The Physics of Neutrino Interactions

This advanced text discusses the fundamental concepts of neutrinos, their properties and interactions with matter, presenting a theoretical framework for describing relativistic particles. It provides a pedagogical description of the field theory of neutrinos, necessary to understand the standard model (SM) of electroweak interactions, and neutrino scattering from leptons and nucleons. Applications of neutrino scattering processes from the nucleons and nuclei are discussed in detail. Nuclear-medium effects in quasielastic scattering, and inelastic and deep inelastic scattering are also covered in depth. A separate chapter on neutrinos in astrophysics highlights the applications of various neutrino processes in the understanding of the universe and its evolution. The text introduces the subject of neutrino oscillations and highlights the need for beyond the standard model (BSM) physics. This topical book will stimulate new ideas and avenues for research, and will form a valuable resource for advanced graduate students and academic researchers in the fields of particle physics and nuclear physics.

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The Physics of Neutrino Interactions

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To

Fouzia Ahmed (wife of MSA)
and
Shashi Singh (wife of SKS)

for their patience, love and support
## Contents

- **List of Figures**  
  xxii
- **List of Tables**  
  xxxv
- **Preface**  
  xxxix
- **Acknowledgments**  
  xliii

### 1 Neutrino Properties and Its Interactions  
1
   1.1 Historical Introduction to Neutrinos  
1.1.1 Neutrino hypothesis  
1.1.2 The problem of energy conservation in $\beta$-decays of nuclei  
1.1.3 Anomalies in the spin–statistics relation for nuclei  
1.1.4 Pauli’s neutron/neutrino vs. Fermi’s neutrino  
2
   1.2 Neutrino Interactions  
5
   1.2.1 Fermi theory of $\beta$-decay  
6
   1.2.2 Gamow–Teller theory  
8
   1.2.3 Parity violation and the two-component neutrino  
10
   1.2.4 Chiral ($\gamma_5$) invariance and $V - A$ theory of $\beta$-decays  
11
   1.2.5 Intermediate vector boson (IVB)  
12
   1.2.6 Weak interactions in strangeness sector: The Cabibbo theory, and the GIM mechanism  
14
   1.2.7 Quark flavors and the CKM matrix  
16
   1.2.8 Nonleptonic weak interaction and CP violation  
17
   1.3 Neutrino Flavors and Universality of Neutrino Interactions  
18
   1.3.1 Experimental discovery of $\bar{\nu}_e$ and $\bar{\nu}_e \neq \nu_e$  
18
   1.3.2 Discovery of muons and muon neutrinos  
20
   1.3.3 Lepton number conservation and $e - \mu$ universality  
21
   1.3.4 Discovery of tau neutrino and $e - \mu - \tau$ universality  
22
   1.4 Properties of Neutrinos  
23
   1.4.1 Weyl, Dirac and Majorana neutrinos  
23
   1.4.2 Neutrino mass  
26
   1.4.3 Neutrino charge and charge radius  
28
1.4.4 Magnetic and electric dipole moments of neutrinos 29
1.4.5 Helicity of neutrino 30
1.5 New Developments 31
1.5.1 Standard model of electroweak interactions and neutral currents 31
1.5.2 Discovery of $W^\pm$, $Z^0$, and Higgs boson 33
1.5.3 Neutrino mass, mixing, and oscillations 34
1.5.4 Matter enhancement of neutrino oscillations and the MSW (Mikheyev–Smirnov–Wolfenstein) effect 37
1.5.5 Three-flavor neutrino oscillations and mass hierarchy 39
1.5.6 Sterile neutrinos and 3+1 flavor mixing 40
1.6 Summary 41

2 Relativistic Particles and Neutrinos 42
2.1 Relativistic Notation 42
2.1.1 Metric tensor 42
2.1.2 Contravariant and covariant vectors 44
2.2 Wave Equation for a Relativistic Particle 45
2.2.1 Klein–Gordon equation for spin 0 particles 45
2.3 Dirac Equation for Spin $\frac{1}{2}$ Particles 49
2.3.1 Spin of a Dirac particle 53
2.3.2 Plane wave solutions of the Dirac equation 54
2.3.3 Normalization of Dirac spinors 59
2.4 Negative Energy States and Hole Theory 61
2.5 Projection Operators 62
2.5.1 Energy projection operators 62
2.5.2 Spin projection operators 63
2.5.3 Helicity and helicity projection operators 64
2.6 Massless Spin $\frac{1}{2}$ Particle and Weyl Equation 65
2.6.1 Equation of motion for massless particles 65
2.6.2 Equation of motion in Weyl representation 67
2.6.3 Chirality and chirality projection operators 68
2.7 Relativistic Spin 1 Particles 69
2.7.1 Massless spin 1 particles 69
2.7.2 Covariant form of Maxwell’s equations and gauge invariance 72
2.7.3 Plane wave solution of photon 74
2.7.4 Massive spin 1 particles 75
2.8 Wave Equation for Particle with Spin $\frac{3}{2}$ 78
2.9 Discrete Symmetry: Parity, Time Reversal, and Charge Conjugation 80
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.9.1 Parity</td>
<td>80</td>
</tr>
<tr>
<td>2.9.2 Dirac equation under parity transformation</td>
<td>82</td>
</tr>
<tr>
<td>2.9.3 Charge conjugation</td>
<td>84</td>
</tr>
<tr>
<td>2.9.4 Time reversal</td>
<td>88</td>
</tr>
<tr>
<td>3 Quantization of Free Particle Fields</td>
<td>96</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>96</td>
</tr>
<tr>
<td>3.2 Lagrangian Formulation for the Dynamics of Particles and Fields</td>
<td>97</td>
</tr>
<tr>
<td>3.2.1 Equation of motion for particles</td>
<td>97</td>
</tr>
<tr>
<td>3.2.2 Quantization of a harmonic oscillator</td>
<td>100</td>
</tr>
<tr>
<td>3.2.3 Equation of motion for fields</td>
<td>102</td>
</tr>
<tr>
<td>3.2.4 Symmetries and conservation laws: Noether’s theorem</td>
<td>103</td>
</tr>
<tr>
<td>3.3 Quantization of Scalar Fields: Klein–Gordon Field</td>
<td>106</td>
</tr>
<tr>
<td>3.3.1 Real scalar field: Creation and annihilation operators</td>
<td>106</td>
</tr>
<tr>
<td>3.3.2 Fock space</td>
<td>110</td>
</tr>
<tr>
<td>3.4 Complex Scalar Field</td>
<td>112</td>
</tr>
<tr>
<td>3.4.1 Creation and annihilation operators</td>
<td>112</td>
</tr>
<tr>
<td>3.4.2 Charge of the complex scalar field: Particles and antiparticles</td>
<td>115</td>
</tr>
<tr>
<td>3.4.3 Covariant commutation relation</td>
<td>116</td>
</tr>
<tr>
<td>3.5 Time-ordered Product and Propagators for Scalar Fields</td>
<td>119</td>
</tr>
<tr>
<td>3.6 Quantization of Spin $\frac{1}{2}$ Fields</td>
<td>123</td>
</tr>
<tr>
<td>3.7 Covariant Anticommutators and Propagators for Spin $\frac{1}{2}$ Fields</td>
<td>127</td>
</tr>
<tr>
<td>3.8 Time-ordered Products and Feynman Propagators</td>
<td>128</td>
</tr>
<tr>
<td>3.9 Quantization of Massless Electromagnetic Fields: Photons</td>
<td>130</td>
</tr>
<tr>
<td>3.10 Commutation Relations and Quantization of $A^\mu(x)$</td>
<td>132</td>
</tr>
<tr>
<td>3.11 Lorenz Condition and Gupta–Bleuler Formalism</td>
<td>134</td>
</tr>
<tr>
<td>3.12 Time-ordered Product and Propagators for Spin 1 Fields</td>
<td>135</td>
</tr>
<tr>
<td>4 Interacting Fields and Relativistic Perturbation Theory</td>
<td>139</td>
</tr>
<tr>
<td>4.1 Introduction</td>
<td>139</td>
</tr>
<tr>
<td>4.2 Simple Forms of Interaction Lagrangians of Fields</td>
<td>140</td>
</tr>
<tr>
<td>4.2.1 Electromagnetic interactions</td>
<td>140</td>
</tr>
<tr>
<td>4.2.2 Weak interactions</td>
<td>141</td>
</tr>
<tr>
<td>4.2.3 Strong interactions</td>
<td>141</td>
</tr>
<tr>
<td>4.2.4 Self interaction of various fields</td>
<td>141</td>
</tr>
<tr>
<td>4.3 Evolution of Physical Systems and the S-matrix</td>
<td>142</td>
</tr>
<tr>
<td>4.3.1 Schrödinger, Heisenberg, and Interaction picture</td>
<td>142</td>
</tr>
<tr>
<td>4.3.2 S-Matrix and relativistic perturbation theory</td>
<td>145</td>
</tr>
</tbody>
</table>
## Contents

4.4 Dyson Expansion and Wick’s Theorem  
4.4.1 Dyson expansion  
4.4.2 Wick’s theorem  
4.5 S-matrix and Feynman Diagrams  
4.6 Invariant Matrix Elements and Feynman Diagrams  
4.6.1 Matrix elements in first order perturbation theory  
4.6.2 Matrix elements in second order perturbation theory  
4.6.3 Matrix elements for the closed loops  
4.6.4 Feynman rules: A summary  
4.7 Scattering Cross Sections and Particle Decay Rates  
4.7.1 Scattering cross sections  
4.7.2 Particle decay rates  

5 Phenomenological Theory I: Nuclear $\beta$-decays and Weak Interaction of Leptons  
5.1 Introduction  
5.2 Development of Phenomenological Theory  
5.2.1 Fermi and Gamow–Teller theories of $\beta$-decays  
5.2.2 General form of the Hamiltonian and the parity violation in $\beta$-decays  
5.2.3 The energy and angular distribution of electrons (positrons) for the $\beta^+$ ($\beta^-$)-decay of unpolarized nuclei  
5.2.4 The longitudinal polarization of $e^- (e^+)$ from $\beta^-(\beta^+)$-decays of unpolarized nuclei  
5.2.5 Helicity of the neutrino  
5.2.6 Spin–momentum correlations in the $\beta$-decay of polarized nuclei  
5.2.7 Mixed $\beta$-transitions and sign of $C_V$  
5.3 Two-component Neutrino and the $V – A$ Theory  
5.4 Weak Interaction of Muon  
5.4.1 Weak decay of muons  
5.4.2 General structure of weak interaction in muon-decay and Michel parameters  
5.4.3 Radiative corrections to $\mu$-decays  
5.5 Inverse Muon-decay and $\nu_\mu$ Scattering  
5.6 Muon Capture and $\mu – e$ Universality  
5.7 Limitations of the Phenomenological Theory  
5.7.1 High energy behavior of $\nu_l – l^-$ scattering and unitarity  
5.7.2 Divergence and renormalization  
5.7.3 Intermediate vector boson (IVB) theory  
5.7.4 Radiative corrections  
5.8 $\tau$ Lepton and Its Weak Decays and $e – \mu – \tau$ Universality  
5.8.1 $\tau$ lepton and its properties
5.8.2 Weak decays of \( \tau \) leptons and \( e - \mu - \tau \) universality 212

6 Phenomenological Theory II: Weak Decays of Hadrons 215

6.1 Introduction 215

6.2 Semileptonic Weak Decays of Hadrons without Strangeness 217

6.2.1 Two-body decay of pions: \( \pi l_2 \) decays 217

6.2.2 Three-body decays of pions: \( \pi l_3 \) decays 221

6.3 Symmetry Properties of the Weak Hadronic Current 222

6.3.1 Lorentz transformation properties and matrix elements 223

6.3.2 Isospin properties of the weak hadronic current 224

6.3.3 T invariance 225

6.3.4 Conserved vector current hypothesis 226

6.3.5 Implications of the CVC hypothesis 226

6.3.6 Partial conservation of axial vector current (PCAC) 228

6.3.7 Implications of PCAC 229

6.3.8 G-parity and second class currents 233

6.4 Semileptonic Weak Decays of Hadrons with Strangeness 234

6.4.1 The Cabibbo theory and the universality of weak interactions 236

6.4.2 The Cabibbo theory in the quark model and quark mixing 237

6.4.3 Applications of Cabibbo theory: K decays 239

6.4.4 Semileptonic decays of hyperons (\( Y \)) 242

6.4.5 Physical observables in semileptonic hyperon decays 244

6.5 Nonleptonic Decays of Strange Particles 247

6.5.1 Nonleptonic decays of \( K \)-mesons 247

6.5.2 Nonleptonic decays of hyperons 249

6.5.3 Radiative weak decays 251

6.6 CP Violation in the Neutral Kaon Sector 253

6.6.1 Neutral kaons, CP eigenstates, and \( K^0 - \bar{K}^0 \) oscillations 253

6.6.2 CP violation in the neutral kaon decays 257

6.7 Flavour Changing Neutral Currents (FCNC) and GIM Mechanism 259

6.7.1 Six quark mixing and CKM matrix 262

6.8 Weak Interaction of Hadrons with Charm and Heavy Flavors 264

6.8.1 Discovery of charm and heavy flavors 264

6.8.2 Weak decays of particles with charm and heavy flavors 266

6.8.3 Weak decays of particles with charm 266

6.8.4 Weak decays of particles with heavy flavors 269

6.9 Limitations of the Phenomenological Theory from the Hadron Sector 269
7 Gauge Field Theories and Fundamental Interactions

7.1 Introduction

7.2 Gauge Invariance in Field Theory

7.3 Local Gauge Symmetries and Fundamental Interactions

8 Unified Theory of Electroweak Interactions

8.1 Introduction

8.2 Description of the Weinberg–Salam Model for Leptons

8.3 Predictions of the Weinberg–Salam Model

8.4 Extension to the Leptons of Other Flavors

8.5 Discovery of Neutral Currents in Electron Scattering

8.6 Discovery of $W^\pm$ and $Z$ Bosons

8.7 Higgs Boson

8.7.1 Discovery of Higgs boson
9 Neutrino and Electron Scattering from Point Particles 362

9.1 Introduction 362
9.2 $e^- + \mu^- \rightarrow e^- + \mu^-$ scattering 363
9.3 $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$ scattering 368
9.4 $\nu_\mu + e^- \rightarrow \nu_\mu + e^-$ scattering 370
9.5 $\nu_e + e^- \rightarrow \nu_e + e^-$ scattering 373
9.5.1 Determination of the magnetic moment of neutrinos 377
9.6 $e^- + e^- \rightarrow \mu^- + \mu^-$ 378

10 Neutrino scattering Cross Sections from Hadrons: Quasielastic Scattering 385

10.1 Introduction 385
10.2 Physical Significance of the Form Factor 386
10.3 $e^- - \pi^\pm$ Elastic Scattering 389
10.4 Electromagnetic Scattering of Electrons with Nucleons 391
10.4.1 Matrix element and form factors 391
10.4.2 Physical interpretation of the form factors 394
10.4.3 Cross sections and the Rosenbluth separation 396
10.4.4 Experimental determination of the form factor 398
10.4.5 Numerical parameterization of the electromagnetic form factors 400
10.5 Quasielastic and Elastic $\nu$ Scattering Processes on Nucleons 402
10.5.1 Introduction 402
10.5.2 Interaction Lagrangian 403
10.5.3 Charged current quasielastic reaction and weak nucleon form factors 404
10.5.4 Neutral current elastic reactions and weak nucleon form factors 407
10.5.5 Symmetry properties of weak hadronic currents and form factors 408
10.5.6 Parameterization of the weak form factors 408
10.5.7 Cross sections for charged current processes 412
10.6 Quasielastic Hyperon Production 414
10.6.1 Matrix elements and form factors 414
10.6.2 Vector form factors 416
10.6.3 Axial vector form factors 416
10.6.4 Cross sections: Experimental results 417
10.7 Polarization of Final Hadrons and Leptons 418
10.7.1 Introduction 418
10.7.2 Polarization of the final hadron 419
10.7.3 Polarization of the final lepton 423
## Contents

### 11 Neutrino Scattering from Hadrons: Inelastic Scattering (I) 425

11.1 Introduction 425
11.2 Inelastic Scattering through CC Excitation of Resonances 430
  11.2.1 CC excitation of spin $\frac{1}{2}$ resonances 430
  11.2.2 CC excitation of spin $\frac{3}{2}$ resonances 446
11.3 Neutral Current Reactions 457
  11.3.1 Excitation of spin $\frac{1}{2}$ resonances 457
  11.3.2 Excitation of spin $\frac{3}{2}$ resonances 461
11.4 Non-resonant Contributions 463
  11.4.1 Chiral symmetry 465
  11.4.2 Transformation of mesons under chiral transformation 466
  11.4.3 Linear sigma model 468
  11.4.4 Explicitly broken chiral symmetry 469
  11.4.5 Non-linear sigma model 470
  11.4.6 Lagrangian for the meson–meson and meson–gauge boson interactions 472
  11.4.7 Lagrangian for the meson–baryon–gauge boson interaction 475

### 12 Neutrino Scattering from Hadrons: Inelastic Scattering (II) 478

12.1 Introduction 478
12.2 Single Pion Production 479
  12.2.1 Charged current 480
  12.2.2 Neutral current 485
  12.2.3 Cross sections 487
12.3 Eta Production 490
  12.3.1 Charged current 491
  12.3.2 Neutral current 492
  12.3.3 Cross section 493
12.4 Associated Production of Strange Particles 493
  12.4.1 Charged current 495
  12.4.2 Cross section 497
12.5 Kaon Production 498
  12.5.1 Cross section 500
12.6 Antikaon Production 501
  12.6.1 Cross section 505

### 13 Neutrino Scattering from Hadrons: Deep Inelastic Scattering 507

13.1 Introduction 507
13.2 Charged Lepton–nucleon DIS 511
13.2.1 Bjorken scaling and parton model 515
13.2.2 Differential scattering cross section in terms of dimensionless variables \( x \) and \( y \) 525
13.3 Deep Inelastic Charged Current \( \nu_l / \bar{\nu}_l - N \) Scattering 527
13.3.1 Relation between electromagnetic and weak structure functions 533
13.3.2 Experimental results of charged current total scattering cross section 534
13.4 Deep Inelastic Neutral Current \( \nu_l / \bar{\nu}_l - N \) Scattering 536
13.5 QCD Corrections 539
13.5.1 Modified parton model 539
13.5.2 NLO evolution 542
13.5.3 TMC effect 544
13.5.4 Higher twist effect 547
13.6 Sum Rules in DIS 548
13.6.1 Adler sum rule 548
13.6.2 Gross–Llewellyn Smith sum rule 549
13.7 Quark–hadron (QH) Duality 550
13.8 Duality in Charged Lepton–nucleon Scattering 555
13.9 Duality in Neutrino–nucleon Scattering 558

14 Weak Quasielastic \( \nu(\bar{\nu}) \)-nucleus Scattering 561
14.1 Introduction 561
14.2 Physics of Nuclear Medium Effects in Quasielastic Scattering 564
14.3 General Considerations 569
14.4 Low Energy Quasielastic Reactions 571
14.4.1 Multipole expansion of the matrix elements 571
14.4.2 Cross sections 575
14.4.3 Single particle matrix element in the shell model of nuclei 577
14.5 Quasielastic \( \nu(\bar{\nu}) \) Reactions at Intermediate Energies 578
14.5.1 Introduction 578
14.5.2 Fermi gas model 580
14.5.3 Local Fermi gas model 582
14.6 Cross Sections and Effect of Nuclear Medium 595
14.7 Nuclear Medium Effects in Neutrino Oscillation Experiments 599

15 Inelastic Scattering of (Anti)neutrinos from Nuclei 603
15.1 Introduction 603
15.2 Charged Current Inelastic Reactions 606
15.2.1 Incoherent meson production 606
<table>
<thead>
<tr>
<th>Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.2.2 Pion production in the delta dominance model</td>
</tr>
<tr>
<td>15.2.3 Quasielastic-like production of leptons</td>
</tr>
<tr>
<td>15.3 Coherent Pion Production</td>
</tr>
<tr>
<td>15.3.1 Introduction</td>
</tr>
<tr>
<td>15.3.2 PCAC based methods</td>
</tr>
<tr>
<td>15.4 Microscopic Model for Coherent Weak Pion Production</td>
</tr>
<tr>
<td>15.4.1 Cross sections</td>
</tr>
<tr>
<td>15.4.2 Final state interactions</td>
</tr>
<tr>
<td>15.5 Results for Cross Sections</td>
</tr>
<tr>
<td>15.6 Pion Production through Hyperon Excitation</td>
</tr>
<tr>
<td>15.6.1 Inelastic production of kaons</td>
</tr>
<tr>
<td>15.6.2 Inelastic production of photons</td>
</tr>
<tr>
<td>16 Deep Inelastic Scattering of (Anti)neutrinos from Nuclei</td>
</tr>
<tr>
<td>16.1 Introduction</td>
</tr>
<tr>
<td>16.2 DIS from Bound Nucleons</td>
</tr>
<tr>
<td>16.3 Extraction of Structure Functions from Cross Section Measurements</td>
</tr>
<tr>
<td>16.4 Phenomenological Study</td>
</tr>
<tr>
<td>16.5 Theoretical Study</td>
</tr>
<tr>
<td>16.5.1 Deep inelastic charged leptons and (anti)neutrino scattering from nuclei</td>
</tr>
<tr>
<td>16.5.2 Aligarh–Valencia model</td>
</tr>
<tr>
<td>16.5.3 Kulagin–Petti model</td>
</tr>
<tr>
<td>16.5.4 Isoscalarity corrections: Phenomenological approach</td>
</tr>
<tr>
<td>16.6 Results and Discussions</td>
</tr>
<tr>
<td>17 Neutrino Sources and Detection of Neutrinos</td>
</tr>
<tr>
<td>17.1 Introduction</td>
</tr>
<tr>
<td>17.2 Solar Neutrinos</td>
</tr>
<tr>
<td>17.2.1 Production</td>
</tr>
<tr>
<td>17.2.2 Detection</td>
</tr>
<tr>
<td>17.3 Atmospheric Neutrinos</td>
</tr>
<tr>
<td>17.3.1 Introduction</td>
</tr>
<tr>
<td>17.3.2 Detection</td>
</tr>
<tr>
<td>17.4 Reactor Antineutrinos</td>
</tr>
<tr>
<td>17.4.1 Detection</td>
</tr>
<tr>
<td>17.5 Supernova Neutrinos</td>
</tr>
<tr>
<td>17.5.1 Introduction</td>
</tr>
<tr>
<td>17.5.2 Neutrino emission in supernova explosions</td>
</tr>
</tbody>
</table>
Contents

17.5.3 Detection 692
17.6 Geoneutrinos 692
17.7 Relic Neutrinos 694
17.8 Accelerator Neutrinos 695

17.8.1 Wide and narrow band of neutrino beams 697
17.9 Neutrinos from the Decay at Rest (DAR) Sources 702
17.10 Neutrinos from Spallation Neutrons 703
17.11 Neutrinos from Muon Storage Ring (MSR) 704
17.12 Beta-beam Neutrinos 704
17.13 Very High Energy Cosmic Neutrinos 706

18 Neutrino Mixing and Oscillations 708

18.1 Introduction 708
18.2 Neutrino Mixing 710

18.2.1 Two-flavor neutrino oscillations in vacuum 710
18.2.2 Three-flavor neutrino oscillation in vacuum 718
18.2.3 Survival probability for $\nu_\alpha \rightarrow \nu_\alpha$ 721
18.2.4 Transition probability for $\nu_\alpha \rightarrow \nu_\beta$ 722
18.2.5 CP violation in the leptonic sector 723
18.2.6 Series expansions for neutrino oscillation probabilities 726
18.2.7 Neutrino mass hierarchy: Normal and inverted 729

18.3 Neutrino Oscillation in Matter 732

18.3.1 Effective potential for neutrino–matter interactions 733
18.3.2 Interaction Hamiltonian in matter 736
18.3.3 Probability for oscillation in matter 739
18.3.4 Resonance condition and level crossing 740

18.4 Neutrino Oscillation: Experimental Status 741
18.5 Sterile Neutrinos 743
18.6 Phenomenology of Sterile Neutrinos 745
18.7 Present and Future Experiments 746

19 Neutrino Astrophysics and the Synthesis of Elements 749

19.1 Introduction 749
19.2 The Big Bang and the Nucleosynthesis of the Lighter Elements 753
19.3 Interstellar Matter 756
19.4 Formation of Stars 757

19.4.1 The Hertzsprung–Russell diagram 758

19.5 Stellar Nucleosynthesis 760
## Contents

19.5.1 Death of a star  760  
19.5.2 Chandrasekhar limit  763  
19.5.3 Death of middle mass stars  763  
19.6 The Supernova Nucleosynthesis: Formation of Heavy Elements  767  
19.6.1 Nucleosynthesis of neutron-rich elements  768  
19.6.2 Nucleosynthesis of proton-rich elements  772  

### 20 Neutrino Interactions Beyond the Standard Model  775  
20.1 Introduction  775  
20.2 Neutrinoless Double-beta Decay  776  
20.2.1 General considerations  776  
20.2.2 Decay rates of $0\nu\beta\beta$  779  
20.2.3 Experiments  785  
20.3 Lepton Flavor Violating Processes  788  
20.4 Flavor Changing Neutral Currents  789  
20.4.1 Particle decay processes  789  
20.4.2 Production of strange and heavy flavored hadrons in scattering experiments  790  
20.5 Nonstandard Interaction (NSI) in High Precision Low Energy Weak Processes  791  
20.6 Summary  794  

### Appendices

A Lorentz Transformation and Covariance of the Dirac Equation  795  
A.1 Lorentz Transformations  795  
A.2 Covariance of Dirac Equation  799  
A.3 Bilinear Covariants  803  
A.4 Nonrelativistic Reduction  804  

B Cabibbo Theory  807  
B.1 Cabibbo Theory, SU(3) Symmetry, and Weak $N–Y$ Transition Form Factors  807  
B.2 Octet Representation of Mesons  814  
B.3 Octet Representation of Baryons  816  

C Some Properties of Pauli and Dirac Matrices and Spin Density Matrices  819  
C.1 Trace Properties of Pauli and Dirac Matrices  819  
C.2 Spin Density Matrix  821
# Contents

D Leptonic and Hadronic Tensors 828

D.1 Contraction of Leptonic Tensors in Electromagnetic Interactions 828
D.2 Contraction of Leptonic Tensors in the Case of Weak Interactions 830
D.3 Contraction of Weak Leptonic Tensor with Hadronic Tensor 831

E General Expression for the Total Scattering Cross Section and Decay Rates 833

E.1 Cross Section 833
E.2 Decay Rate 837

F Expressions of $N(q^2)$, the Coefficients of the Polarization Observables 841

F.1 Expression of $N(q^2)$ in Terms of Mandelstam Variables 841
F.2 Expressions of $A^h(q^2)$, $B^h(q^2)$, and $C^h(q^2)$ 842
F.3 Expressions of $A^l(q^2)$, $B^l(q^2)$ and $C^l(q^2)$ 844

References 847

Index 923