical significance. For example, a problem of chapter 2 reviews the construction of the Kondo Hamiltonian as an illustration of the utility of second quantization. Answers are provided for all questions in the problem sets.

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[1] The reader should not be surprised to find that some of the answers to in-text questions are given ǝpᴉsdn uʍop in footnotes!

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- Lastly, four short **appendices** introduce or review background material referred to in parts of the main text. They include a review of elements of probability theory, a summary of the Fourier transform conventions used in the text, an introduction to modern concepts of differential geometry, and a concise introduction to conformal symmetry.

This third edition of the text responds in part to the changes that have taken place in the research landscape and emphasis since the first edition was published more than a decade ago. Among these changes, the first and foremost reflects revolutionary developments in topological condensed matter physics. The core chapter on topological field theory has been completely rewritten, and two accompanying chapters – one on gauge theory, and another on relativistic quantum matter – have been added. All other chapters have been substantially revised and brought up to date. In particular, we have taken this opportunity to prune material whose prominence and value to future research may have diminished. At the same time, we have eliminated many "typos" and the occasional embarrassing error, many of which have been drawn to our attention by our friends and colleagues in the community (see below)! We fear that the addition of fresh material will have introduced new errors and will do our best to correct them when notified.

Over the years, many people have contributed to this text, either through constructive remarks and insights, or by spotting typos and errors. In this context, it is a great pleasure to acknowledge with gratitude the substantial input of Sasha Abanov, Piet Brouwer, Christoph Bruder, Chung-Pin Chou, Karin Everschor, Andrej Fischer, Sven Gnutzmann, Colin Kiegel, Jian-Lin Li, Tobias Lück, Jakob Müller-Hill, Julia Meyer, Tobias Micklitz, Jan Müller, Patrick Neven, Sid Parameswaran, Achim Rosch, Max Schäfer, Matthias Sitte, Rodrigo Soto-Garrido, Natalja Strelkova, Nobuhiko Taniguchi, Franjo-Frankopan Velic, Matthias Vojta, Jan von Delft, Andrea Wolff, and Markus Zowislok. We finally thank Martina Markus for contributing hand-drawn portraits of some of the great scientists who pioneered the physics discussed in this book.

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