Mathematics and Its Logics

In these essays Geoffrey Hellman presents a strong case for a healthy pluralism in mathematics and its logics, supporting peaceful coexistence despite what appear to be contradictions between different systems, and positing different frameworks serving different legitimate purposes. The essays refine and extend Hellman's modal-structuralist account of mathematics, developing a height-potentialist view of higher set theory which recognizes indefinite extendability of models and stages at which sets occur. In the first of three new essays written for this volume, Hellman shows how extendability can be deployed to derive the axiom of Infinity and that of Replacement, improving on earlier accounts; he also shows how extendability leads to attractive, novel resolutions of the set-theoretic paradoxes. Other essays explore advantages and limitations of restrictive systems – nominalist, predicativist, and constructivist. Also included are two essays, with Solomon Feferman, on predicative foundations of arithmetic.

GEOFFREY HELLMAN is Professor of Philosophy at the University of Minnesota, Twin Cities. His publications include *Mathematics without Numbers: Towards a Modal-Structural Interpretation* (Oxford, 1989), *Varieties of Continua: From Regions to Points and Back* (with Stewart Shapiro, Oxford, 2018), and *Mathematical Structuralism*, Cambridge Elements in Philosophy of Mathematics (with Stewart Shapiro, Cambridge, 2018).

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Geoffrey Hellman
University of Minnesota

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1 "Structuralism without structures," *Philosophia Mathematica* **4**(2) (1996): 100–123.
2 "What is categorical structuralism?," in van Benthem, J., Heinzmann, G., Rebuschi, M., and Visser, H., eds., *The Age of Alternative Logics: Assessing Philosophy of Logic and Mathematics Today* (Dordrecht: Springer, 2006), pp. 151–161.
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