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978-1-108-49260-7 — Topics in Algorithmic Graph Theory
Edited by Lowell W. Beineke , Martin Charles Golumbic , Robin J. Wilson
Frontmatter
More Information

Topics in Algorithmic Graph Theory

Algorithmic graph theory has been expanding at an extremely rapid rate since the middle of the twentieth century, in parallel with the growth of computer science and the accompanying utilization of computers, where efficient algorithms have been a prime goal. This book presents material on developments of graph algorithms and related concepts that will be of value to both mathematicians and computer scientists, at a level suitable for graduate students, researchers and instructors.

The 15 expository chapters, written by acknowledged international experts on their subjects, focus on the development and application of algorithms to solve particular problems. All chapters have been carefully edited to enhance readability and to standardize the chapter structure as well as the terminology and notation. The editors provide basic background material in graph theory, and a chapter written by the book's Academic Consultant, Martin Charles Golumbic (University of Haifa, Israel), provides background material on algorithms connected with graph theory.

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Frontmatter
More Information

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Frontmatter
More Information

In 1895 Gaston Tarry (1843–1913) presented one of the earliest algorithms in graph theory when he wrote ‘Le problème des labyrinthes’ on the tracing of mazes.

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Encyclopedia of Mathematics and its Applications

Topics in Algorithmic Graph Theory

Edited by

LOWELL W. BEINEKE
Purdue University, Indiana

MARTIN CHARLES GOLUMBIC
University of Haifa, Israel

ROBIN J. WILSON
The Open University and University of Oxford

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University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre,
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www.cambridge.org
Information on this title: www.cambridge.org/9781108492607
DOI: 10.1017/9781108592376

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First published 2021

Printed in the United Kingdom by TJ Books Limited, Padstow Cornwall

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data
Names: Beineke, Lowell W., editor. | Golumbic, Martin Charles, editor. |
Wilson, Robin J., editor.
Title: Topics in algorithmic graph theory / Lowell W. Beineke,
Martin Charles Golumbic, and Robin J. Wilson.
Description: New York : Cambridge University Press, 2021. |
Includes bibliographical references and index.
Identifiers: LCCN 2020053205 | ISBN 9781108492607 (hardback)
Subjects: LCSH: Graph algorithms. | Graph theory.
Classification: LCC QA166.245 .B45 2021 | DDC 511/.5–dc23
LC record available at https://lccn.loc.gov/2020053205

ISBN 978-1-108-49260-7 Hardback

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Frontmatter
More Information

Contents

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More Information

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More Information

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Frontmatter
More Information

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Frontmatter
More Information

Foreword

Martin Charles Golumbic

Algorithmic graph theory as a discipline began to develop in the mid-1960s, as computer science began to impact research in optimization, operations research and discrete mathematics. Traditional applications of graph theory expanded their focus from engineering, circuit design and communication networks to new areas such as software development, compilers and bioinformatics. In March 1970 the combinatorics community held its first ‘Southeastern Conference on Combinatorics, Graph Theory, and Computing’ at the University of Louisiana in Baton Rouge, USA, alternating for many years with Florida Atlantic University in Boca Raton, its home for the past several decades. This was soon followed by the ‘Workshop on Graph-Theoretic Concepts in Computer Science’, founded in 1975 and held in Europe each year. Both of these flagship conferences are flourishing today, together with dozens of others throughout the academic world.

As a graduate student in mathematics with a keen interest in computer science, I attended the ‘Complexity of Computer Computations’ symposium organized by Ray Miller and Jim Thatcher at IBM Research at Yorktown Heights in 1972. It was very exciting! At the first coffee break, I introduced myself to Stony Brook professor Charles Fiduccia and learned, for the first time, that two $n \times n$ matrices can be multiplied using $O(n^{2.81})$ arithmetic operations instead of the usual $O(n^3)$-algorithm. ‘Really?’, I asked. ‘Yes, see that fellow over there?’, pointing to Volker Strassen. ‘He did it!’.

The symposium helped to shape my view and interests in discrete applied mathematics and algorithmic graph theory, stressing the importance of designing novel data structures to implement combinatorial algorithms. The speakers at this meeting have become legendary figures in computational complexity, and the diversity of topics emphasized the breadth of research already taking place in mathematics and computer science. Volker Strassen’s lecture was on evaluating rational functions, and Shmuel Winograd’s was about parallel iteration methods. Chuck Fiduccia spoke about upper bounds on the complexity of matrix multiplication, and Ed Reingold gave simple proofs of lower bounds for polynomial evaluation. Michael Rabin presented work on solving linear equations by means of scalar products, and Michael Paterson lectured on efficient iterations for algebraic numbers. Michael J. Fischer spoke

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Frontmatter
More Information

on the efficiency of equivalence algorithms, and Martin Schultz talked about the computational complexity of elliptic partial differential equations.

I was especially impressed by the variety of applications being revolutionized by the development of more efficient algorithms for graphs. John Hopcroft and Robert Tarjan discussed the isomorphism problem on planar graphs, and Richard Brent introduced me to the computational complexity of iterative methods for systems of linear and non-linear equations, which would become relevant when studying chordal graphs. On scheduling applications involving graphs, Vaughan Pratt presented an $O(n \log n)$-algorithm to distribute n records optimally in a sequential access file, Robert Floyd spoke about permuting information in idealized two-level storage and David van Voorhis gave lower bounds for sorting networks.

What struck me most, however, was observing real-time collaborative research at work. Richard Karp presented his famous paper, 'Reducibility among combinatorial problems', showing the computational equivalence of 21 diverse hard problems from number theory, graph theory, optimization, etc. On the second day, during the morning greetings, Karp announced, 'At the hotel last night, we proved these additional problems are NP-complete.' Now that's real-time progress, I thought. And again, on the third day, he announced, 'At the hotel last night, we proved even more problems to be NP-complete.' This exciting experience lit a spark in me, resulting in a career-long obsession of organizing research workshops – over 200 of them.

At the 1972 IBM symposium, all 14 speakers were male, a reflection of a past era. The field has grown to include more women, as is evident by looking at the programmes of current-day conferences and publications. This has made a significant impact on the progress of research.

In his foreword to my first book, *Algorithmic Graph Theory and Perfect Graphs*, Claude Berge wrote, 'The elaboration of new theoretical structures has motivated a search for new algorithms compatible with those structures. The main task for the mathematician is to eliminate the often arbitrary and cumbersome definitions, keeping only the "deep" mathematical problems. In graph theory, it should relate to a variety of other combinatorial structures and must therefore be connected with many difficult practical problems.'

The ensuing years have been an amazingly fruitful period of research in this area. To my great satisfaction, the number of relevant journal articles in the literature has grown a hundredfold. I can hardly express my admiration to all these authors for creating a success story for algorithmic graph theory far beyond my own imagination. With almost 6000 citations, *Algorithmic Graph Theory and Perfect Graphs* has fuelled the development of the field. It continues to convey the message that graph-theoretic models are a necessary and important tool for solving real-world problems – in particular, when focusing on structured graph classes. Moreover, applications provide rich soil for deep theoretical results – stepping stones from which the reader may embark on one of many fascinating research trails.

This new volume on *Topics in Algorithmic Graph Theory* demonstrates the wealth of literature developed over the past 50 years, bringing new points of view to traditional problems in mathematics and presenting emerging contemporary themes

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Frontmatter
More Information

that deserve a special place in the literature. The topics covered here have been chosen to fill a vacuum, and their interrelation and importance will become evident as the reader proceeds through the book.

Chapter 1 opens with a review of basic graph algorithms, including graph search and greedy colouring, leading to applications on structured graph classes. Results on planar graphs and special classes of intersection graphs are featured. Exploiting graph structure is one of the fundamental approaches for designing efficient algorithms to solve important practical problems, and this theme repeats itself in many chapters of this book.

In Chapter 2, Alain Hertz and Bernard Ries present three graph colouring variations – selective colouring, online colouring, and mixed graph colouring – and motivating applications, complexity results and algorithmic developments are discussed for each variation. Chapter 3, by Celina de Figueiredo, is a survey of total colouring, where we assign a colour to each vertex and edge of a graph so that there are no incidence colour conflicts. Both theoretical and algorithmic results are considered for this alternative colouring problem. The total chromatic number has been determined for cycle graphs, complete and complete bipartite graphs, and trees, grids and series-parallel graphs. The total colouring problem is NP-complete, even when restricted to k-regular bipartite graphs. The complexity is unknown for the class of chordal graphs, and only partial results are known for interval graphs, split graphs, cographs, rooted path graphs and dually chordal graphs.

In Chapter 4, Ilan Newman focuses on models and efficient algorithms for testing graph properties. This is the study of deciding the existence of a property in time or space that is significantly smaller than the size of the input, while trading off the accuracy. Property testing has potential applications to 'big-data' scenarios. Chapter 5, by Vadim Lozin, introduces a formal notion for polynomial-time solvable instances of generally NP-hard problems when restricted to graphs with a particular structure. It focuses on finding maximum cliques, graph colouring and satisfiability, exploiting the structure of the input to lower the computational complexity of these problems through a notion of boundary properties and graph operations.

The next four chapters survey work on several classes of perfect graphs. Chapter 6 starts with chordal graphs, their tree representations and algorithms, and then moves up the hierarchy to weakly chordal graphs and chordal probe graphs. Moving down the hierarchy, we then study characterizing properties of block duplicate graphs, strictly chordal graphs, Ptolemaic graphs and laminar chordal graphs, and conclude with a variety of applications of chordal graphs. Chapter 7, by Andreas Brandstädt and myself, presents dually chordal graphs, which are the dual variant of chordal graphs (in a hypergraph sense), and their well-known common hereditary subclass, the strongly chordal graphs. We describe the fundamental tree representation underpinning their structure, as well as various algorithmic applications and alternative hypergraph characterizations. We conclude with the class of chordal bipartite graphs – graphs that are bipartite and weakly chordal – whose structure is closely related to strongly chordal graphs. Chapter 8, by Christian Rosenke, Van Bang Le and Andreas Brandstädt, continues this theme by surveying the leaf power graphs, a subclass of

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Frontmatter
More Information

strongly chordal graphs. Chapter 9, by Karen Collins and Ann N. Trenk, provides an introduction to split graphs, another subclass of chordal graphs. Degree sequences play a crucial role in characterizing split graphs and are studied geometrically using Ferrers diagrams, leading to a formula for counting the number of unlabelled split graphs on n vertices. The chapter concludes with a general theorem on Tyshkevich graph decomposition in which split graphs play a starring role.

In Chapter 10, Martin Milanič begins with a light introduction to strong cliques and stable sets in a graph. A stable set is strong if it intersects every maximal clique, and a strong clique is defined analogously. These concepts played an important role in the study of perfect graphs and are related to other concepts in graph theory, including perfect matchings, well-covered graphs and general partition graphs. The chapter presents related structural and algorithmic results on perfect matchings in graphs and hypergraphs, and exact transversals in hypergraphs.

In Chapter 11, Maximilian Fürst and Dieter Rautenbach present three types of restricted matchings: induced matchings, uniquely restricted matchings, and acyclic matchings. They relate the corresponding matching numbers to each other, and consider their computational complexity, bounds, tractable cases and approximation algorithms.

In Chapter 12, Gila Morgenstern applies the perfect graph approach to geometric covering problems. Geometric covering problems are normally NP-hard, yet under specified restrictions some are reduced to optimization problems on perfect graphs, and so are solvable in polynomial time. This chapter surveys some of these problems and demonstrates the connection between covering problems in geometric domains and the clique cover problem on perfect graphs.

Pavol Hell and Jaroslav Nešetřil devote Chapter 13 to the progress made on establishing the complexity of various homomorphism-related computational problems. This serves as an introduction to graph homomorphisms, in general, and to the complexity of homomorphism problems in particular. The authors challenge the research community by presenting many open questions in this area.

Chapter 14, by Patrice Ossona de Mendez, surveys the basic properties of sparse classes of graphs, from structural, algorithmic and model-theoretic points of view. Finally, in Chapter 15, Serge Gaspers considers extremal vertex-sets in graphs. For a property P, the extremal vertex-sets are either the inclusion-wise minimal or the inclusion-wise maximal vertex-sets with property P. This chapter establishes bounds on the largest number of such extremal vertex-sets that a graph may have, and discusses enumeration algorithms and their use in exponential-time algorithms.

These 15 original chapters, presented here for the first time, are advanced education-oriented surveys, each starting with a familiar theme and developing it through many of the latest results. For those who wish to read more about the topics in books and papers, the references provide many pointers to further reading on aspects that one cannot find in textbooks. I want to express my deep appreciation to my colleagues who have authored these works. We hope that this volume will be a springboard for researchers, and especially for graduate students, to pursue new directions of investigation.

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Preface

The field of graph theory has undergone tremendous growth during the past century, growth that continues at a very rapid pace. As recently as the 1950s, the graph theory community had few members, and most of them were located in Europe and North America. Today there are hundreds of graph theorists, and they span the globe.

By the mid-1970s, the subject had reached the point where we perceived the need for collections of surveys on important topics within graph theory, not only as a resource for established mathematicians, but also for informing students and scholars from other areas of mathematics about this exciting and relatively new field. The result was our three-volume series, *Selected Topics in Graph Theory*, containing chapters written by distinguished experts and then edited into a common style.

Since then the transformation of the subject has continued, with individual branches expanding to the point where they deserved books of their own. This inspired us to conceive of a new series of books, each a collection of chapters within a particular area of graph theory and written by experts within that area. The first four of these books, on algebraic, topological, structural and chromatic graph theory, are companion volumes to the present one, with this volume as the fifth in the series. It is aimed at readers from mathematics, computer science, and other areas that involve algorithms and their application to graphs.

A special feature of these books has been the engagement of academic consultants to advise us on particular topics to be included and authors to be invited from around the world. We believe that this has been successful, with the chapters of each book covering a broad range of topics within the given area. Another important feature is that we have tried to impose uniform terminology and notation throughout the book, in order to ease the passage between chapters.

We hope that these features will facilitate usage of the book in advanced courses and seminars. We thank the authors for cooperating in these efforts, even though it sometimes required their abandoning some of their favourite conventions, and for agreeing to face the ordeal of having their work subjected to detailed critical reading. We believe that the final product is thereby significantly better than it might otherwise have been, as just a collection of individual chapters with differing styles

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Frontmatter
More Information

and terminology. We express our heartfelt appreciation to all of our contributors for their cooperation in these endeavours.

We extend our special thanks to Marty Golumbic for serving as both Academic Consultant and Editor – his advice and contributions have been invaluable. We are also grateful to our copy editor, Alison Durham, and to Cambridge University Press for continuing to publish these volumes; in particular, we thank Roger Astley, Clare Dennison and Anna Scriven for their advice, support, patience and cooperation. Finally we extend our gratitude to our own universities – Purdue University Fort Wayne, the University of Haifa, and the Open University and Oxford University – for the various ways in which they have assisted with our project.

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