

## Introduction

In Chapter 1 the basics of various few-body problems are discussed. These include planetary systems and small stellar clusters. The two-body problem is an important part for understanding few-body dynamics, and hence Chapter 2 discusses this subject. Additionally Chapter 2 covers the mathematics needed to compute the trajectory, orbital elements and some special functions which are convenient to evaluate the movement in elliptical, parabolic and hyperbolic trajectories. The basic classical mathematics of gravitational systems is discussed in Chapter 3. The analytical approximations, especially the ‘variation of parameters’ methods and corresponding differential equations, are the subject of Chapter 4. In Chapter 5 many classical numerical methods are discussed starting from difference methods which are accurate in systems that behave similarly over the studied time interval. In few-body stellar dynamics the systems often change significantly and a constant stepsize cannot be used for long times. The solution has been that of using extrapolation methods in which one uses first simple time-symmetrical integrators with shorter and shorter stepsizes to proceed over a given interval and then one uses polynomial or rational function extrapolation to estimate what the result would be at zero stepsize. In the Solar System, orbits remain mostly similar and therefore the method of symplectic integration (Chapter 6) is found to be a most efficient way for computation. With any computational method it is often necessary to employ coordinate and/or time transformations. There are three ways to proceed here: in Chapter 7 using the KS-transformation (KS) with a time transformation; in Chapter 8, algorithmic regularizations (AR) using the logarithmic Hamiltonian (logH) or the time-transformed leapfrog (TTL), which both are essentially based on regularizing time transformations which give exact trajectories for the two-body problem and generally regular results suitable for extrapolation algorithms. In Chapter 9 the dynamics near black holes is discussed in terms of the post-Newtonian formalism. Also the evolution of

## 2 Introduction

black hole spin is handled here. Finally in Chapter 10 a computational method for calculating the motion of artificial satellites in the complicated gravitational field of the Earth is presented. This can be applied in any field given in the form of an expansion in terms of the spherical functions.

Codes and additional information for this book are available at [www.astro.utu.fi/mikkola/](http://www.astro.utu.fi/mikkola/).