1

Oscillator Fundamentals

At the heart of every phase-locked loop lies an oscillator, playing a critical role in the performance that can be achieved. For this reason, we devote five chapters of this book to oscillator design. This chapter aims to build a solid foundation for general oscillator concepts before we delve into high-performance design in Chapters 3-6. We begin with basic concepts and discover how a negative-feedback system can oscillate. We then extend our view to ring and LC oscillators.

1.1 Basic Concepts

If we release a pendulum from an angle, it swings for a while and gradually comes to a stop. The "oscillation" begins because the original potential energy turns into kinetic energy as the pendulum reaches its vertical position (Fig. 1.1), allowing it to continue its trajectory to the other extreme angle (position 3), at which the

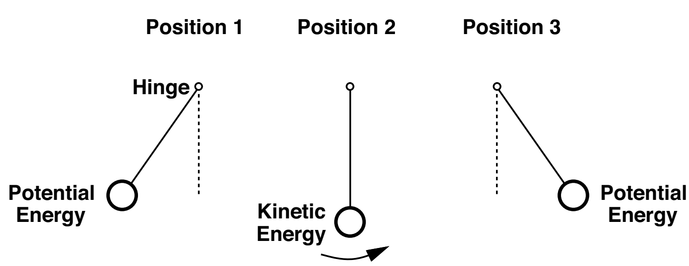

Figure 1.1 A pendulum acting as an oscillatory system.

energy is again in potential form. The oscillation stops because the friction at the hinge and the air resistance convert some of the pendulum's energy to heat in every oscillation period.

In order to sustain the oscillation, we can provide external energy to the pendulum so as to compensate for the loss caused by the hinge and the air. For example, if we give the pendulum a gentle push each time it returns to position 1, it will continue to swing. If the push is too weak, we undercompensate, allowing the oscillation to die; if the push is too strong, we overcompensate, forcing the swing amplitude to increase from one cycle to the next. We also note that the *period* of oscillation is independent of the amplitude.[1]

The above mechanical example points to several ingredients of an oscillatory system: (1) an initial "imbalance," i.e., an initial condition or packet of energy (provided by bringing the pendulum to position 1); (2) a tendency for one type of energy to turn into another and vice versa; and (3) a sustaining mechanism that replenishes the energy lost due to inevitable imperfections. Not all oscillator circuits contain all of these ingredients, but it is helpful to bear these concepts in mind.

[1] This is true only if the pendulum oscillates with a small amplitude.

Example 1.1

Repeat the foregoing experiments with a "lossless" pendulum.

Solution

If released from an angle, such a pendulum oscillates indefinitely. Now, if we give a push each time the pendulum reaches the left end, then the swing keeps increasing due to the additional energy that we inject into the system in each cycle. Note that this indefinite growth does not occur if we give a push at a frequency other than the pendulum's natural oscillation frequency.

The above example serves as a guide in our analysis: if a system has a tendency to oscillate at a frequency ω_0, then it creates a *growing* oscillatory output in response to an external injection at a frequency ω_0. From another perspective, such a system indefinitely amplifies a periodic input at this frequency.

1.2 Oscillatory Feedback System

We know from basic analog design that a negative-feedback system can become unstable. We exploit this property to construct oscillators.

Let us first study oscillation in the frequency domain. Consider the feedback system shown in Fig. 1.2(a), where the negative sign at one adder input signifies negative feedback at low frequencies. Depicted in Fig.

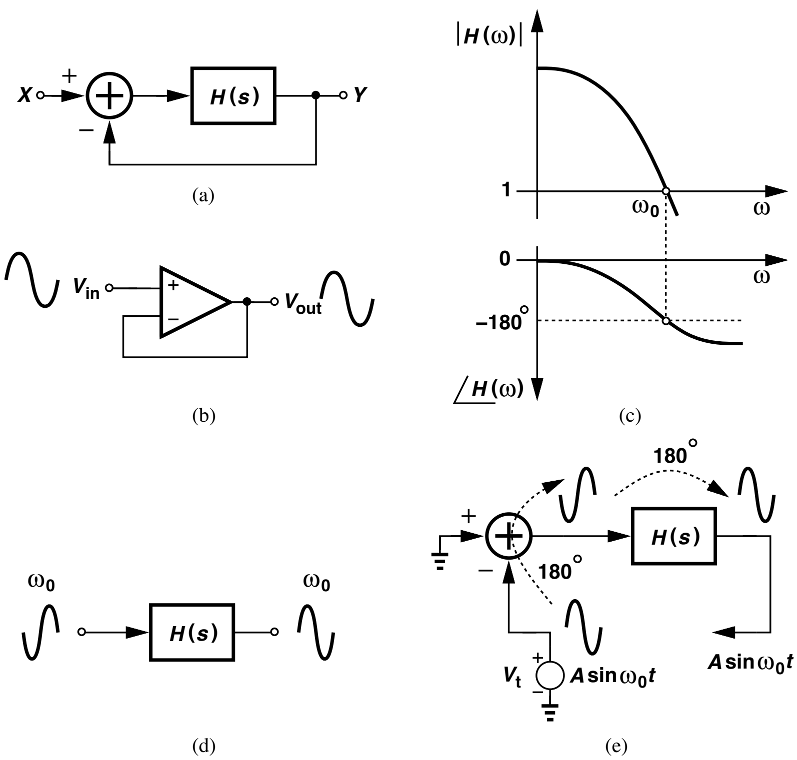

Figure 1.2 (a) Simple feedback system, (b) realization using an op amp, (c) open-loop frequency response showing zero phase margin, (d) signal inversion at ω_0, and (e) propagation of a sinusoid at ω_0 around the loop.

1.2(b) is an implementation example, which, in response to a low-frequency sinusoidal input, simply acts as a unity-gain buffer. Note that the op amp exhibits negligible phase shift at low frequencies.

How can the arrangements in Figs. 1.2(a) or (b) oscillate? Writing the closed-loop transfer function of the former as

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + H(s)}, \tag{1.1}$$

we observe that the denominator falls to zero if $H(s) = -1$ for some value of s. If $X(s)$ is a sinusoid, then $s = j\omega_0$ and we must have $H(j\omega_0) = -1$. The open-loop frequency response thus exhibits a unity magnitude and a 180° phase shift at ω_0 [Fig. 1.2(c)]. We note that $(Y/X)(j\omega_0) \to \pm\infty$, concluding that the system provides an *infinite* gain for such a sinusoid. As surmised in the previous section, this scenario suggests an oscillatory loop.

Let us examine the condition $H(j\omega_0) = -1$ more closely: this equality means that $H(s)$ itself *inverts* the input at this frequency [Fig. 1.2(d)]. That is, $H(s)$ has so much phase shift (or delay) at ω_0 that the overall feedback becomes positive. This can be seen by setting the main input, X, to zero, breaking the loop, and applying a stimulus at this frequency [Fig. 1.2(e)], and following it around the loop. The returned signal is in phase with the test voltage, V_t. We say the loop contains a 180° phase shift due to the nominally negative feedback and another frequency-dependent 180° phase shift arising from $H(s)$. These two phase shifts must not be confused with each other.

The total phase shift of 360° at ω_0 implies that the signal returns to enhance itself as it circulates around the loop. This phenomenon results in amplitude growth because the returned signal is at least as large as the starting signal, i.e., because $|H(j\omega_0)| = 1$. We therefore summarize the conditions for oscillation as

$$|H(j\omega_0)| = 1 \tag{1.2}$$

$$\angle H(j\omega_0) = 180°, \tag{1.3}$$

which are called "Barkhausen's" criteria. We also call $H(j\omega_0) = -1$ the "startup condition." Note that $H(j\omega)$ generally has a complex value at $\omega \neq \omega_0$ and becomes real only at ω_0.

The oscillation buildup can also be studied in the time domain. We begin with the arrangement shown in Fig. 1.3(a) and note that, with $H(j\omega_0) = -1$, the output is equal to the input but shifted by 180°. If the loop

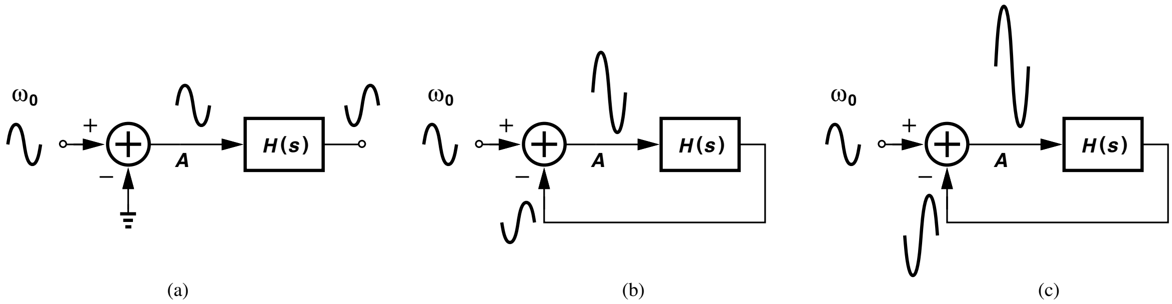

Figure 1.3 Growth of an input sinusoid around the loop with time.

is closed [Fig. 1.3(b)], the output is *subtracted* from the input, yielding a larger swing at A. This signal is again inverted and subtracted from the input, leading to indefinite grow of the amplitude [Fig. 1.3(c)].

In summary, a negative-feedback system can generate a growing periodic output in response to a sinusoidal input if its loop gain reaches -1 at a finite frequency, ω_0. But does such a system oscillate if we apply no input? Yes, the wideband noise of the devices within the loop exhibits a finite energy in the vicinity of ω_0, producing a small component that circulates around the loop and causes oscillation. For example, as shown in Fig. 1.4, a noise source, V_n, at the input of $H(s)$ yields an output given by

$$Y = V_n \frac{H(s)}{1 + H(s)}, \tag{1.4}$$

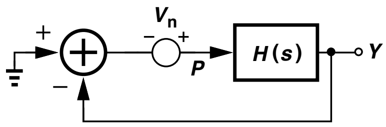

Figure 1.4 Effect of noise injected into a closed-loop system.

thereby experiencing infinite gain at $s = j\omega_0$. That is, even though V_n is infinitesimally small at ω_0, Y can assume a finite swing.

The foregoing analysis suggests that, to test for oscillation, we can inject a sinusoidal input at *any* point and observe the response at *any* point so long as both points are within the loop.[2] In Fig. 1.4, for example, V_n can be placed at the output of $H(s)$. Similarly, the point of observation can be P rather than Y. By the same token, the injection and observation points can be the same, pointing to another method of finding the oscillation conditions that is suited to some circuits. We inject a current at ω_0 into a node within the loop and examine the voltage at that node, i.e., we compute the impedance. If the voltage and hence the impedance go to infinity at ω_0, the circuit can oscillate. Figure 1.5 depicts the concept. We return to this point in Section 1.5.3.

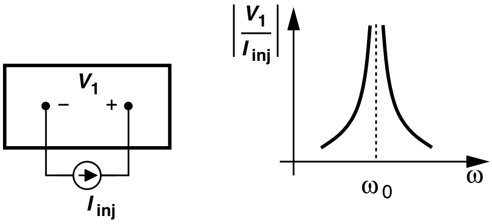

Figure 1.5 Infinite port impedance in an oscillatory circuit.

1.3 A Deeper Understanding

Analysis Methods In the analysis of oscillators, we first wish to determine how the devices and the bias conditions must be chosen so as to guarantee oscillation. Our previous studies point to three methods using the small-signal model of the circuit:

1. Open the loop and enforce the startup condition, $H(j\omega_0) = -1$, thus obtaining the circuit design requirements.
2. In analogy with the ideal pendulum example, release the closed-loop circuit with an initial condition and determine the design parameters for oscillation. The initial condition can be created by an impulse of current injected into a node within the loop or simply by assuming a finite voltage on a capacitor.
3. Inject a sinusoidal current into a node in the closed-loop circuit and compute the conditions necessary for the impedance seen at this node to go to infinity. This method is not universal but still proves helpful.

A given oscillator topology may lend itself more easily to one method than to another. The following examples illustrate these thoughts.

[2] An injection point is considered to be within the loop if the transfer function from that point to the output is not zero.

Example 1.2

A common-source (CS) stage is placed in a feedback loop as shown in Fig. 1.6(a). Can the circuit oscillate?

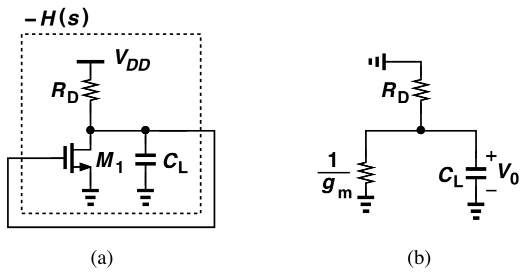

Figure 1.6 (a) Feedback around a CS stage, and (b) equivalent circuit.

Neglect other capacitances and channel-length modulation.

Solution

To retain consistency with the block diagram of Fig. 1.2(a) and noting that the CS stage inverts at low frequencies, we denote the circuit in the dashed box by $-H(s)$ (why?). Applying the first analysis method, we write

$$H(s) = g_m \left(R_D || \frac{1}{C_L s} \right) \tag{1.5}$$

$$= g_m \frac{R_D}{R_D C_L s + 1}. \tag{1.6}$$

Owing to its single pole, $H(s)$ can contribute a maximum phase of $-90°$ (at infinite frequency), disallowing $H(j\omega_0) = -1$. Thus, the loop cannot oscillate.

Let us try the second method by applying an initial condition to C_L. Since M_1 operates as a diode-connected device, the small-signal model reduces to that shown in Fig. 1.2(b), revealing that C_L simply discharges through $R_D||g_m^{-1}$ and no oscillation occurs. Similarly, the third method gives an impedance of $R_D||g_m^{-1}||(C_L s)^{-1}$ seen at the output node, indicating that it cannot go to infinity at any $s = j\omega$.

Example 1.3

The common-source stage studied in the previous example does not exhibit enough phase shift to allow oscillation. It is possible to insert an additional delay in the loop in the form of a delay line as shown in Fig. 1.7(a). Here, a voltage change at the drain takes ΔT seconds to reach the gate. Determine the startup condition and the frequency of oscillation. Neglect all capacitances and channel-length modulation.

Solution

Using our first analysis method, we break the loop as illustrated in Fig. 1.7(b) and write

$$H(s) = -\frac{V_{out}}{V_{in}} \tag{1.7}$$

$$= g_m R_D e^{-s\Delta T}. \tag{1.8}$$

(The transfer function of the ideal delay line is equal to $e^{-s\Delta T}$.) We seek $H(j\omega_0) = -1$, i.e., $g_m R_D \exp(-j\omega_0 \Delta T) = -1$. It follows that, if $|H(j\omega_0)| = 1$ and $\angle H(j\omega_0) = 180°$, then

$$g_m R_D = 1 \tag{1.9}$$

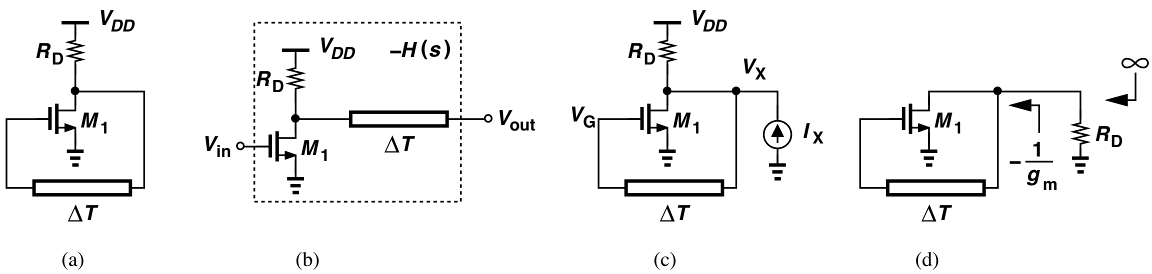

Figure 1.7 (a) CS stage with a feedback delay line, (b) open-loop system, (c) computation of impedance at one node, and (d) illustration of infinite impedance at ω_0.

$$\omega_0 \Delta T = \pi. \tag{1.10}$$

The first equation is the startup condition, and the second can be expressed as

$$f_0 = \frac{1}{2\Delta T}, \tag{1.11}$$

where $f_0 = \omega_0/(2\pi)$. The circuit therefore oscillates with a period equal to $2\Delta T$. Note that the delay line introduces a phase shift of $180°$ at ω_0.

We can apply the third method by computing the closed-loop impedance seen at, for example, the output node. From the arrangement depicted in Fig. 1.7(c), we have $V_G = V_X \exp(-s\Delta T)$ and hence a small-signal drain current of $g_m V_X \exp(-s\Delta T)$. A KCL at the output node gives

$$\frac{V_X}{R_D} + g_m V_X e^{-s\Delta T} = I_X \tag{1.12}$$

and hence

$$\frac{V_X}{I_X} = \frac{R_D}{1 + g_m R_D e^{-s\Delta T}}. \tag{1.13}$$

We observe that if the startup condition, $g_m R_D = 1$ is fulfilled, then the denominator goes to zero for $s = j\omega_0 = j2\pi/(2\Delta T)$. The reader is encouraged to apply the second analysis method as well.

To gain more insight, let us compute the output impedance of the circuit while excluding R_D. If $R_D = \infty$ in Eq. (1.13), we have

$$\frac{V_X}{I_X} = \frac{1}{g_m e^{-s\Delta T}}, \tag{1.14}$$

which reduces to $V_X/I_X = -1/g_m$ at $s = j\omega_0$. Interestingly, the loop comprising M_1 and ΔT presents a negative resistance, which cancels the "loss" due to R_D if $g_m R_D = 1$ [Fig. 1.7(d)].

Oscillation Growth It is important to distinguish between two cases when $H(j\omega_0) = -1$ and the loop is stimulated. In response to an initial condition (or an impulse), the circuit oscillates with a constant amplitude [Fig. 1.8(a)]—as did the ideal pendulum in the previous section. On the other hand, with a sinusoidal excitation at ω_0, the oscillation amplitude continues to grow [Fig. 1.8(b)] (unless some other mechanism, e.g., a nonlinearity, stops the growth).

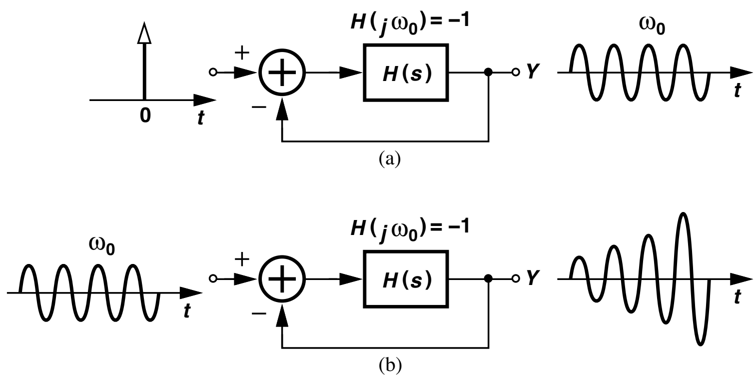

Figure 1.8 Response of an oscillatory system to (a) an impulse, and (b) a sinusoid at ω_0.

Startup Condition Revisited The condition $H(j\omega_0) = -1$ places the feedback loop at the edge of oscillation, failing to hold if process, voltage, and temperature (PVT) variations cause a slight drop in the loop gain. Moreover, this condition prohibits large-signal oscillations: if the oscillation amplitude grows to the extent that the circuit becomes nonlinear, the loop gain may drop below unity, violating the startup condition. The following example illustrates this point.

Example 1.4

Figure 1.9(a) shows a differential realization of the oscillator studied in Example 1.3. If $g_m R_D = 1$, explain

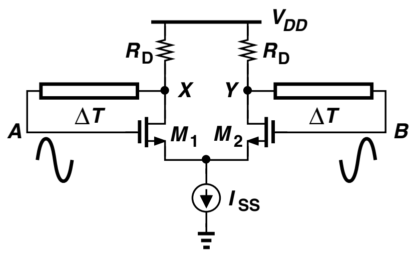

Figure 1.9 Differential pair with feedback delay lines.

why the oscillation amplitude remains small.

Solution

The circuit operates such that V_X and V_Y swing differentially and so do V_A and V_B. If V_A and V_B have small swings, the differential pair exhibits a unity voltage gain, sustaining the oscillation. The circuit cannot operate with large swings because the gain would then drop below unity at the peaks of V_A and V_B.

For the two reasons mentioned above, namely, PVT variations and gain drop due to nonlinearity, oscillators are typically designed with $|H(j\omega_0| > 1$ [and $\angle H(j\omega_0) = 180°$].

Example 1.5

The differential oscillator of Fig. 1.9 is redesigned for voltage swings that are large enough to ensure I_{SS} is entirely steered to the left or to the right. Determine the small-signal loop gain.

Solution

Suppose the four nodes carry a peak-to-peak swing of V_0. For M_1 or M_2 to carry all of I_{SS}, we have $|V_A - V_B|_{max} = V_0 = \sqrt{2}(V_{GS} - V_{TH})$ [1], where $V_{GS} - V_{TH}$ denotes the transistors' overdrive voltage in equilibrium (when $V_A = V_B$). With complete switching of the differential pair, we also have $|V_X - V_Y|_{max} = I_{SS}R_D = |V_A - V_B|_{max}$. It follows that

$$\sqrt{2}(V_{GS} - V_{TH}) = I_{SS}R_D \tag{1.15}$$

and hence

$$g_m R_D = \sqrt{2} \tag{1.16}$$

because $g_m = I_{SS}/(V_{GS} - V_{TH})$ in equilibrium (where $I_{D1} = I_{D2} = I_{SS}/2$).

With $|H(j\omega_0)| > 1$ and $\angle H(j\omega_0) = 180°$ [Fig. 1.10(a)], we face an interesting puzzle. If, for example,

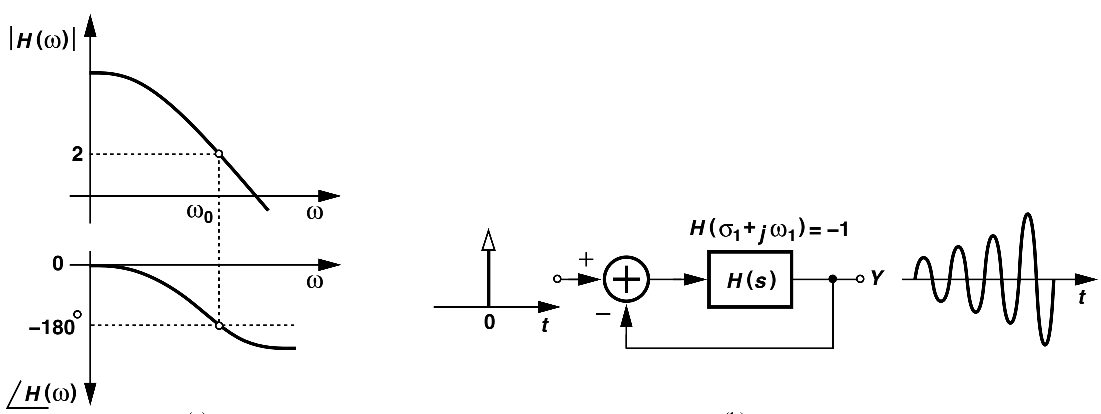

Figure 1.10 (a) Open-loop response with gain greater than unity at ω_0, and (b) impulse response of closed-loop system.

$H(j\omega_0) = -2$, then the closed-loop gain is equal to $H(j\omega_0)/[1 + H(j\omega_0)] = +2$ and not infinity! How then does the circuit oscillate? This result shows that the loop does not oscillate at ω_0. Rather, the circuit may find another value of s such that $H(s)/[1 + H(s)] \to \infty$, i.e., $H(s_1) = -1$, where s_1 has a *complex* value, $\sigma_1 + j\omega_1$, and $\sigma_1 > 0$. We study this case in Appendix I but should mention here that such a value of s leads to a growing sinusoid even with an impulse input [Fig. 1.10(b)], a point of contrast to the situation depicted in Fig. 1.8(a). [As explained in Appendix I, the condition $|H(j\omega_0)| > 1$ does not always guarantee oscillation, but suffices for typical oscillators.]

In the case of $|H(j\omega_0)| > 1$ and $\angle H(j\omega_0) = 180°$, our node impedance test must also be revisited. For example, if $g_m R_D > 1$ in Eq. (1.13), then $|V_X/I_X|$ becomes *real* and *negative* at $s = j\omega_0$ (why?). In particular, the output resistance seen in Fig. 1.7(d) is now "stronger" than $-R_D$, leaving a residual negative component after canceling R_D. This component thus allows the oscillation amplitude to grow.

Positive Feedback at dc We have seen that positive feedback at a finite frequency, ω_0, can cause oscillation. But what happens if we have positive feedback at dc (low frequencies) as well? For example, if we cascade two CS stages as shown in Fig. 1.11 to obtain a greater phase shift, the feedback becomes positive at dc. If the low-frequency loop gain is greater than unity, this circuit latches up. This can be seen by assuming a small upward perturbation in V_X, which leads to a greater, downward change in V_Y. This change in turn causes even a larger upward change in V_X, etc. We say the circuit "regenerates" until V_X reaches V_{DD} and V_Y falls to a low value, turning M_1 off. This circuit is in fact used as a memory cell.

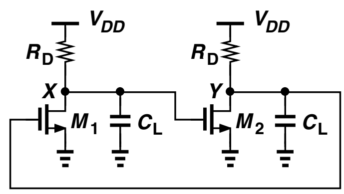

Figure 1.11 Two CS stages in a feedback loop.

To avoid latch-up, we design oscillators such that, at dc, the feedback is negative or, if it is positive, the loop gain is well below unity.

Oscillator Topologies Numerous oscillator topologies have been introduced over the years. Examples include "phase-shift," "Wien bridge," "relaxation," "multivibrator," "ring," and LC oscillators. In this book, we deal with primarily the last two as they are most commonly used in integrated circuit design.

1.4 Basic Ring Oscillators

Ring oscillators are popular in today's phase-locked system for their design flexibility and wide frequency tuning range. This section builds the foundation for these oscillators and Chapters 3 and 4 introduce advanced ring concepts.

We have seen that a single common-source stage does not provide sufficient phase shift to allow $\angle H(j\omega_0) = -180°$. Even a loop employing two CS stages fails to oscillate because $\angle H(j\omega)$ reaches $-180°$ only at $\omega = \infty$. We therefore surmise that a three-stage "ring" can satisfy both $\angle H(j\omega_0) = -180°$ and $|H(j\omega_0)| = 1$. Depicted in Fig. 1.12(a), this simple ring oscillator has negative feedback at low frequencies

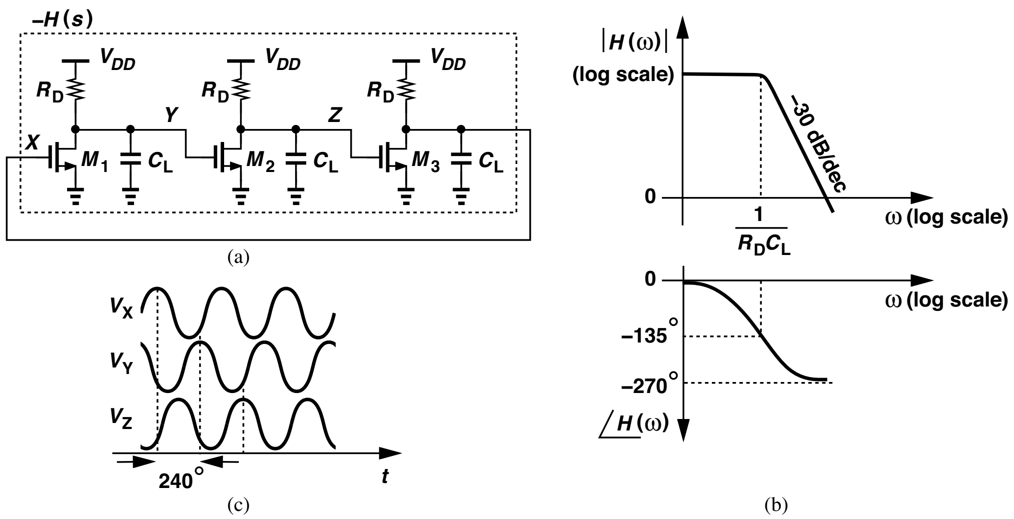

Figure 1.12 (a) Three CS stages in a feedback loop, (b) loop transmission, and (c) node waveforms.

and can be analyzed by assuming identical stages. The circuit in the dashed box is called $-H(s)$ to comply with the negative-feedback system shown in Fig. 1.2(a). We write

$$-H(s) = \left[-g_m\left(R_D||\frac{1}{C_L s}\right)\right]^3, \tag{1.17}$$

where channel-length modulation and other capacitances are neglected. It follows that

$$H(j\omega) = \frac{g_m^3 R_D^3}{(R_D C_L j\omega + 1)^3}. \tag{1.18}$$

Figure 1.12(b) sketches the magnitude and phase behavior. For $|H(j\omega_0)| = 1$, we have

$$\left(\frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega_0^2 + 1}}\right)^3 = 1 \tag{1.19}$$

and hence

$$\omega_0 = \frac{\sqrt{g_m^2 R_D^2 - 1}}{R_D C_L}. \tag{1.20}$$

Also, $\angle H(j\omega_0) = -180°$ yields

$$\tan^{-1}(R_D C_L \omega_0) = 60° \tag{1.21}$$

and

$$\omega_0 = \frac{\sqrt{3}}{R_D C_L}. \tag{1.22}$$

Interestingly, (1.20) and (1.22) give

$$g_m R_D = 2. \tag{1.23}$$

In other words, each stage must provide a low-frequency voltage gain of 2 to guarantee oscillation.

A few properties of the above oscillator are worth noting. First, at ω_0, each stage exhibits a phase shift of 60° arising from its output pole plus 180° due to the low-frequency inversion of a CS amplifier. Thus, the waveforms at X, Y, and Z in Fig. 1.12(a) have a phase separation of 240° (= −120°) [Fig. 1.12(c)]. Second, C_L can represent all of the transistor capacitances with reasonable accuracy. For example, at Y, C_L includes C_{GS2}, C_{DB1}, and the Miller effect of C_{GD2}. Since the waveforms in Fig. 1.12(c) suggest equal swings at the three nodes, we can assume a large-signal voltage gain of −1 from Y to Z and write the Miller capacitance as $C_{mill} = [1 - (-1)]C_{GD2} = 2C_{GD2}$.[3]

Example 1.6

A ring oscillator similar to Fig. 1.12(a) incorporates N identical CS stages, where N is an odd number. Determine the startup condition and the frequency of oscillation.

Solution

With an odd number of stages, the loop provides negative feedback at low frequencies, necessitating a frequency-dependent phase shift of $-180°/N$ per stage for oscillation. That is,

$$-\frac{180°}{N} = -\tan^{-1}(R_D C_L \omega_0) \tag{1.24}$$

and hence

$$\omega_0 = \frac{1}{R_D C_L} \tan\frac{180°}{N}. \tag{1.25}$$

[3] This is an approximation because the phase shift from Y to Z is equal to −120° rather than −180°.