A Guide to Monte Carlo Simulations in Statistical Physics

Dealing with all aspects of Monte Carlo simulation of complex physical systems encountered in condensed-matter physics and statistical mechanics, this book provides an introduction to computer simulations in physics.

This fifth edition contains extensive new material describing numerous powerful algorithms not covered in previous editions, in some cases representing new developments that have only recently appeared. Older methodologies whose impact was previously unclear or unappreciated are also introduced, in addition to many small revisions that bring the text and cited literature up to date. This edition also introduces the use of petascale computing facilities in the Monte Carlo arena.

Throughout the book there are many applications, examples, recipes, case studies, and exercises to help the reader understand the material. It is ideal for graduate students and researchers, both in academia and industry, who want to learn techniques that have become a third tool of physical science, complementing experiment and analytical theory.

DAVID P. LANDAU is the Distinguished Research Professor of Physics and founding Director of the Center for Simulational Physics at the University of Georgia, USA.

KURT BINDER is Professor Emeritus of Theoretical Physics and Gutenberg Fellow at the Institut für Physik, Johannes-Gutenberg-Universität, Mainz, Germany.

CAMBRIDGE

A Guide to Monte Carlo Simulations in Statistical Physics



David P. Landau Center for Simulational Physics, University of Georgia, USA

Kurt Binder Institut für Physik, Johannes-Gutenberg-Universität, Germany





University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India

103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org Information on this title: www.cambridge.org/9781108490146 DOI: 10.1017/9781108780346

© D. P. Landau and K. Binder

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2000 Second edition published 2005 Third edition published 2009 Fourth edition published 2015 Fifth edition published 2021

Printed in the United Kingdom by TJ International Ltd. Padstow Cornwall

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication Data

Names: Landau, David P., author. | Binder, K. (Kurt), 1944– author. Title: A guide to Monte Carlo simulations in statistical physics / David P. Landau, Center for

Simulational Physics, University of Georgia, USA, Kurt Binder, Institut für Physik, Johannes-Gutenberg-Universität, Germany.

Other titles: Monte Carlo simulations in statistical physics

Description: Fifth edition. | Cambridge, United Kingdom; New York, NY : Cambridge University Press, 2020. | Includes bibliographical references and index.

Identifiers: LCCN 2020021931 (print) | LCCN 2020021932 (ebook) | ISBN 9781108490146 (hardback) | ISBN 9781108780346 (epub)

Subjects: LCSH: Monte Carlo method. | Statistical physics.

Classification: LCC QC174.85.M64 L36 2020 (print) | LCC QC174.85.M64 (ebook) | DDC 530.15/8282–dc23

LC record available at https://lccn.loc.gov/2020021931

LC ebook record available at https://lccn.loc.gov/2020021932

ISBN 978-1-108-49014-6 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Contents

| | | | | page |
|---|------|--------|--|------|
| P | refa | ce | | XV |
| 1 | Inti | oduc | tion | 1 |
| | 1.1 | What | is a Monte Carlo simulation? | 1 |
| | 1.2 | A cor | nment on the history of Monte Carlo simulations | 2 |
| | 1.3 | What | problems can we solve with it? | 3 |
| | 1.4 | What | difficulties will we encounter? | 4 |
| | | 1.4.1 | Limited computer time and memory | 4 |
| | | 1.4.2 | Statistical and other errors | 4 |
| | | 1.4.3 | Knowledge that every practitioner should have | 5 |
| | 1.5 | What | strategy should we follow in approaching a problem? | 5 |
| | 1.6 | How | do simulations relate to theory and experiment? | 6 |
| | 1.7 | Persp | ective | 7 |
| | Ref | erence | s | 8 |
| 2 | Son | ne ne | cessary background | 9 |
| | 2.1 | Ther | modynamics and statistical mechanics: a quick reminder | 9 |
| | | 2.1.1 | Basic notions | 9 |
| | | 2.1.2 | Phase transitions | 17 |
| | | 2.1.3 | Ergodicity and broken symmetry | 29 |
| | | 2.1.4 | Fluctuations and the Ginzburg criterion | 30 |
| | | 2.1.5 | A standard exercise: the ferromagnetic Ising model | 31 |
| | 2.2 | Proba | bility theory | 32 |
| | | 2.2.1 | Basic notions | 32 |
| | | 2.2.2 | Special probability distributions and the central | |
| | | | limit theorem | 34 |
| | | 2.2.3 | Statistical errors | 35 |
| | | 2.2.4 | Markov chains and master equations | 36 |
| | 2.3 | The ' | art' of random number generation | 38 |
| | | 2.3.1 | Background | 38 |
| | | 2.3.2 | Congruential method | 39 |
| | | 2.3.3 | Mixed congruential methods | 40 |
| | | 2.3.4 | Shift register algorithm | 40 |
| | | 2.3.5 | Lagged Fibonacci methods | 41 |
| | | 2.3.6 | Tests for quality | 41 |
| | | 2.3.7 | Non-uniform distributions | 44 |
| | | | | |

vi Contents

| | 2.4 | Non-equilibrium and dynamics: some introductory comments | 44 |
|---|-----|---|-----|
| | | 2.4.1 Physical applications of master equations | 44 |
| | | 2.4.2 Conservation laws and their consequences | 46 |
| | | 2.4.3 Critical slowing down at phase transitions | 49 |
| | | 2.4.4 Transport coefficients | 51 |
| | | 2.4.5 Concluding comments: why bother about dynamics | |
| | | when doing Monte Carlo for statics? | 51 |
| | Ref | erences | 52 |
| 3 | Sin | pple sampling Monte Carlo methods | 54 |
| | 3.1 | Introduction | 54 |
| | 3.2 | Comparisons of methods for numerical integration of | |
| | | given functions | 54 |
| | | 3.2.1 Simple methods | 54 |
| | | 3.2.2 Intelligent methods | 56 |
| | 3.3 | Boundary value problems | 57 |
| | 3.4 | Simulation of radioactive decay | 59 |
| | 3.5 | Simulation of transport properties | 60 |
| | | 3.5.1 Neutron transport | 60 |
| | | 3.5.2 Fluid flow | 61 |
| | 3.6 | The percolation problem | 62 |
| | | 3.6.1 Site percolation | 62 |
| | | 3.6.2 Cluster counting: the Hoshen–Kopelman algorithm | 67 |
| | | 3.6.3 Other percolation models | 68 |
| | | 3.6.4 The Lorentz gas and cherry pit models and the | |
| | | localization transition | 69 |
| | | 3.6.5 Explosive percolation | 70 |
| | 3.7 | Finding the groundstate of a Hamiltonian | 72 |
| | 3.8 | Generation of 'random' walks | 73 |
| | | 3.8.1 Introduction | 73 |
| | | 3.8.2 Random walks | 74 |
| | | 3.8.3 Self-avoiding walks | 75 |
| | | 3.8.4 Growing walks and other models | 77 |
| | 3.9 | Final remarks | 78 |
| | Ref | erences | 78 |
| 4 | Im | portance sampling Monte Carlo methods | 80 |
| | 4.1 | Introduction | 80 |
| | 4.2 | The simplest case: single spin-flip sampling for the simple | |
| | | Ising model | 81 |
| | | 4.2.1 Algorithm | 82 |
| | | 4.2.2 Boundary conditions | 85 |
| | | 4.2.3 Finite size effects | 91 |
| | | 4.2.4 Finite sampling time effects | 105 |
| | | 4.2.5 Critical relaxation | 115 |
| | 4.3 | Other discrete variable models | 123 |
| | | 4.3.1 Ising models with competing interactions | 123 |

| | | | Contents | vii |
|---|------------|---------|---|-----|
| | | 4.3.2 | <i>q</i> -state Potts models | 127 |
| | | 4.3.3 | Baxter and Baxter–Wu models | 128 |
| | | 4.3.4 | Clock models | 129 |
| | | 4.3.5 | Ising spin glass models | 130 |
| | | 4.3.6 | Complex fluid models | 131 |
| | 4.4 | Spin- | exchange sampling | 132 |
| | | 4.4.1 | Constant magnetization simulations | 132 |
| | | 4.4.2 | Phase separation | 133 |
| | | 4.4.3 | Diffusion | 135 |
| | | 4.4.4 | Hydrodynamic slowing down | 138 |
| | | 4.4.5 | Interface between coexisting phases | 139 |
| | 4.5 | Micro | ocanonical methods | 140 |
| | | 4.5.1 | Demon algorithm | 140 |
| | | 4.5.2 | Dynamic ensemble | 140 |
| | | 4.5.3 | Q2R | 141 |
| | 4.6 | Gene | ral remarks, choice of ensemble | 141 |
| | 4.7 | Static | s and dynamics of polymer models on lattices | 143 |
| | | 4.7.1 | Background | 143 |
| | | 4.7.2 | Fixed bond length methods | 143 |
| | | 4.7.3 | Bond fluctuation method | 145 |
| | | 4.7.4 | Enhanced sampling using a fourth dimension | 145 |
| | | 4.7.5 | The 'wormhole algorithm' – another method to | – |
| | | | equilibrate dense polymeric systems | 147 |
| | | 4.7.6 | Polymers in solutions of variable quality: θ -point, | – |
| | | | collapse transition, unmixing | 147 |
| | | 4.7.7 | Equilibrium polymers: a case study | 150 |
| | | 4.7.8 | The pruned enriched Rosenbluth method (PERM): | |
| | | | a biased sampling approach to simulate very long | 150 |
| | | 4 7 0 | isolated chains | 153 |
| | 4.0 | 4.7.9 | Perspective | 150 |
| | 4.ð | Some | advice | 150 |
| | Kei | erence | S | 157 |
| 5 | Mo | re on | importance sampling Monte Carlo methods for | |
| | lat | tice sy | stems | 161 |
| | 5.1 | Clust | er flipping methods | 161 |
| | | 5.1.1 | Fortuin–Kasteleyn theorem | 161 |
| | | 5.1.2 | Swendsen–Wang method | 162 |
| | | 5.1.3 | Wolff method | 165 |
| | | 5.1.4 | 'Improved estimators' | 166 |
| | | 5.1.5 | Invaded cluster algorithm | 167 |
| | г э | 5.1.6 | Probability changing cluster algorithm | 167 |
| | 5.2 | Speci | alized computational techniques | 168 |
| | | 5.2.1 | Expanded ensemble methods | 168 |
| | | 5.2.2 | Wuitispin coding | 168 |
| | | 5.2.3 | <i>N</i> -told way and extensions | 169 |

viii

Contents

Cambridge University Press 978-1-108-49014-6 — A Guide to Monte Carlo Simulations in Statistical Physics David Landau , Kurt Binder Frontmatter More Information

5.2.4 Hybrid algorithms 172 5.2.5 Multigrid algorithms 172 5.2.6 Monte Carlo on vector computers 173 5.2.7 Monte Carlo on parallel computers 174 5.3 Classical spin models 175 5.3.1 Introduction 175 5.3.2 Simple spin-tilt method 176 5.3.3 Heatbath method 178 5.3.4 Low temperature techniques 179 5.3.5 Over-relaxation methods 179 5.3.6 Wolff embedding trick and cluster flipping 180 5.3.7 Hybrid methods 181 5.3.8 Monte Carlo dynamics vs. equation of motion dynamics 182 5.3.9 Topological excitations and solitons 182 5.3.10 Finite size scaling for systems with vector 186 order parameters 5.4 Systems with quenched randomness 190 5.4.1 General comments: averaging in random systems 190 5.4.2 Parallel tempering: a general method to better equilibrate systems with complex energy landscapes 194 5.4.3 Random fields and random bonds 195 5.4.4 Spin glasses and optimization by simulated 198 annealing 5.4.5 Aging in spin glasses and related systems 203 5.4.6 Vector spin glasses: developments and surprises 204 5.4.7 The ground state of the Ising spin glass on the square lattice: a case study 204 5.5 Models with mixed degrees of freedom: Si/Ge alloys, a 207 case study 5.6 Methods for systems with long range interactions 209 5.7 Parallel tempering, simulated tempering, and related methods: accuracy considerations 211 214 5.8 Sampling the free energy and entropy 5.8.1 Thermodynamic integration 214 5.8.2 Groundstate free energy determination 216 Estimation of intensive variables: the 5.8.3 chemical potential 216 5.8.4 Lee-Kosterlitz method 217 5.8.5 Free energy from finite size dependence at T_{c} 218 5.9 Miscellaneous topics 218 5.9.1 218 Inhomogeneous systems: surfaces, interfaces, etc. 5.9.2 Anisotropic critical phenomena: simulation boxes with arbitrary aspect ratio 225 5.9.3 Other Monte Carlo schemes 227 5.9.4 Inverse and reverse Monte Carlo methods 229

| | | | Contents | ix |
|-------|----------|---|----------|-----|
| | 5.9.5 | Finite size effects: review and summary | | 231 |
| | 5.9.6 | More about error estimation | | 231 |
| | 5.9.7 | Random number generators revisited | | 233 |
| 5.10 |) Summ | nary and perspective | | 237 |
| Ref | erences | | | 237 |
| 6 Off | -lattice | models | | 243 |
| 6.1 | Fluids | 5 | | 243 |
| | 6.1.1 | NVT ensemble and the virial theorem | | 243 |
| | 6.1.2 | <i>NpT</i> ensemble | | 247 |
| | 6.1.3 | 'Real' microcanonical ensemble | | 251 |
| | 6.1.4 | Grand canonical ensemble | | 252 |
| | 6.1.5 | Near critical coexistence: a case study | | 256 |
| | 6.1.6 | Subsystems: a case study | | 258 |
| | 6.1.7 | Gibbs ensemble | | 264 |
| | 6.1.8 | Widom particle insertion method and varia | nts | 266 |
| | 6.1.9 | Monte Carlo phase switch | | 269 |
| | 6.1.10 | Cluster algorithm for fluids | | 273 |
| | 6.1.11 | Event chain algorithms | | 274 |
| | 6.1.12 | An extension of the 'N-fold way'-algorithm | ı to | |
| | | off-lattice systems | | 277 |
| 6.2 | 'Short | range' interactions | | 278 |
| | 6.2.1 | Cutoffs | | 278 |
| | 6.2.2 | Verlet tables and cell structure | | 278 |
| | 6.2.3 | Minimum image convention | | 279 |
| | 6.2.4 | Mixed degrees of freedom reconsidered | | 279 |
| 6.3 | Treati | nent of long range forces | | 280 |
| | 6.3.1 | Reaction field method | | 280 |
| | 6.3.2 | Ewald method | | 280 |
| | 6.3.3 | Fast multipole method | | 281 |
| | 6.3.4 | Particle–particle particle–mesh (P ³ M) meth | nod | 282 |
| 6.4 | Adsor | bed monolayers | | 283 |
| | 6.4.1 | Smooth substrates | | 283 |
| | 6.4.2 | Periodic substrate potentials | | 283 |
| 6.5 | Comp | lex fluids | | 285 |
| | 6.5.1 | A case study: application of the Liu-Luijte | n | |
| | | algorithm to a binary fluid mixture | | 288 |
| 6.6 | Polym | ers: an introduction | | 289 |
| | 6.6.1 | Length scales and models | | 289 |
| | 6.6.2 | Asymmetric polymer mixtures: a case study | У | 296 |
| | 6.6.3 | Applications: dynamics of polymer melts; t | hin | |
| | | adsorbed polymeric films | | 298 |
| | 6.6.4 | Polymer melts: speeding up bond fluctuation | on | |
| | | model simulations | | 303 |
| 6.7 | Liquio | l crystals; an introduction | | 305 |
| 6.8 | Config | gurational bias and 'smart Monte Carlo' | | 309 |

| Contents |
|----------------|
| Constant and a |

| | 6.9 | Estimation of excess free energies due to walls for fluids | | | |
|---|------|--|--|-----|--|
| | | and so | olids | 313 | |
| | 6.10 | A syn | nmetric, Lennard–Jones mixture: a case study | 316 | |
| | 6.11 | Finite | e size effects on interfacial properties: a case study | 317 | |
| | 6.12 | Outlo | ok | 321 | |
| | Refe | rences | | 321 | |
| 7 | Rew | eighti | ng methods | 326 | |
| | 7.1 | Backg | round | 326 | |
| | | 7.1.1 | Distribution functions | 326 | |
| | | 7.1.2 | Umbrella sampling | 326 | |
| | 7.2 | Single | e histogram method | 329 | |
| | | 7.2.1 | The Ising model as a case study | 330 | |
| | | 7.2.2 | The surface-bulk multicritical point: another | | |
| | | | case study | 338 | |
| | 7.3 | Multi | histogram method | 341 | |
| | 7.4 | Broad | histogram method | 341 | |
| | 7.5 | Trans | sition matrix Monte Carlo | 342 | |
| | 7.6 | Multi | canonical sampling | 342 | |
| | | 7.6.1 | The multicanonical approach and its relationship to | | |
| | | | canonical sampling | 342 | |
| | | 7.6.2 | Near first order transitions | 344 | |
| | | 7.6.3 | Groundstates in complicated energy landscapes | 346 | |
| | | 7.6.4 | Interface free energy estimation | 347 | |
| | 7.7 | A case | e study: the Casimir effect in critical systems | 348 | |
| | 7.8 | Wang | –Landau sampling | 348 | |
| | | 7.8.1 | Basic algorithm | 348 | |
| | | 7.8.2 | Applications to models with continuous variables | 353 | |
| | | 7.8.3 | Two-dimensional Wang-Landau sampling: a | | |
| | | | case study | 354 | |
| | | 7.8.4 | Microcanonical entropy inflection points | 354 | |
| | | 7.8.5 | Back to numerical integration | 356 | |
| | | 7.8.6 | Replica exchange Wang–Landau sampling | 357 | |
| | 7.9 | A case | e study: evaporation/condensation transition | | |
| | | of dro | oplets | 359 | |
| | Refe | rences | | 362 | |
| 8 | Qua | ntum | Monte Carlo methods | 365 | |
| | 8.1 | Intro | luction | 365 | |
| | 8.2 | Feyn | nan path integral formulation | 367 | |
| | | 8.2.1 | Off-lattice problems: low temperature properties | | |
| | | | of crystals | 367 | |
| | | 8.2.2 | Bose statistics and superfluidity | 373 | |
| | | 8.2.3 | Path integral formulation for rotational degrees | | |
| | | | of freedom | 375 | |
| | 8.3 | Lattic | e problems | 377 | |
| | | 8.3.1 | The Ising model in a transverse field | 377 | |
| | | | | | |

| | | | Contents | xi |
|----|------|---------|---|-----|
| | | 8.3.2 | Anisotropic Heisenberg chain: an early case study | 378 |
| | | 8.3.3 | Fermions on a lattice | 382 |
| | | 8.3.4 | An intermezzo: the minus sign problem | 385 |
| | | 8.3.5 | Spinless fermions revisited | 387 |
| | | 8.3.6 | Cluster methods for quantum lattice models | 389 |
| | | 8.3.7 | Continuous time simulations | 391 |
| | | 8.3.8 | Decoupled cell method | 392 |
| | | 8.3.9 | Handscomb's method and the stochastic series | |
| | | | expansion (SSE) approach | 393 |
| | | 8.3.10 | Wang–Landau sampling for quantum models | 394 |
| | | 8.3.11 | Fermion determinants | 396 |
| | 8.4 | Monte | e Carlo methods for the study of | |
| | | groun | dstate properties | 398 |
| | | 8.4.1 | Variational Monte Carlo (VMC) | 398 |
| | | 8.4.2 | Green's function Monte Carlo methods (GFMC) | 400 |
| | 8.5 | Quant | tum Monte Carlo in nuclear physics | 402 |
| | 8.6 | Towa | rds constructing the nodal surface of off-lattice, | |
| | | many- | -Fermion systems: the 'survival of the | |
| | | fittest | algorithm' | 404 |
| | 8.7 | Bypas | sing the minus sign problem: phase transitions in | |
| | | antife | rromagnetic metals | 408 |
| | 8.8 | Concl | uding remarks | 411 |
| | Ref | erences | 8 | 412 |
| 9 | Mo | nte Ca | arlo renormalization group methods | 416 |
| | 9.1 | Introd | luction to renormalization group theory | 416 |
| | 9.2 | Real s | pace renormalization group | 420 |
| | 9.3 | Mont | e Carlo renormalization group | 421 |
| | | 9.3.1 | Large cell renormalization | 421 |
| | | 9.3.2 | Ma's method: finding critical exponents and the fixed | |
| | | | point Hamiltonian | 423 |
| | | 9.3.3 | Swendsen's method | 424 |
| | | 9.3.4 | Location of phase boundaries | 426 |
| | | 9.3.5 | Dynamic problems: matching time-dependent | |
| | | | correlation functions | 428 |
| | | 9.3.6 | Inverse Monte Carlo renormalization | |
| | | | group transformations | 428 |
| | Ref | erences | 5 | 429 |
| 10 | No | n-equi | librium and irreversible processes | 430 |
| | 10. | 1 Intro | oduction and perspective | 430 |
| | 10. | 2 Driv | ven diffusive systems (driven lattice gases) | 431 |
| | 10. | 3 Crys | stal growth | 434 |
| | 10.4 | 4 Don | nain growth | 437 |
| | | 10.4 | 1 Phase separation in mixtures | 437 |
| | | 10.4 | 2 A case study: growth of domains and aging | |
| | | | phenomena in spin glasses | 440 |

| xii | Contents |
|-----|----------|
| X11 | Contents |

| | 10.5 | Polymer growth | 443 |
|----|---|---|--|
| | | 10.5.1 Linear polymers | 443 |
| | | 10.5.2 Kinetic gelation: a case study | 443 |
| | 10.6 | Growth of structures and patterns | 445 |
| | | 10.6.1 Eden model of cluster growth | 445 |
| | | 10.6.2 Diffusion limited aggregation | 445 |
| | | 10.6.3 Cluster-cluster aggregation | 448 |
| | | 10.6.4 Cellular automata | 448 |
| | 10.7 | Models for film growth | 449 |
| | | 10.7.1 Background | 449 |
| | | 10.7.2 Ballistic deposition | 450 |
| | | 10.7.3 Sedimentation | 451 |
| | | 10.7.4 Kinetic Monte Carlo and MBE growth | 452 |
| | 10.8 | Transition path sampling | 454 |
| | | 10.8.1 What is transition path sampling? | 454 |
| | | 10.8.2 A case study: the disk to slab transition in the | |
| | | two-dimensional Ising model | 455 |
| | 10.9 | Forced polymer pore translocation: a case study | 457 |
| | 10.10 | The Jarzynski non-equilibrium work theorem and its | |
| | | application to obtain free energy differences | |
| | | from trajectories | 460 |
| | 10.11 | Outlook: variations on a theme | 462 |
| | Refer | ences | 462 |
| | | | |
| 11 | Lattie | ce gauge models: a brief introduction | 465 |
| 11 | Lattie | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory | 465 465 |
| 11 | Lattie 11.1 11.2 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters | 465 465 467 |
| 11 | Lattie 11.1 11.2 11.3 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for $Z(N)$ lattice gauge models | 465 465 467 467 |
| 11 | Lattie 11.1 11.2 11.3 11.4 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for $Z(N)$ lattice gauge models Compact U(1) gauge theory | 465 465 467 467 468 |
| 11 | Lattie 11.1 11.2 11.3 11.4 11.5 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for $Z(N)$ lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory | 465 465 467 467 468 469 |
| 11 | Lattic 11.1 11.2 11.3 11.4 11.5 11.6 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for $Z(N)$ lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory Introduction: quantum chromodynamics (QCD) and phase | 465 465 467 467 468 469 |
| 11 | Lattic 11.1 11.2 11.3 11.4 11.5 11.6 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for $Z(N)$ lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory Introduction: quantum chromodynamics (QCD) and phase transitions of nuclear matter | 465 465 467 467 468 469 470 |
| 11 | Lattie 11.1 11.2 11.3 11.4 11.5 11.6 11.7 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for $Z(N)$ lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory Introduction: quantum chromodynamics (QCD) and phase transitions of nuclear matter The deconfinement transition of QCD | 465 465 467 467 468 469 470 473 |
| 11 | Lattic 11.1 11.2 11.3 11.4 11.5 11.6 11.7 11.8 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for $Z(N)$ lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory Introduction: quantum chromodynamics (QCD) and phase transitions of nuclear matter The deconfinement transition of QCD Finite size scaling based on Polyakov loops: a case study | 465 465 467 467 468 469 470 473 475 |
| 11 | Lattic 11.1 11.2 11.3 11.4 11.5 11.6 11.7 11.8 11.9 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for $Z(N)$ lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory Introduction: quantum chromodynamics (QCD) and phase transitions of nuclear matter The deconfinement transition of QCD Finite size scaling based on Polyakov loops: a case study Towards quantitative predictions | 465 465 467 467 468 469 470 473 475 478 |
| 11 | Lattic 11.1 11.2 11.3 11.4 11.5 11.6 11.7 11.8 11.9 11.10 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for $Z(N)$ lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory Introduction: quantum chromodynamics (QCD) and phase transitions of nuclear matter The deconfinement transition of QCD Finite size scaling based on Polyakov loops: a case study Towards quantitative predictions Density of states in gauge theories | 465 467 467 467 468 469 470 473 475 478 481 |
| 11 | Lattic 11.1 11.2 11.3 11.4 11.5 11.6 11.7 11.8 11.9 11.10 11.11 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for $Z(N)$ lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory Introduction: quantum chromodynamics (QCD) and phase transitions of nuclear matter The deconfinement transition of QCD Finite size scaling based on Polyakov loops: a case study Towards quantitative predictions Density of states in gauge theories Perspective | 465 467 467 468 469 470 473 475 478 481 481 |
| 11 | Lattic 11.1 11.2 11.3 11.4 11.5 11.6 11.7 11.8 11.9 11.10 11.11 Refere | the gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for $Z(N)$ lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory Introduction: quantum chromodynamics (QCD) and phase transitions of nuclear matter The deconfinement transition of QCD Finite size scaling based on Polyakov loops: a case study Towards quantitative predictions Density of states in gauge theories Perspective ences | 465 467 467 468 469 470 473 475 478 481 481 482 |
| 11 | Lattic 11.1 11.2 11.3 11.4 11.5 11.6 11.7 11.8 11.9 11.10 11.11 Reference | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for $Z(N)$ lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory Introduction: quantum chromodynamics (QCD) and phase transitions of nuclear matter The deconfinement transition of QCD Finite size scaling based on Polyakov loops: a case study Towards quantitative predictions Density of states in gauge theories Perspective ences | 465 465 467 467 468 469 470 473 475 478 481 481 482 484 |
| 11 | Lattie 11.1 11.2 11.3 11.4 11.5 11.6 11.7 11.8 11.9 11.10 11.11 Reference A brie 12.1 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for $Z(N)$ lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory Introduction: quantum chromodynamics (QCD) and phase transitions of nuclear matter The deconfinement transition of QCD Finite size scaling based on Polyakov loops: a case study Towards quantitative predictions Density of states in gauge theories Perspective ences | 465 465 467 467 468 469 470 473 475 478 481 481 482 484 484 |
| 11 | Lattie 11.1 11.2 11.3 11.4 11.5 11.6 11.7 11.8 11.9 11.10 11.11 Reference A brie 12.1 12.2 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for Z(N) lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory Introduction: quantum chromodynamics (QCD) and phase transitions of nuclear matter The deconfinement transition of QCD Finite size scaling based on Polyakov loops: a case study Towards quantitative predictions Density of states in gauge theories Perspective ences ef review of other methods of computer simulation Introduction Molecular dynamics | 465 465 467 467 468 469 470 473 475 478 481 481 481 482 484 484 |
| 11 | Lattic 11.1 11.2 11.3 11.4 11.5 11.6 11.7 11.8 11.9 11.10 11.11 Reference 12.1 12.2 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for $Z(N)$ lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory Introduction: quantum chromodynamics (QCD) and phase transitions of nuclear matter The deconfinement transition of QCD Finite size scaling based on Polyakov loops: a case study Towards quantitative predictions Density of states in gauge theories Perspective ences ef review of other methods of computer simulation Introduction Molecular dynamics 12.2.1 Integration methods (microcanonical ensemble) | 465 467 467 468 469 470 473 475 478 481 481 481 482 484 484 484 |
| 11 | Lattic 11.1 11.2 11.3 11.4 11.5 11.6 11.7 11.8 11.9 11.10 11.11 Reference 12.1 12.2 | cc gauge models: a brief introductionIntroduction: gauge invariance and lattice gauge theorySome technical mattersResults for Z(N) lattice gauge modelsCompact U(1) gauge theorySU(2) lattice gauge theoryIntroduction: quantum chromodynamics (QCD) and phasetransitions of nuclear matterThe deconfinement transition of QCDFinite size scaling based on Polyakov loops: a case studyTowards quantitative predictionsDensity of states in gauge theoriesPerspectiveencesef review of other methods of computer simulationIntroductionMolecular dynamics12.2.1Integration methods (microcanonical ensemble)12.2.2Other ensembles (constant temperature, constant | 465 467 467 468 469 470 473 475 478 481 481 482 484 484 484 484 |
| 11 | Lattie 11.1 11.2 11.3 11.4 11.5 11.6 11.7 11.8 11.9 11.10 11.11 Reference 12.1 12.2 | cc gauge models: a brief introductionIntroduction: gauge invariance and lattice gauge theorySome technical mattersResults for Z(N) lattice gauge modelsCompact U(1) gauge theorySU(2) lattice gauge theoryIntroduction: quantum chromodynamics (QCD) and phasetransitions of nuclear matterThe deconfinement transition of QCDFinite size scaling based on Polyakov loops: a case studyTowards quantitative predictionsDensity of states in gauge theoriesPerspectiveencesef review of other methods of computer simulationIntroductionMolecular dynamics12.2.1Integration methods (microcanonical ensemble)12.2.2Other ensembles (constant temperature, constant pressure, etc.) | 465 465 467 467 468 469 470 473 475 478 481 481 482 484 484 484 484 484 484 |
| 11 | Lattie 11.1 11.2 11.3 11.4 11.5 11.6 11.7 11.8 11.9 11.10 11.11 Reference 12.1 12.2 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for Z(N) lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory Introduction: quantum chromodynamics (QCD) and phase transitions of nuclear matter The deconfinement transition of QCD Finite size scaling based on Polyakov loops: a case study Towards quantitative predictions Density of states in gauge theories Perspective ences ef review of other methods of computer simulation Introduction Molecular dynamics 12.2.1 Integration methods (microcanonical ensemble) 12.2.2 Other ensembles (constant temperature, constant pressure, etc.) | 465 465 467 467 468 469 470 473 475 478 481 481 481 482 484 484 484 484 484 484 484 484 |
| 11 | Lattie 11.1 11.2 11.3 11.4 11.5 11.6 11.7 11.8 11.9 11.10 11.11 Reference 12.1 12.2 | ce gauge models: a brief introduction Introduction: gauge invariance and lattice gauge theory Some technical matters Results for Z(N) lattice gauge models Compact U(1) gauge theory SU(2) lattice gauge theory Introduction: quantum chromodynamics (QCD) and phase transitions of nuclear matter The deconfinement transition of QCD Finite size scaling based on Polyakov loops: a case study Towards quantitative predictions Density of states in gauge theories Perspective ences ef review of other methods of computer simulation Introduction Molecular dynamics 12.2.1 Integration methods (microcanonical ensemble) 12.2.3 Non-equilibrium molecular dynamics 12.2.4 Hybrid methods (MD + MC) | 465 465 467 467 468 469 470 473 475 478 481 481 481 482 484 484 484 484 484 484 484 484 481 481 |

| | Contents | xiii |
|--------|--|-------|
| | 12.2.5 <i>Ab initio</i> molecular dynamics | 492 |
| | 12.2.6 Hyperdynamics and metadynamics | 493 |
| | 12.2.7 Molecular dynamics for systems with | 170 |
| | open boundaries | 494 |
| 12.3 | Ouasi-classical spin dynamics | 495 |
| | 12.3.1 Combined molecular dynamics-spin dynamics | |
| | (MD–SD) simulations | 499 |
| 12.4 | Langevin equations and variations (cell dynamics) | 500 |
| 12.5 | Micromagnetics | 501 |
| 12.6 | Dissipative particle dynamics (DPD) | 502 |
| 12.7 | Lattice gas cellular automata | 503 |
| 12.8 | Lattice Boltzmann equation | 504 |
| 12.9 | Multiscale simulation | 505 |
| 12.10 | Multiparticle collision dynamics | 507 |
| 12.11 | Active matter | 509 |
| 12.12 | Machine learning | 512 |
| Refe | rences | 515 |
| 13 Mon | to Carlo simulations at the periphery of physics | |
| and l | beyond | 510 |
| 13.1 | Commentary | 510 |
| 13.1 | Astrophysics | 510 |
| 13.2 | Materials science | 520 |
| 13.5 | Chemistry | 520 |
| 13.1 | 'Biologically inspired' physics | 524 |
| 15.5 | 13.5.1 Commentary and perspective | 524 |
| | 13.5.2 Lattice proteins | 524 |
| | 13.5.3. Cell sorting | 527 |
| 13.6 | Biology | 527 |
| 13.0 | Mathematics/statistics/computer science | 527 |
| 13.7 | Socionhysics | 530 |
| 13.0 | Fcononhysics | 530 |
| 13.10 | Conspirysies ('Traffic' simulations | 531 |
| 13.10 | Medicine | 533 |
| 13.11 | Networks: what connections really matter? | 534 |
| 13.12 | Finance | 535 |
| Refe | rences | 536 |
| 14.35 | | 540 |
| 14 Mon | te Carlo studies of biological molecules | 540 |
| 14.1 | Introduction | 540 |
| 14.2 | Protein folding | 541 |
| | 14.2.2 II introduction | 541 |
| | 14.2.2 How to best simulate proteins: Monte Carlo or | F 4 0 |
| | molecular dynamics | 542 |
| | 14.2.3 Generalized ensemble methods | 543 |
| | 14.2.4 Globular proteins: a case study | 545 |
| | 14.2.5 Simulations of membrane proteins | 545 |

xiv Contents

| | 14.3 Monte Carlo simulations of RNA structures | 548 |
|----|--|-----|
| | 14.4 Monte Carlo simulations of carbohydrates | 549 |
| | 14.5 Determining macromolecular structures | 551 |
| | 14.6 Outlook | 551 |
| | References | 552 |
| 15 | Emerging trends | 554 |
| | Index | 556 |

Preface

Historically physics was first known as 'natural philosophy' and research was carried out by purely theoretical (or philosophical) investigation. True progress was obviously limited by the lack of real knowledge of whether or not a given theory really applied to nature. Eventually experimental investigation became an accepted form of research although it was always limited by the physicist's ability to prepare a sample for study or to devise techniques to probe for the desired properties. With the advent of computers it became possible to carry out simulations of models which were intractable using 'classical' theoretical techniques. In many cases computers have, for the first time in history, enabled physicists not only to invent new models for various aspects of nature but also to solve those same models without substantial simplification. For a number of years computer power has increased quite dramatically, with access to computers becoming both easier and more common (e.g. with personal computers and workstations), and computer simulation methods have also been steadily refined. As a result computer simulations have become another way of doing physics research. They provide another perspective; in some cases simulations provide a theoretical basis for understanding experimental results, and in other instances simulations provide 'experimental' data with which theory may be compared. There are numerous situations in which direct comparison between analytical theory and experiment is inconclusive. For example, the theory of phase transitions in condensed matter must begin with the choice of a Hamiltonian, and it is seldom clear to what extent a particular model actually represents real materials on which experiments are done. Since analytical treatments also usually require mathematical approximations whose accuracy is difficult to assess or control, one does not know whether discrepancies between theory and experiment should be attributed to shortcomings of the model, the approximations, or both. The goal of this text is to provide a basic understanding of the methods and philosophy of computer simulations research with an emphasis on problems in statistical thermodynamics as applied to condensed matter physics or materials science. There exist many other simulational problems in physics (e.g. simulating the spectral intensity reaching a detector in a scattering experiment) which are more straightforward and which will only

CAMBRIDGE

Cambridge University Press 978-1-108-49014-6 — A Guide to Monte Carlo Simulations in Statistical Physics David Landau , Kurt Binder Frontmatter <u>More Information</u>

xvi Preface

occasionally be mentioned. We shall use many specific examples and, in some cases, give explicit computer algorithms, but we wish to emphasize that these methods are applicable to a wide variety of systems including those which are not treated here at all. As computer architecture changes, the methods presented here will in some cases require relatively minor reprogramming and in other instances will require new algorithm development in order to be truly efficient. We hope that this material will prepare the reader for studying new and different problems using both existing as well as new computers.

At this juncture we wish to emphasize that it is important that the simulation algorithm and conditions be chosen with the physics problem at hand in mind. The *interpretation* of the resultant output is critical to the success of any simulational project, and we thus include substantial information about various aspects of thermodynamics and statistical physics to help strengthen this connection. We also wish to draw the reader's attention to the rapid development of scientific visualization and the important role that it can play in producing *understanding* of the results of some simulations.

This book is intended to serve as an introduction to Monte Carlo methods for graduate students, and advanced undergraduates, as well as more senior researchers who are not vet experienced in computer simulations. The book is divided up in such a way that it will be useful for courses which only wish to deal with a restricted number of topics. Some of the later chapters may simply be skipped without affecting the understanding of the chapters which follow. Because of the immensity of the subject, as well as the existence of a number of very good monographs and articles on advanced topics which have become quite technical, we will limit our discussion in certain areas, e.g. polymers, to an introductory level. Many existing Monte Carlo programs and related subprograms are in FORTRAN and will be available to the student from libraries, journals, etc. (FORTRAN has also evolved dramatically over its more than 60 years of existence, and the newest versions are efficient and well suited for operations involving arrays and for parallel algorithms. Object oriented languages, like C++, while useful for writing complex programs, can be far more difficult to learn. Programs written in popular, non-compiler languages, like Java or MATLAB, can be more difficult to debug and run relatively slowly. Nevertheless, all the methods described in this book can be implemented using the reader's 'language of choice'.) A number of sample problems are suggested in the various chapters; these may be assigned by course instructors or worked out by students on their own. Our experience in assigning problems to students taking a graduate course in simulations at the University of Georgia over a more than 35-year period suggests that for maximum pedagogical benefit, students should be required to prepare cogent reports after completing each assigned simulational problem. Students were required to complete seven 'projects' in the course of the semester for which they needed to write and debug programs, take and analyze data, and prepare a report. Each report should briefly describe the algorithm used, provide sample data and data analysis, draw conclusions, and add comments. (A sample program/output should be included.) In this way, the students

CAMBRIDGE

Cambridge University Press 978-1-108-49014-6 — A Guide to Monte Carlo Simulations in Statistical Physics David Landau , Kurt Binder Frontmatter <u>More Information</u>

Preface xvii

obtain practice in the summary and presentation of simulational results, a skill which will prove to be valuable later in their careers. For convenience, many of the case studies that are described have been simply taken from the research of the authors of this book – the reader should be aware that this is by no means meant as a negative statement on the quality of the research of numerous other groups in the field. Similarly, selected references are given to aid the reader in finding more detailed information, but because of length restrictions it is simply not possible to provide a complete list of relevant literature. Many coworkers have been involved in the work which is mentioned here, and it is a pleasure to thank them for their fruitful collaboration. We have also benefited from the stimulating comments of many of our colleagues and we wish to express our thanks to them as well.

The pace of developments in computer simulations continues at breakneck speed. This fifth edition of our 'guide' to Monte Carlo simulations updates some of the references and includes numerous additions reflecting new algorithms that have appeared since work on the fourth edition was completed. The emphasis on the use of Monte Carlo simulations in biologically related problems in the fourth edition proved to foretell the future, as the use of Monte Carlo methods for the study of biological molecules has continued to expand. Similarly, the use of Monte Carlo methods in 'non-traditional' areas of research has continued to grow. (Monte Carlo methods are now widely used in industry for applications as diverse as risk management, optimization of manufacturing tool selection, and modeling turbomolecular pumps. Since our present focus is on the use of Monte Carlo methods in statistical physics, we direct the interested reader elsewhere for information on these topics.) There have been exciting new developments in computer hardware; in particular, the use of GPUs in scientific computing has dramatically altered the price/performance ratio for many algorithmic implementations. In particular, almost all of the most powerful supercomputers are now hybrid combinations of many multi-core CPUs and GPUs. Because of advances in computer technology and algorithms, new results often have much higher statistical precision than some of the older examples in the text. Nonetheless, the older work often provides valuable pedagogical information for the student and may also be more readable than more recent, and more compact, papers. An additional advantage is that the reader can easily reproduce some of the older results with only a modest investment of modern computer resources. Of course, newer, higher resolution studies that are cited often permit yet additional information to be extracted from simulational data, so striving for higher precision should not be viewed as 'busy work'. While earlier editions of this text included an Appendix with sample codes written in Fortran77, students are now often introduced to programming with Python and many research level codes are now written in Fortran 90/95 or C++. In the future it is likely that other languages will also enter the fray although the algorithms themselves will remain unchanged. For this reason we decided to eliminate the sample programs from this Edition but have made program listings available on the website of the Center for Simulational Physics

xviii Preface

www.csp.uga.edu/MCbook for the interested reader. We hope that this guide will help impart to the reader not only an understanding of the methodology of Monte Carlo simulations but also an appreciation for the new science that can be uncovered with the Monte Carlo method.