

Introduction to the Standard Model and Beyond

The Standard Model of particle physics is an amazingly successful theory describing the fundamental particles and forces of nature. This text, written for a two-semester graduate course on the Standard Model, develops a practical understanding of the theoretical concepts it's built upon, to prepare students to enter research. The author takes a historical approach to demonstrate to students the process of discovery which is often overlooked in other textbooks, presenting quantum field theory and symmetries as the necessary tools for describing and understanding the Standard Model. He develops these tools using a basic understanding of quantum mechanics and classical field theory, such as Maxwell's electrodynamics, before discussing the important role that Noether's theorem and conserved charges play in the theory. Worked examples feature throughout the text, while homework exercises are included for the first five parts, with solutions available online for instructors. Inspired by the author's own teaching experience, suggestions for independent research topics have been provided for the second half of the course, which students can then present to the rest of the class.

Stuart Raby is a professor of physics at The Ohio State University. He is among the original proponents of the supersymmetric extension of the Standard Model and a pioneer of supersymmetric grand unified theories. His work focuses on the experimental consequences of physics beyond the Standard Model, ranging from collider experiments to proton decay searches, dark matter candidates and questions in cosmology. His first book, *Supersymmetric Grand Unified Theories*, was published by Springer in 2017.

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Introduction to the Standard Model and Beyond

Quantum Field Theory, Symmetries and Phenomenology

STUART RABY

The Ohio State University



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Preface

The goal of the book is to develop a practical understanding of the theoretical concepts inherent in the Standard Model and to enable students to enter a research program in this exciting field. Note that the topics discussed in this book are also useful for students with interests in nuclear and astro/particle physics.

This book bears a strong resemblance to similar texts. However, it provides an introduction to field theory and the Standard Model which I believe may be more accessible to both experimentalists and theorists. Also, by following the development of the theory historically, it may allow students to understand the process of discovery, which seems to be missing in more recent texts. The development of the Standard Model relied heavily on symmetries, which can be both continuous and discrete, global and local, softly broken and exact. Therefore the book focuses on an understanding of symmetries in particle physics; their consequences and tests thereof.

This book is designed as a text for a one-year course on the Standard Model. The prerequisites include advanced courses on classical mechanics, electrodynamics and quantum mechanics. However, it does not require quantum field theory as a prerequisite. Homework problems and solutions are included in the book and have been used in the first half of the course. However, in the latter half of the course we have found that, firstly, there are too many directions in particle physics for which there is insufficient time to discuss in class, and, secondly, students have relished the independent research they do in discovering the details of some of these directions on their own. Thus, I provide a list of possible topics for independent research at the end of the book. Students typically present two research papers and lectures on these topics in class. In developing the material in the book we acknowledge some texts which have had a significant influence on our presentation. These include *Elementary Particle Physics* (Wiley, 1966) by S. Gasiorowicz, *Particle Physics and Introduction to Field Theory* (CRC Press, 1981) by T. D. Lee and *Lie Algebras in Particle Physics*, 2nd edition (CRC Press, 2018) by H. Georgi.

Note, the material in the book is presented step by step: introducing subjects as foundations which are then built upon and used over and over. For students who have already had a field theory course, the material in Part II may be encouraged as a refresher. Chapters 8, 9 and 10, in particular, on the symmetries of complex scalars and fermions, should be emphasized. Then instruction might cover the material in Parts III–X.

What is Particle Physics?

The 20th century witnessed the most amazing discovery of what we now call the Standard Model of Particle Physics. The Standard Model describes the results of hundreds of thousands of experiments, with of order 28 arbitrary parameters that are fit by experiment. Particle physics can be defined as humankind's search for an understanding of the basic building blocks of matter and their interactions. As such it began with the earliest musing about space, time and atoms by Aristotle and Democritus, with the study of gravitational phenomena by Galileo and Newton and the study of electric and magnetic phenomena by the likes of Coulomb, Ampere, Faraday and Maxwell. By the end of the 19th century, physicists were wondering whether all of nature was understood and, according to Maxwell in his Introductory Lecture on Experimental Physics, held at Cambridge in October 1871,

... the opinion seems to have got abroad, that in a few years all the great physical constants will have been approximately estimated, and that the only occupation which will then be left to men of science will be to carry on these measurements to another place of decimals. ... But we have no right to think thus of the unsearchable riches of creation, or of the untried fertility of those fresh minds into which these riches will continue to be poured. ... But the history of science shews that even during the phase of her progress in which she devotes herself to improving the accuracy of the numerical measurement of quantities with which she has long been familiar, she is preparing the materials for the subjugation of the new regions, which would have remained unknown if she had been contented with the rough methods of her early pioneers. I might bring forward instances gathered from every branch of science, shewing how the labour of careful measurement has been rewarded by the discovery of new fields of research, and by the development of new scientific ideas.

Soon after Maxwell made these comments, a period of highly significant scientific breakthroughs began with the discovery of radio waves by Hertz (1886–1889), followed quickly by those X-rays by Roentgen (1895) and nuclear radiation by Becquerel (1896). It was these discoveries which put into motion the dramatic events of the 20th century.

Beginning with the discovery of the electron by Thomson in 1897 and the seminal work of Rutherford, Geiger and Marsden in 1911, physicists have investigated the atom using particle beams (electrons, protons and alpha particles) as probes. They developed new detection methods: the geiger counter, scintillators, cloud and then bubble chambers. This new paradigm for probing matter and new detectors led to many discoveries.

The mystery of the atom was unraveled. With Bohr's quantum theory and Pauli's exclusion principle, a complete theory of the periodic table of the elements was developed, leading to the fields of physical chemistry and, eventually, microbiology.

The nucleus was shattered and the strong interactions, holding protons and neutrons inside the core, were investigated. Hence the beginning of nuclear physics.

These new detectors were then pointed to the heavens, unveiling a shower of energetic charged particles emanating from distant galaxies. New particles, such as the muon, the pion and the positron (also known as an anti-electron) were

discovered in the cosmic ray shower. You know, it would take a lead umbrella to stop this eternal shower.

By the late 1950s, a major step forward came with the advent of particle accelerators at Brookhaven in New York and CERN in Geneva. With controlled particle beams (made of strongly interacting particles, such as pions, kaons and protons), experiments revealed a virtual particle zoo, with many more species of strongly interacting particles – all called hadrons. It became evident that the hadronic zoo was immense and it was unlikely that hadrons were elementary or fundamental particles at all.

To make a long story short, by 1974, the chaos of discovery led to the Standard Model describing all observed particle phenomena in terms of three fundamental forces (four, including gravity) and the fundamental building blocks of matter, i.e. quarks and leptons.

Only now, after the dust of this chaotic discovery settles, are we able with hindsight to recognize the underlying principles which define the theory we call the Standard Model. It is these principles, and their logical extension, which I will attempt to describe in this course. Particle physics has relied on amazing new methods of experiment to explore nature. Moreover, it has relied on *symmetries* and their consequences to describe nature. It is this hand-in-hand method which has propelled us into the present state of our understanding. Once again, we are at a moment in history where the role of scientists appears to be just to measure the fundamental constants to the next decimal point. But like Maxwell, more than 100 years ago, we can expect that new experimental techniques and measurements will lead to the next epoch of discovery.

Acknowledgements

This book is based on a Standard Model course that I have given at Ohio State University over the course of some 30 years. The material presented here has greatly benefited from the many graduate students who have taken the course and asked many tough questions. I would not have been able to write this book without their help. I also want to thank Professor Leonard Susskind for his strong support which kept me in the field. Finally, I want to thank my wife, Michele, whose guidance over these 50 years has always kept me grounded.