

Part I

Modelling

1 • *What Is a Model?*

If one put this question to travellers walking through Leipzig Main Station it's likely that quite a few of them would mention the names of some popular fashion models. But some others would probably point to a glass case in the middle of the hall which contains a small landscape with plastic houses, cars, people and trains running around in circuits.

While installed to entertain people, the glass case may help to give us a first answer to the question 'what is a model?' The items in the case are obviously meant to represent a socio-technical system that can be found in the real world. The model railway is smaller than the real-world system and also lacks various details, such as the smoke that would come out of a real steam engine, and the human agents in the landscape seem to be frozen in their activities. Thus, the model railway abstracts from a number of details present in the real world. However, it is easy for the spectator to imagine the smoke of the engines and understand what the human agents would be doing if they could move. So the level of abstraction is obviously well chosen: if there were much more detail and less abstraction, the construction of the model would have been too costly and the model too difficult to run; while if there were much more abstraction, running and watching the model would probably be boring and the model would not fulfil its purpose of entertaining people.

The model railway includes several of the issues and features that characterise a model after Baumgärtner et al. (2008, p. 389): namely, that a model is designed to serve a certain purpose and that it is an abstract representation of a real system. To capture the remaining characteristics of such a model one might leave Leipzig Main Station and walk to the nearby tram stop. Several maps are installed there to guide locals and tourists. One of these maps looks similar to Fig. 1.1, showing streets, buildings and other features of Leipzig's city centre.

Like the model railway, the map contains an appropriate level of abstraction to fulfil a given purpose: transmitting information about the spatial structure of Leipzig's city centre from the producer of the map to

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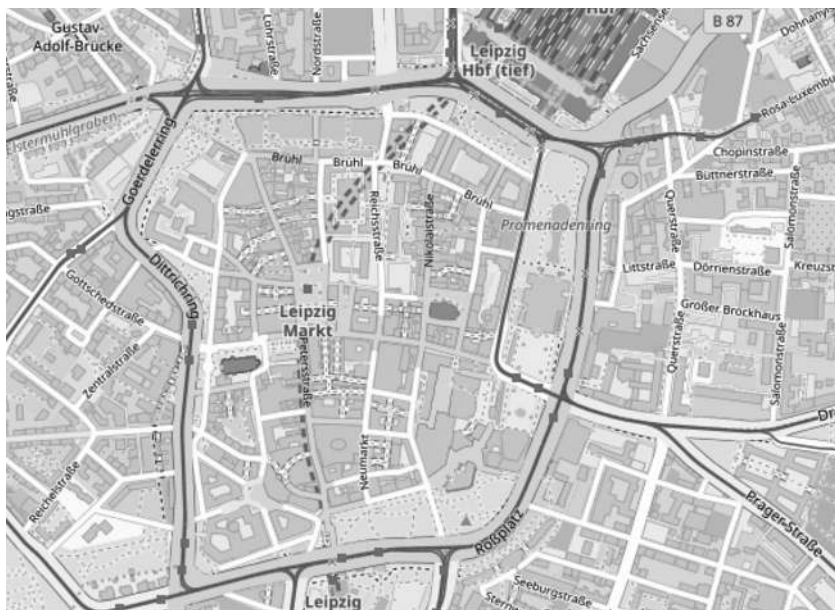


Figure 1.1 Map of the city centre of Leipzig. Source: OpenStreetMap (URL: www.openstreetmap.org/#map=15/51.3418/12.3787; last access 4 February 2019)

its users. In order to approach Baumgärtner et al.'s final model characteristic, one may note that the map in Fig. 1.1 is probably readable only for a person who has been in a city before. Only a person with this experience will understand that the linear structures on the map are streets while the closed shapes depicted in grey represent buildings. In contrast, for someone who has lived their entire life in a remote place without any contact with the Western world, this will be much less obvious. This demonstrates that a model (in the present case a map) can only be understood if the user of the model has an understanding of the construction of the world (in the present case cities) similar to that of the developer of the model. This brings us to the final characteristic of a model: that it 'is based on the concepts within a ... community's basic construction of the world' (Baumgärtner et al. 2008, p. 389).

While most people on earth share the concepts that are required to read a map, concepts may strongly differ among different communities when scientific models are considered. Ecological models are often based on the ecological concept of a population, which in turn is based on the concepts of individuals and species, since a population by definition can be formed only by individuals of the same species. In economics, the

concept of a market that mediates demand and supply for goods and services is a central one.

Concepts further differ among the disciplines with regard to the quantities characterising the state of a system. Physics and economics often employ the concept of equilibrium. A system is in equilibrium when all forces or processes cancel each other and there is no change in any component of the system. In economics, such an equilibrium may be reached in a market when demand and supply for a good are equal, so that the quantity of the good and its price are constant. For a certain time, the concept of equilibrium was popular in ecology, too. However, it soon came to be regarded as insufficient, since ecological systems are usually not in equilibrium. Instead, stability concepts such as persistence (how long does a system persist within a certain state?) and resilience (how difficult is it to push the system away from a certain state to another state through some external driving force, and how long does it take for the system to resume its original state after the driving force has been turned off?) have been developed and are more appropriate to characterise the state and dynamics of an ecological system (Grimm and Wissel 1997). Over the past 100 years or so, these other stability concepts have become accepted in physics, too, and more recently also in economics (Perrings 2006).

Altogether, the concepts within a scientific community's basic construction of the world differ between disciplines and partly overlap. When building interdisciplinary models such as ecological-economic models, it is necessary to take this issue into account carefully. To summarise and conclude this chapter, I now combine the characteristics of a model that have already been introduced and quote Baumgärtner et al. (2008, p. 389):

A model is an abstract representation of a system under study, explicitly constructed for a certain purpose, and based on the concepts within a scientific community's basic construction of the world that are considered relevant for the purpose.

2 • *Purposes of Modelling*

Chapter 1 emphasised the important role of purpose in the adequate design of a model. It concluded that the clear formulation of purpose is an important step before and during the development of a model.

Model purposes are manifold. Baumgärtner et al. (2008) distinguish nine different purposes: theory development, generalisation, theory testing, understanding, explanation, prediction, decision support, communication and teaching. I will add two further purposes: integration of knowledge and mediation between scales. Lastly, models may be distinguished by being general or specific and by being used in positive or in normative analysis. I will introduce each of these purposes using examples from physics, ecology and economics.

2.1 Theory Development

The following three sections address the relationship between models and theories. The topics of these three sections – theory development, generalisation and theory testing – are interrelated. In the development of a theory it is important to consider that the theory needs to be able to generalise specific observations into a coherent framework and that it can be tested against real-world observations, because only then will it be of value.

Models can form an intermediate element between an abstract theory and the observable world, or as Morrison and Morgan (1999a) put it, models act as mediators between theory and the world. This is possible because they contain elements from both sides (Morrison and Morgan 1999b). Using a term from computer science, I would add that a model can serve as an instance, that is, a realisation, of the theory in the observable world.

To explain these definitions of a model I will use a theory from the realm of physics. Before discussing the theory, however, I want to illustrate it by reference to a social phenomenon that can be observed

in some cities: that people form distinct neighbourhoods or groups with strong similarities within groups but strong dissimilarities between groups. The first and probably most famous model analysis that addressed this issue was by Schelling (1969), who wondered why many cities in the United States consist of distinct black and white neighbourhoods. Schelling hypothesised that this could be due to people's preferences to be surrounded by their like, and that people who are currently in the minority within their neighbourhood would move to another neighbourhood in which their colour is the majority. If we code the colours black and white into two numbers, such as +1 and −1, and assume that a person's 'happiness' increases by an amount g for each neighbour of the same colour and decreases by g for each neighbour of a different colour, we can write the happiness h of citizen i with colour s_i ($s_i \in \{-1, +1\}$) as

$$h_i(s_i) = g \sum_{j \in J_i} s_i s_j, \quad (2.1)$$

where index j applies to everyone j in the neighbourhood J_i . Each match ($s_i = s_j = +1$ or $s_i = s_j = -1$) increases $h_i(s_i)$ by an amount g and each mismatch ($s_i \neq s_j$) reduces it by g . The 'total happiness' in society may thus be calculated as the sum of the individual happiness of all:

$$H = \sum_i h_i(s_i). \quad (2.2)$$

Eqs (2.1) and (2.2) are in fact much older than Schelling's work and were first formulated by the physicist Ernst Ising (1900–98) to model the phenomenon of ferromagnetism. Iron and various other metals are so-called ferromagnets characterised by their ability to assume two different phases or states, a magnetic and a non-magnetic phase. The two phases are separated by a critical temperature T_c , so that at temperatures T below T_c the ferromagnet is magnetic and above it is not. The Ising model represents the ferromagnet by a (usually square) grid in D dimensions where on each grid point i a spin s_i is situated (Chaikin and Lubensky 1995; Hohenberg and Krekhov 2015). This spin can be imagined as a small elementary magnet (like the needle of a tiny compass). While in a real ferromagnet a spin can assume any direction, a spin in the Ising model can, for simplicity, assume only one of two possible directions: up or down, or mathematically more conveniently: $s_i = +1$ or $s_i = -1$.

In a ferromagnet, neighbouring spins try to point in the same direction, so neighbouring s_i tend to be either both +1 or both −1. To model

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this, Ising formulated the so-called Hamiltonian of the system which represents the system's energy, as

$$H = -g \sum_{\langle i,j \rangle} s_i s_j, \quad (2.3)$$

where the sum runs over all pairs of neighbouring spins (in a two-dimensional grid there would be four neighbours [north, south, east, west], while in a three-dimensional grid each spin would have six neighbours). Parameter g is a positive constant measuring the strength of the interaction between neighbouring spins. Due to the positivity of g , quantity H is smaller when two neighbouring spins have the same sign (both $+1$ or both -1) than when they have opposite signs (one $+1$ and the other one -1).

Using the mathematical technique of functional integration, and ignoring fluctuations (see, e.g., Chaikin and Lubensky 1995; Hohenberg and Krehov 2015), the so-called free energy of the system with volume Ω can be deduced to

$$F = \Omega [a\Psi^2 + b\Psi^4] \quad (2.4)$$

with parameter

$$a = \frac{D}{T} (T - 2gD), \quad (2.5)$$

where D is the number of dimensions of the grid and T is temperature (the meaning of parameter $b > 0$ is irrelevant in the present context). Quantity Ψ is the level of magnetisation in the system (Negele and Orland 1988). The free energy is defined as the energy that can be converted into work at a constant temperature and volume, and physical systems attempt to assume a state that minimises their free energy. For $a > 0$ (cf. Fig. 2.1, solid line) the free energy after Eq. (2.4) has only a single minimum located at $\Psi = 0$ (zero magnetisation), while for $a < 0$ there are two minima with non-zero magnetisation, $\Psi \neq 0$. The value $a = 0$ is the critical point that separates the magnetic phase, $\Psi \neq 0$, from the non-magnetic one, $\Psi = 0$.

Setting a of Eq. (2.5) to zero and solving for T yields the critical temperature,

$$T_c = 2gD, \quad (2.6)$$

so that for low temperatures $T < T_c$ we have $a < 0$ and the system is magnetic, while for $T > T_c$ we have $a > 0$ and the system is non-magnetic. Quantity T_c is the critical temperature at which the

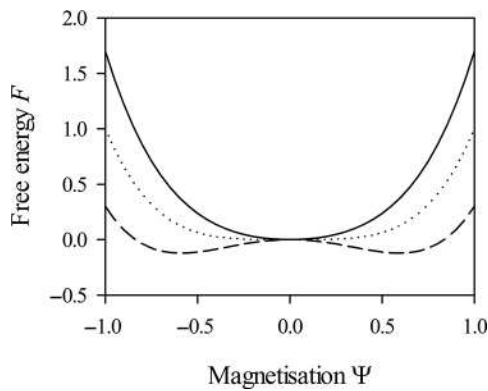


Figure 2.1 Free energy F as a function of the magnetisation Ψ , as given in Eq. (2.4). The parameters are $b = 1$, and $a = 0.7$ (solid line), $a = 0$ (dotted line) and $a = -0.7$ (dashed line).

ferromagnet changes between the magnetic and the non-magnetic phases, a process termed phase transition. As Eq. (2.6) shows, T_c is positively related to the interaction strength g , so that increasing g increases T_c .

The approach of writing the free energy F of a system as a polynomial of some macroscopic system property such as the magnetisation Ψ , solving for the value Ψ^* that minimises F and discussing under which circumstances Ψ^* is non-zero, is the core of the so-called Ginzburg–Landau Theory (GL Theory), named after the two Russian physicists Vitaly Lazarevich Ginzburg and Lev Davidovich Landau. The Ginzburg–Landau Theory provides an intuitive and mathematically relatively simple description of phase transitions in complex systems with many interacting particles or agents (Chaikin and Lubensky 1995). Mathematical calculations allow that theory to be derived from the Ising model, so the model may be regarded as a realisation of the theory. In Section 2.2 we will see that the Ginzburg–Landau Theory has applications in many different types of complex systems.

2.2 Generalisation

In Section 2.1 the Ginzburg–Landau Theory was presented as a suitable tool to describe the phase transition in a ferromagnet. However, the same theory can be used to describe many other phase transitions in very different physical systems, such as the transition from vapour to liquid

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or vice versa or superconductivity where electrical currents flow without any resistivity if the temperature is below a critical level (Chaikin and Lubensky 1995; Hohenberg and Krekhov 2015). All these systems share the feature that below the critical temperature a macroscopic system property (magnetism in the case of the ferromagnet, the difference between the liquid and gas densities in a vapour–liquid system, and the density of the supercurrent component in a superconductor) is positive.

Phase transitions can also be observed in biological and social systems. The Schelling model is a prominent example. By comparing Eqs (2.1) and (2.2) with Eq. (2.3) it becomes obvious that the ‘happiness’ H of the people in the Schelling model is (except for the minus sign) formally identical to the Hamiltonian H of the ferromagnet. And in fact, if randomness is added to the people’s behaviour in the Schelling model and identified with a temperature (which is quite obvious, considering that temperature results from the random motion of physical particles), the Schelling model exhibits a behaviour very similar to that of the Ising model of the ferromagnet (Stauffer 2007).

These examples demonstrate that the behaviour of very different systems can be described by the same theory and structurally similar models. Although the modelled systems differ (as do the models used for their description: e.g. the Hamiltonian used to model a superconductor differs considerably from the Ising model, Eq. (2.3), above), they can all be cast into the same Ginzburg–Landau Theory. Similarly, in reverse, one may also say that the different models can be regarded as instances of the GL Theory in the observable world.

The Ising model is an example of a model that was originally developed for a physical system but has been subsequently applied to social systems. Another example of such a transfer between disciplines can be found in the analysis of forest fires. The dynamics of forest fires has been a matter of research by both physicists and ecologists (Zinck and Grimm 2009). ‘Physical’ wildfire models belong to the research fields of statistical physics that deal with the dynamics of complex systems with many interacting particles or agents. These ‘physical’ wildfire models are usually based on strongly simplifying assumptions about the ecological processes affecting the spatio-temporal dynamics of wildfires and thus have largely been ignored in the ecological literature. ‘Ecological’ wildfire models, in contrast, consider the ecological processes in much greater detail, which allegedly leads to more realistic model behaviour and better predictions of the fire dynamics in real-world forests. However, the ecological wildfire models are usually too complex and too specific to

generate a general understanding of wildfire dynamics or to reveal the general principles governing them.

Zinck and Grimm (2009) compared the dynamics produced by two ‘ecological’ wildfire and two ‘physical’ models. All models are grid-based, where each grid cell may be empty or contain one or more trees in a particular condition (more details on grid-based models can be found in Section 6.2). Furthermore, in all models a wildfire can spread through the forest, because a burning grid cell (of course, it’s the trees in the cell that burn) can ignite a neighbouring grid cell. In the two ‘ecological’ models the probability of a grid cell being ignited by a burning neighbour depends on the time since the last fire. This is plausible, because the longer the time since the last fire, the more biomass will have accumulated and the more easily the trees in the grid cell will burn.

The two ‘physical’ models do not include this burning condition. Instead, in one of these models a grid cell is ignited with certainty if one of its neighbours is burning and if it contains a tree. The second model is a so-called dynamic percolation model (for more details on percolation models, see Section 2.4) in which, as in the first physical wildfire model, grid cells can be ignited by neighbouring grid cells, but where, in contrast, any cell can be ignited – be it occupied by a tree or not.

The authors evaluated and compared the four models with regard to several landscape-scale variables: (i) the relationship between the size A of the burnt area and the frequency $f(A)$ of observing such a fire size, (ii) several parameters (e.g. edge) characterising the geometric shape of a burned area and (iii) the landscape diversity measured by the Shannon–Wiener index (cf. Begon et al. 1990, ch. 17.2.1) of the successional states (which are characterised by the time since the last fire) in the model landscape. It turns out that all models except for the percolation model show the same frequency distribution of burnt areas, which follows a power law:

$$f \sim A^{-\gamma}, \quad (2.7)$$

meaning that larger fires are less frequent than smaller ones. The exponent γ lies between 1.16 and 1.17 for all three models and only the percolation model substantially deviates, with $\gamma = 0.78$. The shape parameters of the burnt areas agree extraordinarily well among all four models and in addition nearly perfectly agree with empirical observations from a forest in Canada. Lastly, the first three models produce nearly the same landscape diversity for many different model parameterisations. Again, only the percolation model deviates slightly from the others.