Notation Index

General notation:

$\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, \mathbb{T}$: the usual number sets (natural, integral, rational, real,. complex, complex unimodular)

1: the unit element of an algebra; whenever it appears in a tensor product of algebras, it stands for the scalar algebra $\mathbb{C} \cdot 1$

ι: the identity mapping

$\mathscr{A}(G)$ (18.7); $\mathfrak{U}_\varphi$ (2.12); $\mathfrak{U}'_\varphi$ (2.12); $A_{\tau,u}$ (27.4); $\mathrm{Ad}(u)$ (2.23); $\mathrm{Ad}(\sigma)$ (15.15); $\mathrm{ad}(\delta)$ (15.15); $Aut(\mathscr{M})$ (2.23); $Aut_\varphi(\mathscr{M})$ (2.25); $Aut(F^{\mathscr{M}})$ (25.1)

$\mathbb{B}$ (28.10); $\mathscr{B}_w(\mathscr{M})$ (2.23); $\mathscr{B}(\mathscr{X}, \mathscr{Y})$ (13.1); $\mathscr{B}_w(\mathscr{X}, \mathscr{Y})$ (13.1); $\mathscr{B}_w(\mathscr{X}, \mathscr{Y})_*$ (13.1); $B(F^{\mathscr{M}})$ (26.1)

$c(a,b)$ (20.2); $c(e,f)$ (17.15); $c_\varphi(\psi)$ (24.2); co (convex hull); $\overline{co}^w$ (w-closed convex hull)

$D(A)$, $Dom(A)$ (domain of an operator, A.1); $D(\mathscr{H}, \psi)$ (7.1); $d(\varphi_1, \varphi_2)$ (6.10); $\mathrm{d}g$, $\mathrm{d}\gamma$ (18.8); $\mathrm{d}\mu/\mathrm{d}\nu$ (Radon–Nikodym derivative); $[DF : DE]_t$ (11.15); $[D\psi : D\varphi]_t$ (3.1); $[D\psi : D\varphi]_c$ (26.3)

$E^\varphi_{\mathscr{N}}$ (9.8, 12.18); $E^{\mathscr{N}}_\varphi$ (10.4); e_n (4.4); $e_n(A)$ (A.1); $\{e_{ij}\}_{1 \le i,j \le n}$ (system of matrix units, 3.2); $\mathscr{M}^\sigma_{exp}$ (11.10)

F (18.8); $\mathfrak{F}(\sigma)$ (16.2); $\mathscr{F}_n$ (the type I_n factor); F_k (22.6); $F_\lambda = F^{\mathscr{M}}_\lambda$ (24.1); $\mathfrak{F}^{\mathscr{M}}_\lambda$ (24.9); $\mathfrak{F}_E$ (11.5); $\mathfrak{F}_\varphi$ (1.1); f_α (1.5)

$\mathbf{G}$ (18.5); $\hat{\mathbf{G}}$ (18.7); $G(\mathscr{N})$ (25.7)

$\mathscr{H}_\varphi$ (1.2); $H(F^{\mathscr{M}})$ (26.1)

$\mathscr{I}(a,b)$ (20.2); $\mathscr{I}^U_x$ (14.2); $Int(\mathscr{M})$ (3.2); $Int\ \sigma$ (16.4)

J_φ (2.12); j_φ (2.12); $J_{\psi,\varphi}$ (3.11); $\mathscr{J}(\mathscr{H}, \psi)$ (7.1); J_G (18.4); j_G (18.7); $\mathscr{J}_0(F)$ (14.1); $\mathscr{J}_{00}(F)$ (14.1)

$\mathscr{K}(G)$ (18.4); K_G (18.4); k_G (18.5); $\mathscr{K}(F)$ (14.1); $Ker\ \sigma$ (16.4)

$\mathfrak{L}(G)$ (18.4); $\mathscr{L}^\infty(G)$ (18.4); L^0_ξ, L_ξ (2.12); $\mathscr{L}^p(\mathscr{H}, \psi)$ (7.16); $\mathbf{l}(\varphi)$ (4.19); $\mathbf{l}(x)$ (left support of an element, [L])

$\mathscr{M}(G)$ (13.2); $\mathfrak{M}_\varphi$ (1.1); $\mathfrak{M}_E$ (11.5); $\mathbf{m}(g)$ (18.8); $\mathbf{m}(\gamma)$ (18.9); $mod(\sigma)$ (25.1); $Mat_2(\mathscr{M})$ (matrix algebra over $\mathscr{M}$, 3.1, 3.3)

$\mathscr{N}(E)$ (10.17); $\mathfrak{N}_\varphi$ (1.1); $\mathfrak{N}_E$ (11.5)

$\mathfrak{o}_{\mathscr{M}}$ (3.2); $Out(\mathscr{M})$ (3.2)

P_δ (18.19, 19.7); P_σ (18.20), 19.16); $P_{\hat{\sigma}}$ (19.8, 22.2); P_φ (24.6); $(\mathscr{P}_{\mathscr{M}}, F^{\mathscr{M}})$ (25.1); $\mathscr{P}_\lambda$ (22.16); $\mathfrak{P}_\varphi$ (2.23); $p_u(\cdot)$ (14.10); $p(\sigma)$ (17.2); p_φ (24.5); p_σ (25.1); $PV \int_{-\infty}^{+\infty}$ (A.12); $Proj(\mathscr{M})$ (the projection lattice of $\mathscr{M}$; p_ξ (cyclic projection); $P(\mathscr{M}, \mathscr{N})$ (11.5)

$\mathscr{Q}(\sigma; E)$ (15.5); $q(\sigma; E)$ (15.5); q^σ_t (15.6); q^σ_∞ (15.6)

$\mathfrak{R}(G)$ (18.4); R^0_η, R_η (2.12); $\mathscr{R}(\mathscr{M}, \delta)$ (19.1); $\mathscr{R}(\mathscr{M}, \sigma)$ (19.1, 22.1); R^ψ_η (7.1); $\mathbb{R}_d$ (28.10); $\mathscr{R}_\lambda$ (A.17); $\mathscr{R}(\{\lambda_k\}_{k \in \mathbb{N}}$ (A.17); $r(\varphi, \psi)$ (23.17); $\mathfrak{r}(G)$ (28.6); $\mathbf{r}(\varphi)$ (4.19); $\mathbf{r}(x)$ (right support of an element, [L]); $\mathscr{R}\{\mathscr{X}\}$ (von Neumann algebra generated by $\mathscr{X}$)

$S(\mathscr{M})$ (18.2); $\mathfrak{S}_\varphi$ (6.5); S^0_φ, S_φ (2.12); $S^0_{\psi,\varphi}, S_{\psi,\varphi}$ (3.11); S^φ_t (7.6); $Sp\ U$ (14.5); $Sp_U(x)$ (14.2); $Sp(x), Sp_{\mathscr{A}}(x)$ (usual spectrum, 14.6); $Sp(\varphi_k/\mathscr{M}_k)$ (A.17); $S(\infty)$ (22.6); $s(e,f)$ (17.15); $\mathbf{s}(\varphi)$ (2.1,4.19); $\mathbf{s}(E)$ (11.5); $\mathbf{s}(x)$ (support of an element); $\mathscr{S}(A)$ ($\mathscr{S}_A$) [L, 9.9)])

$T(\mathscr{M})$ (27.1); $\mathfrak{T}_\varphi$ (2.12); $\mathscr{T}'_\varphi$ (2.4); T^σ_f (18.21); T^σ_a (22.1); tr (the canonical trace on $\mathscr{B}(\mathscr{H})$)

$U(\mathscr{H})$ (2.23); $U(\mathscr{M})$ (3.2); u_σ (2.23)

V_G (18.4); $V_{\psi,\varphi}$ (3.16)

W_G (18.4); $W_n(\mathscr{M})$ (§23); $W_{ns}(\mathscr{M})$ (§23); $W_{nsf}(\mathscr{M})$ (6.9), §23); $W_{int}(\mathscr{M})$ (24.1); $W^\infty_{int}(\mathscr{M})$ (24.1); $W(\omega)$ (25.4); $W(H,K)$ (30.8)

$Z_\sigma(G; \mathscr{M})$ (5.1, 20.1); $Z_\sigma(G; U(\mathscr{M}))$ (20.3); $Z(F^{\mathscr{M}})$ (26.1); $Z(\mathscr{J})$ (14.1); $\mathscr{Z}(\mathscr{M})$ (centre of $\mathscr{M}$).

$\Gamma(\sigma)$ (16.1); $\Gamma(\mathscr{M})$ (30.10)

Δ_G (18.4); Γ_φ (2.12); $\Delta_{\psi\varphi}$ (3.11); $\Delta(\varphi/\psi)$ (7.3)

δ_G (18.7); δ^*_G (18.7); $\delta_{\mathscr{M}}$ (3.2, 26.4); δ_x, δ_a (10.29, 15.13); δ^s_t (Kronecker symbol, 22.1)

$\partial = \partial_\sigma$ (25.3); $\partial : U(\mathscr{P}_{\mathscr{M}}) \to Z(F^{\mathscr{M}})$ (26.1)

∇_G (18.4)

χ_I (characteristic function of I)

Λ_f (18.22)

$\boldsymbol{\lambda}$ (18.4)

μ_G (18.4)

ν_φ (24.7)

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π_G (18.5); π_φ (1.2); π_σ (18.6); π_φ^δ (18.10)
$\boldsymbol{\rho}$ (18.4)
$\sum_n^\otimes \psi_n$ (23.15)
σ_t^φ (2.12); σ_c^φ (26.3); σ_α^φ (2.14); σ_t^E (11.15); $\sigma_t^{\psi,\varphi}$ (3.10); $\sigma_\alpha^{\psi,\varphi}$ (3.12); $\sigma(\mathscr{X},\mathscr{Y})$ (weak topology defined by $\mathscr{Y}$ on $\mathscr{X}$)
$\theta(\varphi,\psi)$ (3.1)
τ_φ (24.6)
ω_G (18.4)

Other symbols:

$\approx$: isomorphism
$\sim$: $e \sim f$ (equivalence of projections); $\sigma \sim \tau$ (15.11); $(\mathscr{M},\sigma) \sim (\mathscr{N},\tau)$ (20.13); $a \sim b$ (20.2); $\sim$: $\mathscr{H} \bar{\otimes} \mathscr{K} \to \mathscr{K} \bar{\otimes} \mathscr{H}$ (18.1); $\tilde{\sim}$: $\mathscr{M} \bar{\otimes} \mathscr{N} \to \mathscr{N} \bar{\otimes} \mathscr{M}$ (18.1)
$\precapprox$: $a \precapprox b$ (20.2); $\psi \precapprox \varphi$ (23.1)
$\lesssim$: $a \lesssim b$ (20.2); $\psi \lesssim \varphi$ (23.1)
$\leq$: $A \leq B$ (A.4); $\varphi_2 \leq \varphi_1(\lambda)$ (6.9)
$<$: $H < K$ (30.4)
$\wedge$: Fourier transform $\hat{f}$, $\hat{\mu}$ (14.1), $\hat{\xi}$ (18.8), $\hat{f}$ (28.1), $\hat{x}(k)$ (16.17), $\hat{x}(\gamma)$ (21.3); dual group $\hat{G}$ (14.1); dual homomorphism $\hat{\varphi}$ (14.9); dual action $\hat{\sigma}$ (19.3), $\hat{\delta}$ (19.4); dual weight $\hat{\varphi}$ (19.8, 91.17); $\hat{E}$ (1.6)
$\check{}$: $\check{b}$ (20.7); $\check{\varphi}$ (23.15)
$*$: predual space $\mathscr{M}_*$ (A.16); $\mathscr{B}_w(\mathscr{X},\mathscr{Y})_*$ (13.1); convolution (18.4), 18.22)
$\sharp$: involution (18.4, 18.22)
$+$: positive part $\mathscr{X}^+$, $\mathscr{X}_+^*$ (1.6); $\overline{\mathscr{M}^+}$ (11.1); sum $A \hat{+} B$ (A.11)
: closure $\bar{A}$, $\bar{q}$ (A.9); $\overline{\mathscr{M}^+}$ (11.1); $\bar{t}$ (26.2)
$\perp$: $E^\perp$ (16.4); $H^\perp$ (21.5); orthogonality
$\natural$: canonical centre valued trace (12.14)
[ ]: full group $[G]$ (17.3); matrix $[x_{ij}]$
$\cdot$: $\varphi(\cdot a)$, $\varphi(a \cdot)$ (2.13); $h \cdot k$, $\varphi \cdot k$ (18.3); $k(\cdot)$ (18.7)
$\circ$: k^0 (18.3)
' : φ' (2.12); commutant
$\uparrow$: $A_k \uparrow A$ (A.5)
∞ : $\mathscr{M}_\infty^\varphi$ (2.15)
$\times$: $G \times_\sigma T$ (22.10)
$\otimes$: various tensor products (3.9, 8.1, 8.2, 9.4, 12.8, 20.4, A.17, [L]); $\xi \otimes \eta$ (4.23)
$\| \ \|$: $\|x\|_\varphi$, $\|x\|_\varphi^\sharp$ (1.2, 7.20); $\|x_1\|$, $\|x_2\|$ (17.17)
$(|)$: $(a|b)_\varphi$ (1.2)
$\langle , \rangle$: $\langle t, \gamma \rangle$ (14.1); $\langle x, \varphi \rangle$ (18.1)
[,): $[H, K)$ (30.8)
juxtaposition: φ_a (4.1); φ_A (4.4); A_ε (A.1); ${}_a\sigma$ (20.1); a^p (20.1); a_φ (1.2); σ^e (15.4); φ_e (2.21); $\mathscr{M}_e$ (reduced algebra); $\mathscr{M}^\varphi$ (2.21); $\mathscr{M}^a$ (20.1); $\mathscr{N}^\lambda$ (25.7); U_μ, U_f (13.2); φ_ν (2.21); $\mathscr{X}^U$ (14.3); $\mathscr{X}(U;E)$ (14.3); $\mathscr{X}_0(U;E)$ (14.3); $\mathscr{X}_{00}(U;E)$ (14.3)

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