Game Theory

Second Edition

Covering both noncooperative and cooperative games, this comprehensive introduction to game theory also includes some advanced chapters on auctions, games with incomplete information, games with vector payoffs, stable matchings, and the bargaining set. Mathematically oriented, the book presents every theorem alongside a proof. The material is presented clearly and every concept is illustrated with concrete examples from a broad range of disciplines. With numerous exercises the book is a thorough and extensive guide to game theory from undergraduate through graduate courses in economics, mathematics, computer science, engineering, and life sciences to being an authoritative reference for researchers.

MICHAEL MASCHLER was a professor in the Einstein Institute of Mathematics and the Center for the Study of Rationality at the Hebrew University of Jerusalem in Israel. He greatly contributed to cooperative game theory and to repeated games with incomplete information.

EILON SOLAN is a professor in the School of Mathematical Sciences at Tel Aviv University in Israel. The main topic of his research is repeated games. He serves on the editorial board of several academic journals.

SHMUEL ZAMIR is a professor emeritus in the Department of Statistics and the Center for the Study of Rationality at the Hebrew University of Jerusalem in Israel. The main topics of his research are games with incomplete information and auction theory. He is the editor-in-chief of the *International Journal of Game Theory*.

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MICHAEL MASCHLER
EILON SOLAN
SHMUEL ZAMIR

Translated from Hebrew by Ziv Hellman

English Editor Mike Borns

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To Michael Maschler

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NOTATIONS

The book makes use of a large number of notations; we have striven to stick to accepted notations and to be consistent throughout the book. The coordinates of a vector are always denoted by a subscript index, $x = (x_i)_{i=1}^n$, while the indices of the elements of sequences are always denoted by a superscript index, $x^1, x^2, \ldots$ The index of a player in a set of players is always denoted by a subscript index, while a time index (in repeated games) is always denoted by a superscript index. The end of the proof of a theorem is indicated by □, the end of an example is indicated by ◂, and the end of a remark is indicated by ♦.

For convenience we provide a list of the mathematical notations used throughout the book, accompanied by a short explanation and the pages on which they are formally defined. The notations that appear below are those that are used more than once.

0	chance move in an extensive-form game	50
$\vec{0}$	origin of a Euclidean space	579
$\emptyset$	strategy used by a player who has no decision vertices in an extensive-form game	5
$\mathbf{1}_A$	function that is equal to 1 on event A and to 0 otherwise	595
2^Y	collection of all subsets of Y	336
$\|X\|$	number of elements in finite set X	603
$\|\|x\|\|_\infty$	L_∞ norm, $\|\|x\|\|_\infty := \max_{i=1,2,\ldots,n} \|x_i\|$	539
$\|\|x\|\|$	norm of a vector, $\|\|x\|\| := \sqrt{\sum_{l=1}^d (x_l)^2}$	579
$A \vee B$	maximum matching (for men) in a matching problem	942
$A \wedge B$	maximum matching (for women) in a matching problem	943
$A \subseteq B$	set A contains set B or is equal to it	
$A \subset B$	set A strictly contains set B	
$\langle x, y \rangle$	inner product	579
$\langle\langle x^0, \ldots, x^k \rangle\rangle$	k-dimensional simplex	965
$\succsim_i$	preference relation of player i	13
$\succ_i$	strict preference relation of player i	10
$\approx_i$	indifference relation of player i	10, 944
$\succsim_P$	preference relation of an individual	905
$\succ_Q$	strict preference relation of society	905
$\approx_Q$	indifference relation of society	905
$x \geq y$	$x_k \geq y_k$ for each coordinate k, where x, y are vectors in a Euclidean space	676
$x > y$	$x \geq y$ and $x \neq y$	676

$x \gg y$	$x_k > y_k$ for each coordinate k, where x, y are vectors in a Euclidean space	676
$x + y$	sum of vectors in a Euclidean space, $(x+y)_k := x_k + y_k$	676
xy	coordinatewise product of vectors in a Euclidean space, $(xy)_k := x_k y_k$	676
$x + S$	$x + S := \{x + s \colon s \in S\}$, where $x \in \mathbb{R}^d$ and $S \subseteq \mathbb{R}^d$	676
xS	$xS := \{xs \colon s \in S\}$, where $x \in \mathbb{R}^d$ and $S \subseteq \mathbb{R}^d$	676
cx	product of real number c and vector x	676
cS	$cS := \{cs \colon s \in S\}$, where c is a real number and $S \subseteq \mathbb{R}^d$	676
$S + T$	sum of sets; $S + T := \{x + y \colon x \in S, y \in T\}$	676
$\lceil c \rceil$	smallest integer greater than or equal to c	542
$\lfloor c \rfloor$	largest integer less than or equal to c	542
$x^\top$	transpose of a vector, column vector that corresponds to row vector x	580
$\mathrm{argmax}_{x \in X} f(x)$	set of all x where function f attains its maximum in the set X	124, 676
$a(i)$	producer i's initial endowment in a market	751
A	set of actions in a decision problem with experts	609
A	set of alternatives	904
A_i	player i's action set in an extensive-form game, $A_i := \cup_{j=1}^{k_i} A(U_i^j)$	232
A_k	possible outcome of a game	13
$A(x)$	set of available actions at vertex x in an extensive-form game	44
$A(U_i)$	set of available actions at information set U_i of player i in an extensive-form game	54
b_i	buyer i's bid in an auction	91, 474
$b(S)$	$b(S) = \sum_{i \in S} b_i$ where $b \in \mathbb{R}^N$	719
$\mathrm{br}_{\mathrm{I}}(y)$	Player I's set of best replies to strategy y	125
$\mathrm{br}_{\mathrm{II}}(x)$	Player II's set of best replies to strategy x	124
B_i	player i's belief operator	402
B_i^p	set of states of the world in which the probability that player i ascribes to event E is at least p, $B_i^p(E) := \{\omega \in Y \colon \pi_i(E \mid \omega) \geq p\}$	435
$\mathrm{BZ}_i(N; v)$	Banzhaf value of a coalitional game	828
$\mathcal{B}$	coalitional structure	723
$\mathcal{B}_i^T$	set of behavior strategies of player i in a T-repeated game	533
$\mathcal{B}_i^\infty$	set of behavior strategies of player i in an infinitely repeated game	546
c	coalitional function of a cost game	711
c_+	maximum of c and 0	887
c_i	$c_i(v_i) := v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}$	509
C	function that dictates the amount that each buyer pays given the vector of bids in an auction	474

 Notations

$C(x)$	set of children of vertex x in an extensive-form game	5
$\mathcal{C}(N,v)$	core of a coalitional game	736
$\mathcal{C}(N,v;\mathcal{B})$	core for a coalitional structure	780
$\text{conv}\{x_1,\ldots,x_K\}$	smallest convex set that contains the vectors $\{x_1,\ldots,x_K\}$, also called the convex hull of $\{x_1,\ldots,x_K\}$	538, 676, 963
d	disagreement point of a bargaining game	676
d_i	debt to creditor i in a bankruptcy problem	881
d^t	distance between average payoff and target set	589
$d(x,y)$	Euclidean distance between two vectors in Euclidean space	580
$d(x,S)$	Euclidean distance between point and set	580
$\mathcal{D}(\alpha,x)$	collection of coalitions whose excess is at least α, $\mathcal{D}(\alpha,x) := \{S\subseteq N, S\neq\emptyset: \quad e(S,x)\geqslant\alpha\}$	866
$e(S,x)$	excess of coalition S, $e(S,x) := v(S)-x(S)$	850
E	set of vertices of a graph	41, 43
E	estate of bankrupt entity in a bankruptcy problem	881
E	set of experts in a decision problem with experts	609
F	set of feasible payoffs in a repeated game	538, 587
F	social welfare function	905
F_i	cumulative distribution function of buyer i's private values in an auction	474
$F_i(\omega)$	atom of the partition $\mathcal{F}_i$ that contains ω	335
F^N	cumulative distribution function of joint distribution of vector of private values in an auction	474
$\mathcal{F}$	collection of all subgames in the game of chess	5
$\mathcal{F}$	family of bargaining games	676
$\mathcal{F}^N$	family of bargaining games with set of players N	701
$\mathcal{F}_d$	family of bargaining games in $\mathcal{F}$ where the set of alternatives is comprehensive and all alternatives are at least as good as the disagreement point, which is $(0,0)$	694
$\mathcal{F}_i$	player i's information in an Aumann model of incomplete information	334
g^T	average payoff up to stage T (including) in a repeated game	581
G	graph	41
G	social choice function	912
h	history of a repeated game	532
h_t	history at stage t of a repeated game	610
$H(t)$	set of t-stage histories of a repeated game	532, 609
$H(\infty)$	set of plays in an infinitely repeated game	546
$H(\alpha,\beta)$	hyperplane, $H(\alpha,\beta) := \{x\in\mathbb{R}^d: \langle\alpha,x\rangle=\beta\}$	586, 989
$H^+(\alpha,\beta)$	half-space, $H^+(\alpha,\beta) := \{x\in\mathbb{R}^d: \langle\alpha,x\rangle\geq\beta\}$	586, 989
$H^-(\alpha,\beta)$	half-space, $H^-(\alpha,\beta) := \{x\in\mathbb{R}^d: \langle\alpha,x\rangle\leq\beta\}$	586, 989
i	player	
$-i$	set of all players except player i	

I	function that dictates the winner of an auction given the vector of bids	474
J	number of lotteries that compose a compound lottery	14
$J(x)$	player who chooses a move at vertex x of an extensive-form game	44
$-k$	player who is not k in a two-player game	580
k_i	number of information sets of player i in an extensive-form game	54
K	number of outcomes of a game	16
K_i	player i's knowledge operator	336
$\mathcal{KS}, \mathcal{KS}(S)$	Kalai–Smorodinsky solution to bargaining games	699
L	lottery: $L = [p_1(A_1), p_2(A_2), \ldots, p_K(A_K)]$	13
L	number of commodities in a market	751
$\widehat{L}$	compound lottery: $\widehat{L} = [q_1(L_1), \ldots, q_J(L_J)]$	14
$\mathcal{L}$	set of lotteries	13
$\widehat{\mathcal{L}}$	set of compound lotteries	15
$m(\epsilon)$	minimal coordinate of vector ε	275, 279
m_i	number of pure strategies of player i	146
$m_i(S)$	highest possible payoff to player i in a bargaining game	694
M	maximal absolute value of a payoff in a game	528
$M_{m,l}$	space of matrices of dimension $m \times l$	213
$M(\epsilon)$	maximal coordinate of vector ε	275, 279
$\mathcal{M}(N; v; \mathcal{B})$	bargaining set for coalitional structure $\mathcal{B}$	834
n	number of players	77
n	number of buyers in an auction	474
n_x	number of vertices in subgame $\Gamma(x)$	5
N	set of players	43, 881, 710
N	set of buyers in an auction	474
N	set of individuals	904
N	set of producers in a market	751
$\mathbb{N}$	set of natural numbers, $\mathbb{N} := \{1, 2, 3, \ldots\}$	
$\mathcal{N}$	$\mathcal{N}(S, d)$, Nash's solution to bargaining games	681
$\mathcal{N}(N; v)$	nucleolus of a coalitional game	853
$\mathcal{N}(N; v; \mathcal{B})$	nucleolus of a coalitional game for coalitional structure $\mathcal{B}$	853
$\mathcal{N}(N; v; K)$	nucleolus relative to set K	852
O	set of outcomes	13, 43
p	common prior in a Harsanyi game with incomplete information	358
p_k	probability that the outcome of lottery L is A_k	13
p_x	probability distribution over actions at chance move x	50
P	binary relation	905

P	set of all weakly balancing weights for collection $\mathcal{D}^*$ of all coalitions	749
$\mathbf{P}$	common prior in an Aumann model of incomplete information	345
$\mathbf{P}_\sigma(x)$	probability that the play reaches vertex x when the players implement strategy vector σ in an extensive-form game	265
$\mathbf{P}_\sigma(U)$	probability that the play reaches a vertex in information set U when the players implement strategy vector σ in an extensive-form game	284
P^N	vector of preference relations	905
$PO(S)$	set of efficient (Pareto optimal) points in S	678
$PO^W(S)$	set of weakly efficient points in S	678
$\mathcal{P}(A)$	set of all strict preference relations over a set of alternatives A	905
$\mathcal{P}(N)$	collection of nonempty subsets of N, $\mathcal{P}(N) := \{S \subseteq N, S \neq \emptyset\}$	720, 749
$\mathcal{P}^*(A)$	set of all preference relations over a set of alternatives A	905
$\mathcal{PN}(N;v)$	prenucleolus of a coalitional game	853
$\mathcal{PN}(N;v;\mathcal{B})$	prenucleolus of a coalitional game for coalitional structure $\mathcal{B}$	853
q	quota in a weighted majority game	714
$q(w)$	minimal weight of a winning coalition in a weighted majority game, $q(w) := \min_{S \in \mathcal{W}^m} w(S)$	876
$\mathbb{Q}_{++}$	set of positive rational numbers	
r_k	total probability that the result of a compound lottery is A_k	18
$R_1(p)$	set of possible payoffs when Player 1 plays mixed action p, $R_1(p) := \{puq^\top : q \in \Delta(\mathcal{J})\}$	585
$R_2(p)$	set of possible payoffs when Player 2 plays mixed action q, $R_2(p) := \{puq^\top : q \in \Delta(\mathcal{I})\}$	585
$\mathbb{R}$	real line	
$\mathbb{R}_+$	set of nonnegative numbers	
$\mathbb{R}_{++}$	set of positive numbers	
$\mathbb{R}^n$	n-dimensional Euclidean space	
$\mathbb{R}^n_+$	nonnegative orthant in an n-dimensional Euclidean space, $\mathbb{R}^n_+ := \{x \in \mathbb{R}^n : x_i \geq 0, \quad \forall i = 1, 2, \ldots, n\}$	
$\mathbb{R}^S$	$\|S\|$-dimensional Euclidean space, where each coordinate corresponds to a player in S	719
range(G)	range of a social choice function	918
s	strategy vector	45
$\mathfrak{s}$	function that assigns a state of nature to each state of the world	334
s^t	action vector played at stage t of a repeated game	532
s_i	strategy of player i	45, 56

s_t	state of nature that corresponds to type vector t in a Harsanyi game with incomplete information	358
$\mathfrak{s}^{-1}(C)$	set of states of the world that correspond to a state of nature in C, $\mathfrak{s}^{-1}(C) := \{\omega \in Y\colon s(\omega) \in C\}$	341
S	set of all vectors of pure strategies	77
S	set of states of nature in models of incomplete information	334
S	set of states of nature in a decision problem with experts	609
S	set of alternatives in a bargaining game	676
S_i	set of player i's pure strategies	77
Sh	Shapley value	802
supp	support of a probability distribution	216
supp	support of a vector in $\mathbb{R}^n$	971
t_i	player i's type in models of incomplete information	461
T	set of vectors of types in a Harsanyi model of incomplete information	358
T	number of stages in a finitely repeated game	536
T_i	player i's type set in a Harsanyi model of incomplete information	358
u	payoff function in a strategic-form game	43, 609
u_i	player i's utility function	14
u_i	player i's payoff function	77
u_i	producer i's production function in a market	751
u_i^t	payoff of player i at stage t in a repeated game	535
u^t	vector of payoffs at stage t in a repeated game	535
$u(s)$	outcome of a game under strategy vector s	45
U_i^j	information set of player i in an extensive-form game	54
U_i	mixed extension of player i's payoff function	146
$U(C)$	uniform distribution over set C	
$U[\alpha]$	scalar payoff function generated by projecting the payoffs in direction α in a game with payoff vectors	596
v	value of a two-player zero-sum game	114
v	coalitional function of a coalitional game	710
$\underline{v}$	maxmin value of a two-player non-zero-sum game	112
$\overline{v}$	minmax value of a two-player non-zero-sum game	112
$\overline{v}$	maximal private value of buyers in an auction	479
v_0	root of a game tree	42, 43
v_i	buyer i's private value in an auction	91
v^*	superadditive closure of a coalitional game	780
$\underline{v}_i$	player i's maxmin value in a strategic-form game	103, 103, 175
$\overline{v}_i$	player i's minmax value in a strategic-form game	175, 537
$\mathrm{val}(A)$	value of a two-player zero-sum game whose payoff function is given by matrix A	596
V	set of edges in a graph	41, 43

Notations

V	set of individually rational payoffs in a repeated game	538
V_0	set of vertices in an extensive-form game where a chance move takes place	43
V_i	set of player i's decision points in an extensive-form game	43
V_i	random variable representing buyer i's private value in an auction	475
$\mathbb{V}$	buyer's set of possible private values in a symmetric auction	479
$\mathbb{V}_i$	buyer i's set of possible private values	474
$\mathbb{V}^N$	set of vectors of possible private values: $\mathbb{V}^N := \mathbb{V}_1 \times \mathbb{V}_2 \times \cdots \times \mathbb{V}_n$	474
w_i	player i's weight in a weighted majority game	714
$\mathcal{W}^m$	collection of minimal winning coalitions in a simple monotonic game	873
x_{-i}	$x_{-i} := (x_j)_{j \neq i}$	85
$x(S)$	$x(S) := \sum_{i \in S} x_i$, where $x \in \mathbb{R}^N$	719
X	$X := \times_{i \in N} X_i$	2
X_k	space of belief hierarchies of order k	451
X_{-i}	$X_{-i} := \times_{j \neq i} X_j$	85
$X(n)$	standard $(n-1)$-dimensional simplex, $X(n) := \{x \in \mathbb{R}^n : \sum_{i=1}^n x_i = 1, \quad x_i \geq 0 \quad \forall i\}$	981
$X(N; v)$	set of imputations in a coalitional game, $X(N; v) := \{x \in \mathbb{R}^n : x(N) = v(N), x_i \geq v(i) \quad \forall i \in N\}$	724, 850
$X^0(N; v)$	set of preimputations, $X^0(N; v) := \{x \in \mathbb{R}^N : x(N) = v(N)\}$	852
$X(\mathcal{B}; v)$	set of imputations for coalitional structure $\mathcal{B}$, $X(\mathcal{B}; v) := \{x \in \mathbb{R}^N : x(S) = v(S) \quad \forall S \in \mathcal{B}, x_i \geq v_i \quad \forall i\}$	724
$X^0(\mathcal{B}; v)$	set of preimputations for coalitional structure $\mathcal{B}$, $X^0(\mathcal{B}; v) := \{x \in \mathbb{R}^N : x(S) = v(S) \quad \forall S \in \mathcal{B}\}$	852
Y	set of states of the world	334, 345
$\widetilde{Y}(\omega)$	minimal belief subspace in state of the world ω	411
$\widetilde{Y}_i(\omega)$	minimal belief subspace of player i in state of the world ω	412
Z_k	space of coherent belief hierarchies of order k	454
$Z(P, Q; R)$	preference relation in which alternatives in R are preferred to alternatives not in R, the preference over alternatives in R is determined by P, and the preference over alternatives not in R is determined by Q	914
$Z(P^N, Q^N; R)$	preference profile in which the preference of individual i is $Z(P_i, Q_i; R)$	914
β_i	buyer i's strategy in an auction	475
β_i	buyer i's strategy in a selling mechanism	502
β_i^*	buyer i's strategy in a direct selling mechanism in which he reports his private value	503
Γ	extensive-form game	43, 50, 54

Γ	extension of a strategic-form game to mixed strategies	146
Γ_T	T-stage repeated game	536
Γ_λ	discounted game with discount factor λ	552
Γ_∞	infinitely repeated game	547
$\Gamma(x)$	subgame of an extensive-form game that starts at vertex x	5, 45, 55
$\Gamma^*(p)$	extended game that includes a chance move that selects a vector of recommendations according to the probability distribution p in the definition of a correlated equilibrium	316
$\Delta(S)$	set of probability distributions over S	145
ε	vector of constraints in the definition of perfect equilibrium	275
ε_i	vector of constraints of player i in the definition of perfect equilibrium	275
$\varepsilon_i(s_i)$	minimal probability in which player i selects pure strategy s_i in the definition of perfect equilibrium	274
$\theta(x)$	vector of excesses in decreasing order	850
θ_i^k	$A_k \approx [\theta_i^k(A_K), (1-\theta_i^k)(A_0)]$	20
λ	discount factor in a repeated game	551
λ_α	egalitarian solution with angle α of bargaining games	691
μ^k	belief hierarchy of order k	451
χ^S	incidence vector of a coalition	741
Π	belief space: $\Pi = (Y, \mathcal{F}, s, (\pi_i)_{i\in N})$	474
π_i	player i's belief in a belief space	397
σ	strategy in a decision problem with experts	609
σ_i	mixed strategy of player i	145
σ_{-k}	strategy of the player who is not player k in a two-player game	580
Σ_i	set of mixed strategies of player i	146
τ_i	strategy in a game with an outside observer $\Gamma^*(p)$	316
τ_i	player i's strategy in a repeated game	533, 546
τ_i^*	strategy in a game with an outside observer in which player i follows the observer's recommendation	317
$\varphi, \varphi(S,d)$	solution concept for bargaining games	677
φ	solution concept for coalitional games	723
φ	solution concept for bankruptcy problems	881
Ω	universal belief space	462