

Gravitational Few-Body Dynamics

Using numerical integration, it is possible to predict the individual motions of a group of a few celestial objects interacting with each other gravitationally. In this introduction to the few-body problem, a key figure in developing more efficient methods over the past few decades summarizes and explains them, covering both basic analytical formulations and numerical methods.

The mathematics required for celestial mechanics and stellar dynamics is explained, starting with two-body motion and progressing through classical methods for planetary system dynamics. This first part of the book can be used as a short course on celestial mechanics.

The second part develops the contemporary methods for which the author is renowned – symplectic integration and various methods of regularization. This volume explains the methodology of the subject for graduate students and researchers in celestial mechanics and astronomical dynamics with an interest in few-body dynamics and the regularization of the equations of motion.

SEPPO MIKKOLA is a senior lecturer at the University of Turku and staff member at Tuorla Observatory. He has made important contributions to the regularization of equations of N -body motion. Since he invented ‘algorithmic regularization’ of few-body system dynamics, it has become the foundation of many simulations worldwide.

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A Numerical Approach

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**Dedicated to my wife Elisa,
my daughters Virpi and Juulia, and
grand-daughters Minja, Siina and Ronja.**

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Preface

This book contains an exposition of the most important subjects in celestial mechanics that I have been working on. The book also includes some traditional material, which is necessary for anyone who wants to learn/study the gravitational few-body problem. Some basic subjects are repeated in different chapters/sections to avoid the need to read the entire book if one is interested only in some special problem.

Since 1972 I have been interested in the regularization of equations of motion for the gravitational few-body problem. The starting point was the excellent book by Stiefel and Scheifele (1971). In the days when I started my PhD Thesis work on the four-body problem I learned how to use the so-called KS-transformation (Kustaanheimo and Stiefel, 1965) on the few-body problem. My first achievement was the programming of the complete regularization of the N -body problem due to Heggie (1974). Later I developed a different method, the chain method, in collaboration with Sverre Aarseth during my first extended visit to Cambridge, UK (Mikkola and Aarseth, 1990).

More than a decade later I invented, completely accidentally, a new regularization method, which is called algorithmic regularization (AR). After completing the idea with Kiyotaka Tanikawa, the method was published in 1999 (Mikkola and Tanikawa, 1999a) but is not yet a textbook subject and is therefore a quite important part of this book.

My first astronomy professor Liisi Oterma (1915–2001) taught me how to compute orbits of asteroids from observations. This was the starting point of my interest in celestial mechanics, especially in the numerical methods.

Professor Mauri Valtonen suggested the four-body problem as the topic of my PhD work. That led me to learn the practical use of the KS-transformation.

Soon after that I visited Sverre Aarseth in Cambridge. This visit led to a long-time collaboration with Aarseth and, after some time, led to the invention of the chain method and later to a generalization of the AR method, i.e. the time-transformed leapfrog (TTL).

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Professor Kiyotaka Tanikawa was involved with the invention of the AR and in addition we were working on the mathematics of the three-body problem, especially in the one- and two-dimensional cases which are often non-physical but interesting mathematically.

Work with David Merritt led to the writing of the Algorithmic Regularization Chain code (ARCHAIN) and to the invention of methods that made it possible to include velocity-dependent perturbation into AR, in which it was originally excluded.

For the introduction to satellite dynamics and collaboration I thank Phil Palmer.

I also would like to thank Kimmo Innanen (1937–2011) who encouraged me to study the dynamics of asteroids. This led me to familiarize myself with the symplectic integrators, especially with the Wisdom–Holman method (Wisdom and Holman, 1991). An additional consequence of that was my later invention of a partial regularization of the Wisdom–Holman method with the help of a time transformation.

I am grateful to all the above-mentioned teachers and colleagues, Kimmo Innanen, Sverre Aarseth, David Merritt, Kiyotaka Tanikawa, Phil Palmer, Mauri Valtonen and Liisi Oterma for their support and interest in my work. Particularly I thank Kimmo Innanen, Sverre Aarseth, Bo Reipurth and Kiyotaka Tanikawa for their help with the writing of this book.

Seppo Mikkola