

New Spaces in Mathematics

After the development of manifolds and algebraic varieties in the previous century, mathematicians and physicists have continued to advance concepts of space. This book and its companion explore various new notions of space, including both formal and conceptual points of view, as presented by leading experts at the *New Spaces in Mathematics and Physics* workshop held at the Institut Henri Poincaré in 2015.

The chapters in this volume cover a broad range of topics in mathematics, including diffeologies, synthetic differential geometry, microlocal analysis, topos theory, infinity-groupoids, homotopy type theory, category-theoretic methods in geometry, stacks, derived geometry, and noncommutative geometry. It is addressed primarily to mathematicians and mathematical physicists, but also to historians and philosophers of these disciplines.

MATHIEU ANEL is a Visiting Assistant Professor at Carnegie Mellon University. His research interests include higher category theory, algebraic topology, and topos theory.

GABRIEL CATREN is Permanent Researcher in philosophy of physics at the French National Centre for Scientific Research (CNRS). His research interests include the foundations of classical and quantum mechanics, and the foundations of gauge theories.

New Spaces in Mathematics
Formal and Conceptual Reflections

Edited by

MATHIEU ANEL
Carnegie Mellon University

GABRIEL CATREN
CNRS - Université de Paris



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